Optimization of the Bending Strength of Asymmetric Gears by better Design of the Joint Profile

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Abstract: *The Asymmetric gears are involute profile teeth gears, designed as nonstandard gears with the aim of obtain better functional parameters like the efficiency, the transmission error, the specific sliding, the contact stress, the bending stress or another, resulting from those mentioned above, as a sum of terms multiplied by some coefficient which take in consideration how important for the designing objective is those one. For improve the behavior of the gears in meshing we have been developed many computer applications in order to design and also to study the geometrical and functional parameters. That offers the possibility, only by running the applications without any other costs, to change many times the designing parameters, analyzing the performances calculated, in order to find the perfect combination for better results. In this paper are presented some aspect about the influence of the joint profile design, of the pinion and of the gear, on the bending stress at the base of the asymmetric teeth.*

Keywords: *parametrical design, design applications, bending stress, joint profile*

1 INTRODUCTION

The calculus algorithm that was at the base of design application packet construction has the advantage of using the method of direct design of the gears [8]. So have been designed first the pinion and the gear and, based on those involute profiles and dimensions, second have been designed the generation gear rack [7].

Based on the profiles and dimensions established for the generation gears rack, have been wrote the equations of the joint profiles, different at the base of the direct profile and of the inverted profile [6]. Another important aspect, that can offer more advantages was that for the joint profiles it was possible to use two different method, the generation of the pinion and the gear on the base of the same gear rack or on the base of two different gear rack, which dimensions are different and so the root profile are different and, as a consequence, the properties of the gears can be analyzed in order to choose the better solution.

If the objective of the designer is to improve the bending strength of the teeth it is was also necessary to establish a calculus relations for the values of the bending stress, at the booth joint profiles, and for the booth possibilities of using asymmetric gears, having the direct profile as active profile or having the inverted profile as active profile. All these mentioned possibilities have been taken in consideration for establishing the geometrical and functional parameters calculus algorithm and, having the mathematical equations, for writing, in Matlab, the computer calculus and analyzing application [1], [2], [5]. After the profiles design it was necessary the development, for representing at real dimensions the pinion, the gear and booth in meshing, of Autolisp applications [9]. Those offer many opportunities for verifying, for the designed asymmetric gears, the correct meshing, only on the computer, without any material consumption. Also so was possible to representing the tooth of the gears at real dimension and using the finite elements method for obtain preliminary result about the stress and the displacements of the teeth under the loads [3], [4].

For illustrate the final results in relation with the geometrical parameters calculated and used for gears representation, in figure 1 are presented the 2D models of the gears for an asymmetric gear designed, noted with the code *A_as_18_25_120_40_20_0_1*, for have information on the number of teeth, centroid distance, the profile angles, and on the gear rack design. In figure are presented also the calculated values for the diameters of the base circles of the two different involute profile, two of the gear and two of the pinion.

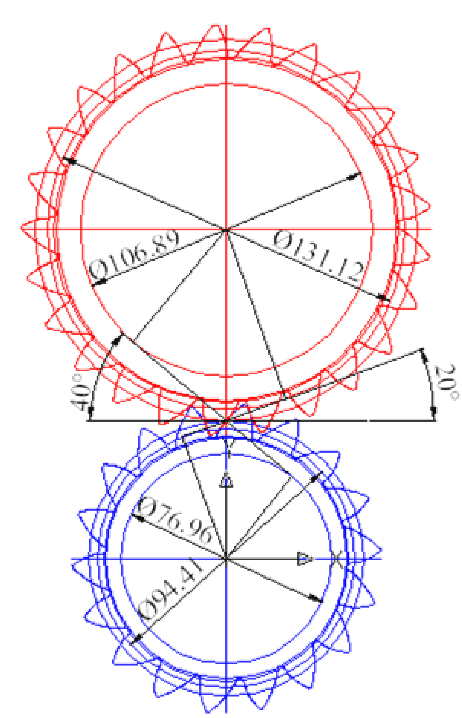

Fig. 1. Representation of asymmetric gears in meshing with the direct profile as active profile and the two different line of action

2 JOINT PROFILES DESIGN AND TOOTH BASE CROSS SECTION

The maximum bending stress was first determined considering the asymmetric teeth like a short beam fixed in the body of the gear and with the meshing load acting in the last point of the involute profile, considering that situation the most unfavorable.

In order to determine the mathematical expression for the bending stress at the bottom of the involute teeth it was necessary to be established a method for determined the cross section of the tooth. The bending moment was determined in relation with the centroid of the cross section of the tooth, those position was established after have been determined the dimensions of the cross section, on the base of the equations of the joint profiles. In figure 2 the first point and the last point of the joint profiles are noted with (Ia,Fa) for the direct profile and (Ii,Fi) for the inverted profile. For writing the equations of joint profile it have been used axes systems having the origin in the center of the wheel, the y axes containing the Ia point, respectively Ii point and the x axes oriented, in opposite directions, to the tip of the tooth.

The joint profile equations have been wrote considering that those are obtained by generation with the top circle of the asymmetric gear rack profile.

The direct joint profile parametrical equations are given in relation of a parameter α which varies within the boundaries: $\alpha_{dc} \le \alpha \le 90^o$.

$$x_{2d}(\alpha) = \left(\frac{d}{\tan\alpha} + R\cdot\cos\alpha\right)\cdot\cos\left(\frac{d}{r_{rc}\cdot\tan\alpha}\right) + \left(d - r_{rc} + R\cdot\sin\alpha\right)\cdot\sin\left(\frac{d}{r_{rc}\cdot\tan\alpha}\right);$$

$$y_{2d}(\alpha) = \left(\frac{d}{\tan\alpha} + R\cdot\cos\alpha\right)\cdot\sin\left(\frac{d}{r_{rc}\cdot\tan\alpha}\right) - \left(d - r_{rc} + R\cdot\sin\alpha\right)\cdot\cos\left(\frac{d}{r_{rc}\cdot\tan\alpha}\right). \quad (1)$$

The inverted joint profile parametrical equations are given in relation of a parameter α which varies within the boundaries: $\alpha_{ic} \le \alpha \le 90^o$. The limits of variation are depending of the gear rack profiles angles.

$$x_{2d}(\alpha) = \left(\frac{d}{\tan\alpha} + R\cdot\cos\alpha\right)\cdot\cos\left(\frac{d}{r_{rc}\cdot\tan\alpha}\right) + \left(d - r_{rc} + R\cdot\sin\alpha\right)\cdot\sin\left(\frac{d}{r_{rc}\cdot\tan\alpha}\right);$$

$$y_{2d}(\alpha) = \left(\frac{d}{\tan\alpha} + R\cdot\cos\alpha\right)\cdot\sin\left(\frac{d}{r_{rc}\cdot\tan\alpha}\right) - \left(d - r_{rc} + R\cdot\sin\alpha\right)\cdot\cos\left(\frac{d}{r_{rc}\cdot\tan\alpha}\right). \quad (2)$$

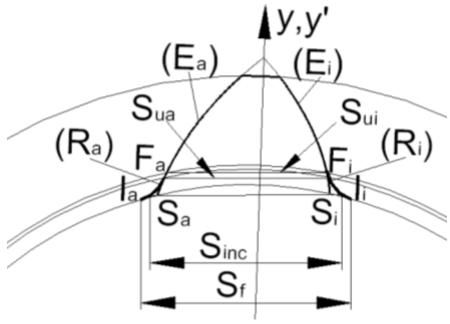

Fig. 2. Representation of asymmetric profiles and the cross section of the tooth

The rectangular cross section of the tooth base, considered as the section where the bending stress have maximum values have one of dimensions equal with the wheel width and the second dimension given by the following relation:

$$\overline{S_a S_i} = \sqrt{\left(x_{S_a} + x'_{S_i}\right)^2 + \left(y_{S_a} - y'_{S_i}\right)^2} = h. \quad (3)$$

If the coefficient of asymmetry of the gears is modified, by modifying the angles of the asymmetrical teeth profiles, the dimensions of the cross section varies and that influence the bending stress at the base of the tooth, to the gear and in the same time to the pinion, which cross section is also modified.

For emphasize the variations of cross section geometrical characteristics values in relation with the coefficient of asymmetry are given, for comparison, some values in the following tables.

Table 1.Ggeometrical characteristics-pinion

Gears	k	h1[mm]	A1[mm^2]	W1[mm^3]
P1. 25	1.03	9.93	357.78	592.63
P2. 30	1.08	10.06	362.41	608.09
P3. 35	1.14	10.23	368.37	628.23
P4. 40	1.22	10.39	374.07	647.82

Table 2.Ggeometrical characteristics-gear

Gears	k	h2[mm]	A2[mm^2]	W2[mm^3]
G1. 25	1.03	10.01	360.56	601.87
G2. 30	1.08	10.43	375.69	653.45
G3. 35	1.14	10.80	389.02	700.64
G4. 40	1.22	11.14	401.37	745.85

Table 3.Ggeometrical characteristics-gear (with one single generation gear rack)

Gears	k	h2[mm]	A2[mm^2]	W2[mm^3]
G1. 25	1.03	9.40	338.74	531.23
G2. 30	1.08	10.04	361.62	605.42
G3. 35	1.14	10.56	380.30	669.58
G4. 40	1.22	11.02	396.92	729.40

In the case when it has been used in the design relation the method with the same generation rack for the pinion and for the gear, the results are more significant.

The values have been calculated for the number of pinion teeth $z_1 = 20$ and gear teeth $z_2 = 41$, the centre distance $a = 120$ mm and the profile angles are $\alpha_i = 20^o$ for the inverted profile and for the direct profile, in the same order with the lines in the tables, the values are $\alpha_d = [25, 30, 35, 40]$.

3 EFFORTS VARIATION IN THE LINE OF ACTION AND STRESS COMPONENTS

The meshing loads being normal to the tooth profile on the contact points, the components in relation with the normal direction of the cross section are variable on the line of action, also the efforts that give the stress components.

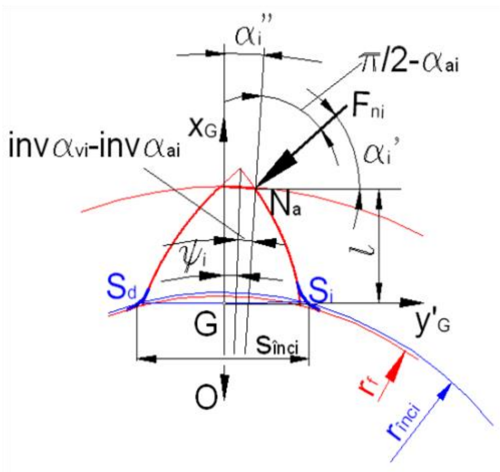

Fig. 3. The angles used for establishing the position of the load, for inverted active profile

In figure 3 it is represented the involute tooth with the force acting in the point Na of the inverted profile. So results the angles necessary for establishing the load position in relation with the tooth cross section.

For the force acting in any other point of the inverted profile the position of the force F_{nji} may be determined in relation with the pressure angle (α_{ji}) coresponding with the j contact point of the i profile.

$$\alpha'_i = \pi/2 - ((\pi/2 - \alpha_{ai}) + \alpha''_i) = \alpha_{ai} - \alpha''_i \quad (4)$$

$$\alpha''_i = (\text{inv}\alpha_{vi} - \text{inv}\alpha_{ai}) + \psi_i \quad (5)$$

$$\alpha'_i = \alpha_{ai} - (\text{inv}\alpha_{vi} - \text{inv}\alpha_{ai}) - \psi_i \quad (6)$$

$$\alpha'_{ji} = \pi/2 - ((\pi/2 - \alpha_{ji}) + \alpha''_{ji}) = \alpha_{ji} - \alpha''_{ji} \quad (7)$$

$$\alpha''_{ji} = (\text{inv}\alpha_{vi} - \text{inv}\alpha_{ji}) + \psi_i \quad (8)$$

$$\alpha'_{ji} = \alpha_{ji} - (\text{inv}\alpha_{vi} - \text{inv}\alpha_{ji}) - \psi_i \quad (9)$$

The efforts components based on which have been determined the stresses components and those variation on the line of action have been also expressed using (α_{ji}) the pressure angle for the j contact point.

$$N_{ji} = F_{nji} \cdot \sin\alpha'_{ji} \quad (10)$$

$$T_{ji} = F_{nji} \cdot \cos\alpha'_{ji} \quad (11)$$

$$M_{iji} = F_{nji} \cdot \cos\alpha'_{ji} \cdot x_G^{Nj} - F_{nji} \cdot \sin\alpha'_{ji} \cdot y'^{Nj}_G \quad (12)$$

The diagrams of variations of the stress at the pinion and the gear with asymmetric teeth emphasize that the best behavior, having as objective the minimization of bending stress, can be obtain with the inverted asymmetric gears designed with two different generation gear rack. It has been obtained also, for having comparing date, the variation diagrams for the symmetric teeth gears, designed considering that those are only a particular case of asymmetric gears.

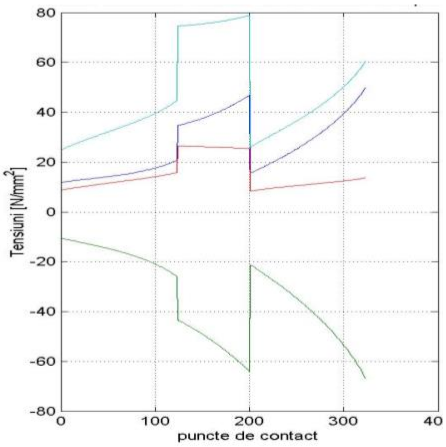

Fig. 4. The stress variation diagrams at the pinion tooth base for the inverted gears

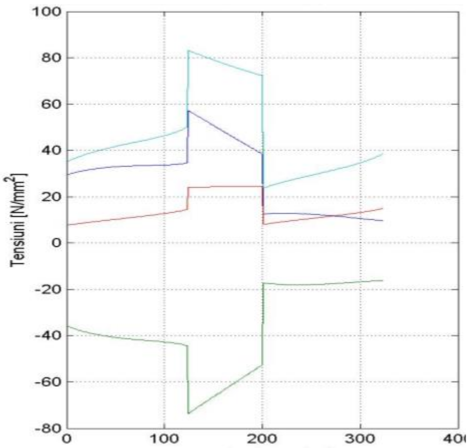

Fig. 5. The stress variation diagrams at the gear tooth base for the inverted gears

On the base of stress components it is possible to determine and study the variation of the equivalent stress, considering in the same time the effect of axial force, shear force and bending moment that acting on the gear tooth.

In the following figures are given the variation diagrams of the equivalent stress, for the pinion and gear, obtained with the developed computer application, for the inverted gears and also for the symmetric involute teeth gears, designed having the same initial date. The stress values show the better behavior of the inverted gears in relation with the direct gears and symmetric gears.

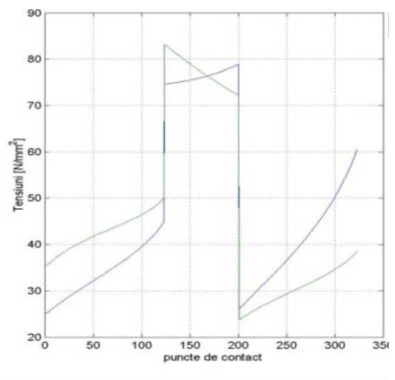

Fig. 6. The equivalent stress variation diagrams to the pinion and gear of the inverted asymmetric gears.

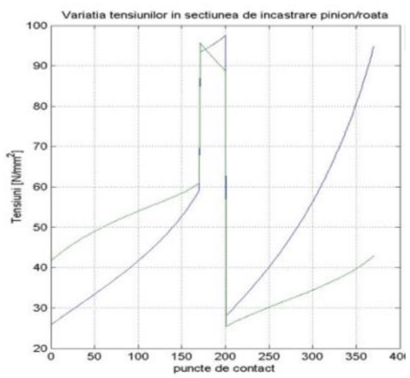

Fig. 7. The angles used for establishing the position of the load, with inverted active profile

Another design parameter that have influence on the joint profile, and thus the tooth cross section in relation with the bending stress, is the direct gear rack profile angle, implicit the inverted gear rack profile angle. Studying the relation between the gear rack angles and the dimensions of the base tooth cross section it has been observed that when the direct gear rack profile angle increase the dimension of the tooth cross section at the gear increase and at the pinion decrease, thus give very different values of the stress. To get close-up values it is useful to choose smaller values for the generation gear rack angles.

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