Energy Simulation Based on a Mathematical Model of a Residential Building

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Abstract: *In this paper, we have developed a new approach mathematical model for determining the heat loss of a real residential building, which also takes into account the boundary structures used and installed, heat gain from solar radiation, filtration losses, and heat gain from fixtures in the building. Previous approaches did not take into account the different temperatures of each delimiting space and its fluctuations. It was calculated with only a constant temperature. However, during the construction of the model, it turned out that this fluctuation causes a serious discrepancy. The construction documentation of the building was available for the creation of the model, and the built-in materials and structures as well as the geometrical dimensions were also checked on site. Temperature measurements were performed at several points in the living space, in the attic and basement, as well as in the outdoor environment. The measurement results were performed when not in use, so that we could only inspect the building itself, thus ruling out the influence of the wall user. Based on these measurements, model identification will be performed later. This type of approach also takes into account the temperature of the boundary spaces, which were previously ignored, and their temperature fluctuations. The individual characteristic physical elements where the different heat derivations take place have been determined. We wrote the applied mathematical models on these. The aim was to create a mathematical model in which it will be possible to study the hydraulic connections of different heating and cooling systems from the point of view of energy efficiency.*

Keywords: *Energy simulation, heating, mathematical model, residential building*

1 INTRODUCTION

The calculation methods and specifications used in building energy tests are only suitable for estimating the expected consumption of the building. From the calculation method in Hungary, Decree 7/2006 TNM also adopts two calculation methods. Simplified and detailed calculations. During the design phase, scaling software or individual calculations are typically used. This is appropriate for determining the energy rating of buildings, but cannot be used for further research. Therefore, we use the necessary mathematical modeling. (Nishesh, 2021) Fortunately, many simulation software programs are available today. However, the physical meaning and mathematical description of the elements used are essential for their use. (Qinglong, 2017). (Farkas, 1999.) He uses a blackbox model suitable for modeling solar radiation in a university note. We can get a more complete picture of the expected behavior of our built environment through modeling and simulation (Mourshed, 2012). An important factor is a change in the outside temperature, and thus the determination of the equilibrium temperature. The built-in structures have a significant effect on the behavior of the building. (Vadiee, 2019)

1.1 Material and method

The examined building (Fig. 1.) is a family house. Main parts: basement, living space, attic. To calculate heat loss and heat gain, the building was grouped according to its main parts. Different characteristics apply to each space.

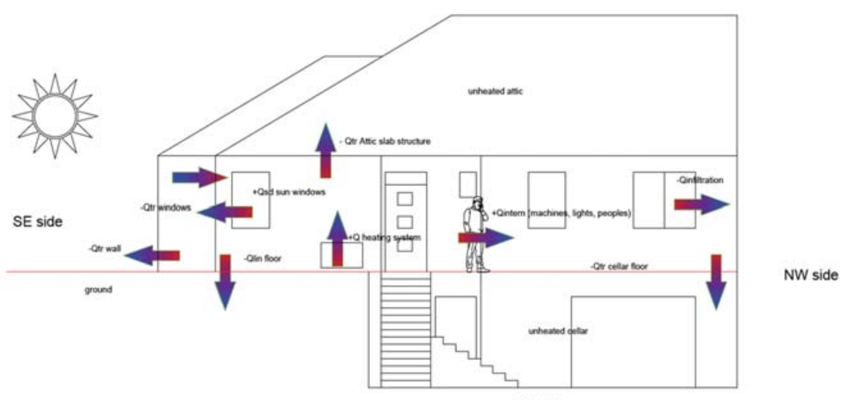

Fig. 1. Typical heat transfers in the examined residential building

The primary purpose of the building is to provide the desired comfort to the users. All this so that you can ensure it regardless of the external weather conditions. To do this, the building must be cooled below the heating limit temperature and the building must be

cooled above the cooling limit temperature. So the main focus is the living space, here we want to provide the right temperature, humidity, and fresh air. We examined the living space, which was broken down into additional elements. This is a standard procedure during modeling. We divided the living space into the following parts: external walls, doors, and windows, floor lying on the ground, basement floor, attic floor. These elements are known for their material properties. Their thermal conductivity, heat storage mass, etc. The heat loss/heat gain of the building is also affected by the air exchange as well as the internal heat production. Internal heat production is the heat dissipation of building users and fixtures. A steady state is ensured by the heating/cooling system. From the point of view of the living space, therefore, it has delimiting structures with three different temperature spaces: Outdoor space, which means directly the outside temperature. Attic space is a space with strongly fluctuating temperatures, which is also affected by solar radiation (roof warming) and filtration. The cellar space, which can be characterized by a minimal heat fluctuation due to the large heat storage mass, must also be considered for this space.
To set up the appropriate mathematical model, we divided the components affecting each heat loss and heat gain into "parts". Let's look at the mathematical model of each of the major elements for the living space.

1.2 Results

1.2.1. Elements of the residential buildings math model

To describe the mathematical model required for the energy calculation of a building, it must be broken down into parts. In the case of physically delimiting surfaces, several types of heat transfer occur.

1.2.2. Conduction

Mathematical relation for the calculation of heat transfer by heat conduction: The Fourier law is used to describe heat conduction, the general form of which is as follows:

$$\dot{Q} = k_r \cdot \frac{A}{D}(T_i - T_e)$$

where

$\dot{Q}$	Heat flow	[W]
k_r	Material resulting	[W/m2K]
A	Area normal to the heat flow direction	[m2]
D	Distance between layers	[m]
Ti	Temperatures of the layers inside	[K]
Te	Temperatures of the layers outside	[K]

1.2.3. Radiation

Part of the heat transfer takes place with heat radiation

Heat transfer by convection represents the transfer of heat between two bodies, which is formed by the movement of fluid. This transfer is described by Newton's law of cooling with the following equation:

$$\dot{Q} = k_r \cdot A \cdot (T_i - T_e)$$

where

$\dot{Q}$	Heat flow	[W]
k	Convection heat transfer coefficient	[W/m2K]
A	Surface area	[m2]
Ti	Temperatures of the layers inside	[K]
Te	Temperatures of the layers outside	[K]

1.2.4. Thermal mass

The heat storage mass must also be taken into account in these systems.

Which is the thermal mass, which is the ability of a substance or combination of substances to store internal energy. This property is characterized by the weight and specific heat of the material. The thermal mass is described by the following equation:

$$\dot{Q} = c \cdot m \cdot \frac{dT}{dt}$$

where

$\dot{Q}$	Heat flow	[W]
c	Specific heat of mass material	[J/kgK]
m	Mass	[kg]
T	Temperature	[K]
t	Time	[s]

1.2.5. Linear conduction

In the case of a residential building, special heat transport sites, eg line heat transfer along with the floors. This is a rearranged form of the thermal conductivity equation, where the thermal conductivity coefficient is projected over a length instead of a surface. This can be described by the following equation:

$$\dot{Q} = \varphi_l \cdot L \cdot (T_i - T_e)$$

where

$\dot{Q}$	Heat flow	
Ψ_l	Linear heat transfer coefficient [W/mK]	
L	Length of the wall section in contact with the ground [m]	
T_i	Temperatures of the layers inside	[K]
T_e	Temperatures of the layers outside	[K]

1.2.6. Filtration

Filtration losses are losses when fresh air is introduced into a building. We describe it with the following equation:

$$\dot{Q} = \frac{n}{3600} \cdot V \cdot \rho_{air} \cdot c \cdot (t_i - t_e)$$

where

$\dot{Q}$	Heat flow – filtration	[W]
n	Air exchange ratio	[1/h]

ρ_{air}	Air density	[kg/m^3]
c	Specific heat of air	[J/kgK]
T_i	Temperatures inside	[K]
T_e	Temperatures outside	[K]

1.2.7. Heat gains

The elements that reduce the heat demand for heating are the heat gain from solar radiation and the heat gain from other sources. The heat gain from solar radiation is negligible, given that it is not available in all cases.

1.2.7. Solar heat gain

When calculating the heat demand, we can also take into account the heat gain from solar radiation with the following equation:

$$\dot{Q} = A_w \cdot \dot{q}_s$$

where

$\dot{Q}$	Heat flow – solar heat gain	[W]
A_w	Surface area window	[m2]
$\dot{q}_s$	The specific energy flow of solar radiation [W/m2]	

1.2.8. Internal heat gain

In the case of a residential building, the internal heat gains are also typical. These are the heat given off by people and machines. It can be calculated either by aggregation per unit or by specific values. The calculation with specific values is described by the following equation:

$$\dot{Q} = A_b \cdot \dot{q}_i$$

where

$\dot{Q}$	Heat flow – internal heat gain	[W]
A_w	Building floor area	[m2]
$\dot{q}_i$	specific internal heat gain	[W/m2]

1.3 Discussion

In the case of a residential building, the elements presented above can be used to determine the energy characteristics. To determine the heating output, we get the following equation.

$$\dot{Q}_{heat\ demand} = \dot{Q}_{conduction\ heat\ loss} + \dot{Q}_{radiation\ heat\ loss} + \dot{Q}_{linear\ conduction\ hat\ loss} + \dot{Q}_{filtration\ heat\ loss} - \dot{Q}_{solar\ heat\ gain} - \dot{Q}_{internal\ heat\ gain}$$

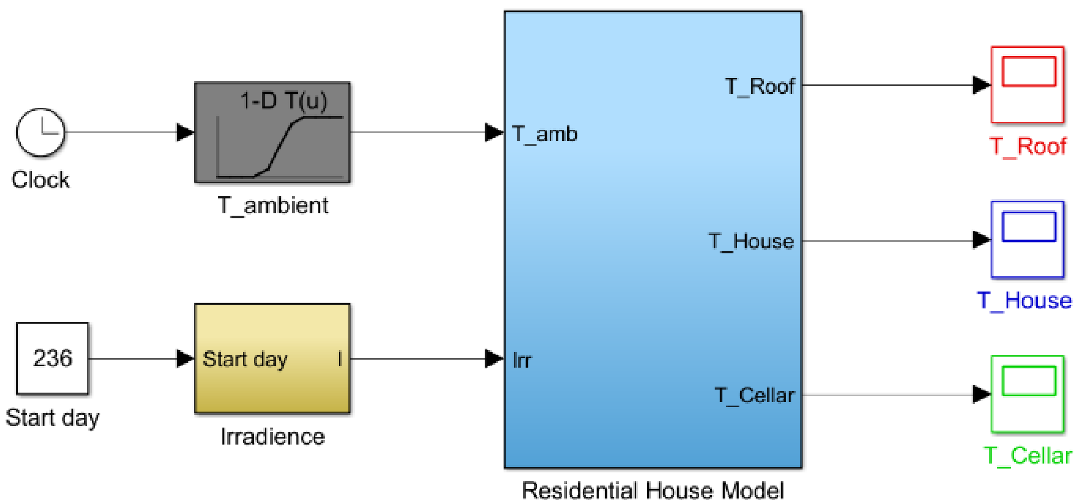

Fig. 2. Matlab Simulink model of residential house

In the case of the model, the input signal is the ambient temperature and the heat gain from the radiation. It is also suitable for modeling outdoor temperature and solar radiation with a mathematical equation and measurement results. The model of the residential house includes the structural materials typical of the building. Complemented by heat gains and heat losses from solar radiation, internal heat sources, and filtration. During the modeling, the output signal is the temperature of the zones that can be clearly distinguished in terms of the individual functions of the building: attic space, living space, and cellar.

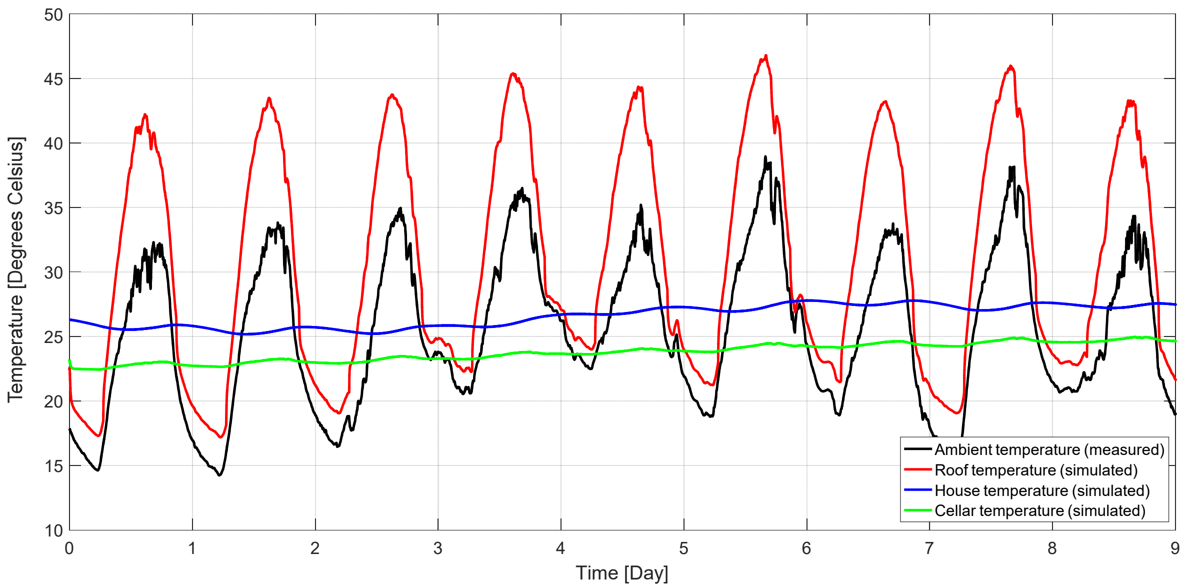

Fig. 3. The result of the simulation: residential house temperatures

Figure 3. shows the simulation result. Based on the results of the ambient temperature measurements, it is included as an input to the model. House temperatures were calculated using the MATLAB SIMULINK simulation model figures 2. based on the mathematical model described.

1.4 Conclusion

In the case of residential buildings, the mathematical models required to perform energy calculations are based on a physical basis. These can be used as a white-box model to set up the model. With the help of the model, serenity analysis of different scenarios of a given building can be performed easily and quickly. Whether it's modifying wall structures or optimizing them. The weather parameters specific to a given building can also be easily taken into account. Using these mathematical models, we get closer to more accurate modeling of buildings.

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