MODELING THE CHANGE IN MODULUS FACTOR UNDER TENSILE-TENSILE FATIGUE LOAD

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Abstract: *In order to predict the life expectancy of a polymer specimen under fatigue load a new fatigue model, based on our laboratory experiments, will be presented in this paper. Our fatigue model provides an approximate description of the three life stages of polypropylene specimens without taking into consideration the stochastic effects. With an additional stochastic module containing the probabilistic calculations our fatigue model will serve as basis for further qualitative analysis.*
Keywords: *fatigue, modulus factor, fiber bundle, polypropylene, Burgers*

1. INTRODUCTION

Last year we have conducted a series of fatigue experiments on polypropylene specimens in the Biomechanical Research Centre of the Budapest University of Technology and Economics (BME) [1]. After evaluating the research data we could observe the three stages of the damage process, but although several research groups have observed the same stiffness retardation no mechanical models were developed previously to describe it. Van Paepegem and Degrieck [2] have found that mechanical models can describe the process quantitatively, although Shokreigh and Lessard [3] have defined a fatigue criteria for seven different damage types. The damage mechanisms are too complex both in geometry and evaluation process. In the Department of Polymer Engineering and Textile Technology BME Vas and Rácz [4] have studied the effects of the microscopic fiber orientation on the macro-scale material properties. The fiber bundle theory is based on the probability assumption that a fibrous structure is built of close fiber assemblies called bundles. The original static tensile and bending tests were conducted [5] on unidirectionally reinforced composites (UDC). To apply the fiber bundle theory on homogenous polymer we suppose that long molecule fibres in the polymer act as oriented reinforcing fibres in an UDC, and these polymer molecule chains have the same connections between each other like reinforcing fibres in a composite.

2. RESEARCH COURSE

It is well known, that long molecule chains are the building blocks of the polymers. These molecule chains are locally straightening by external tensile load, these straightened bundles are causing material inhomogenities [4, 5]. These local discontinuities are expanding as tensile load increases, which cause the molecule chains to brake. As more and more chains are braking crack propagation can be observed first in microscopic, then macroscopic level. This crack propagation is the basic cause of the fatigue in the specimen.

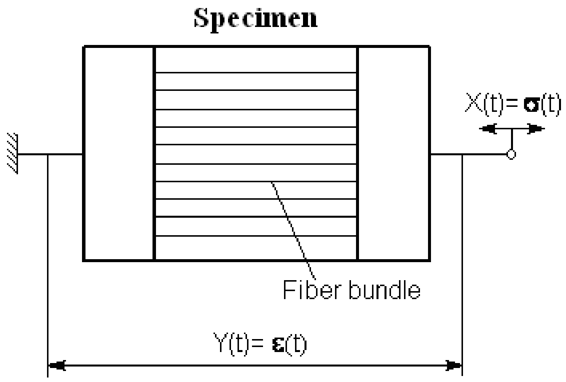

Figure 1 – Fiber bundle model of fatigue damage

The straightened and stretched molecule chains oriented at one direction can be treated as fibers which form idealized bundles (Figure 1). By modeling the amount of torn fibers in a bundle [6, 7, 8] as a function of load cycles we can model the inner structure of the polymer and the damage mechanism as well.

3. METHOD USED

To model the strain response of the polymer specimen we have used a generalized Burger's viscoelastic (Figure 2) model. It contains multiple Maxwell filaments simulating the stress-strain relationships of the molecule chains, and one Kelvin-Voight segment modeling the viscoelastic behavior of the connection between the fibers.

The effect of the sinusoid tensile load (eq. 1) is constantly increasing tensile strain in the whole Burger's model, as well as in the Maxwell bundle [3, 4, 9]. We can assume that there is no strain limit on the Kelvin-Voight element, but every Maxwell filament breaks after reaching its random strain limit. If we assume that the Maxwell bundle is modeling the reinforcing fibers and the Kelvin-Voight element is to model the adhesion between the matrix and the fibres our model can be adopted to an unidirectionally reinforced composite material.

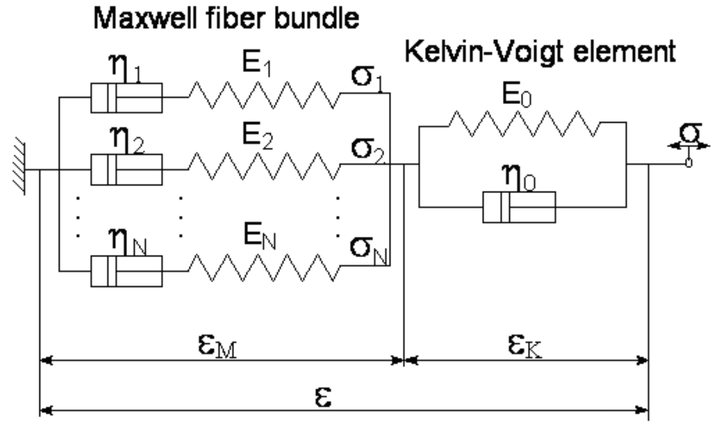

Figure 2 – Analogous mechanical model

To simulate the damage process we have calculated the strain response of the deterministic model (Figure 2) for the stress excitation:

$$\sigma = \sigma_0 + \sigma_A \sin \varpi t \quad \textbf{(1)}$$

$$\dot{\sigma} = \sigma_A \varpi \ \ \cos \varpi t \quad \textbf{(2)}$$

$$\ddot{\sigma} = -\sigma_A \varpi^2 \sin \varpi t \quad \textbf{(3)}$$

Hence this is a linear model consisting of one Kelvin-Voight element and N Maxwell elements at t=0, the strain response of the whole model can be calculated as sum of the strain response of the Maxwell bundle and the Kelvin-Voight element. After defining the boundary conditions the differential equations of the Burgers model can be solved deterministically.

We get the strain of the Kelvin-Voigt element (eq. 5) by solving the stress-strain differential equation (eq. 4):

$$\sigma_0 + \sigma_A \sin \varpi t = E_0 \varepsilon_K + \eta_0 \dot{\varepsilon}_K \quad \textbf{(4)}$$

$$\varepsilon_K = \frac{\sigma_0}{E_0} + c_K e^{-\frac{E_0}{\eta_0}t} + \frac{\sigma_A E_0}{E_0^2 + \eta_0^2 \varpi^2} \sin \varpi t - \frac{\sigma_A \eta_0 \varpi}{E_0^2 + \eta_0^2 \varpi^2} \cos \varpi t \quad \textbf{(5)}$$

By using the stress-strain differential equation (eq. 6), the strain of a single Maxwell fiber (eq. 7) can be evaluated:

$$\sigma_{Ai} \varpi \cos \varpi t + \frac{E_i}{\eta_i}\left(\sigma_{0i} + \sigma_{Ai} \sin \varpi t\right) = E_i \dot{\varepsilon}_{Mi} \quad \textbf{(6)}$$

$$\varepsilon_{Mi} = \frac{\sigma_{0i}}{\eta_i} t + \frac{\sigma_{Ai}}{E_i} \sin \varpi t - \frac{\sigma_{Ai}}{\eta_i \varpi} \cos \varpi t + c_M \quad \textbf{(7)}$$

The constants of the stress-strain functions can be calculated after defining the boundary conditions:

$$c_K = \frac{\sigma_A \eta_0 \varpi}{E_0^2 + \eta_0^2 \varpi^2} + \frac{\eta_0}{E_0 n}\sum_{i=1}^{n}\frac{\sigma_{0i}}{\eta_i} + \frac{\eta_0 \varpi}{E_0 n}\sum_{i=1}^{n}\frac{\sigma_{Ai}}{E_i} - \frac{\sigma_A \eta_0 \varpi}{E_0 E_{stat}} \quad \textbf{(8)}$$

$$c_M = \frac{\sigma_0}{E_{stat}} + \frac{\sigma_A \eta_0 \varpi}{E_0^2 + \eta_0^2 \varpi^2} + \frac{1}{n\varpi}\sum_{i=1}^{n}\frac{\sigma_{Ai}}{\eta_i} \quad \textbf{(9)}$$

The stress of the i[th] Maxwell fiber is a function of the number of the intact fibers $0<n\leq N$. (N is the number of the Maxwell fibers in the Maxwell bundle before the start of the first cycle.) By assuming that the fiber breakage intensity is a much slower process than the periodical time of the stress function we can state that the number of the intact fibers in a cycle is arbitrary constant, and the stress in single Maxwell fiber can be calculated:

$$\sigma_{0i} = \frac{\sigma_0}{n}, \text{ and } \sigma_{Ai} = \frac{\sigma_A}{n} \quad \textbf{(10)}$$

We can simplify the deterministic calculations by switching from time based analysis to cycle based one. To do so we have to consider the following assumptions:

1. We can only measure the stress and strain values when the stress function has it's local peak, so in the strain functions (eq. 5 and 7) $\sin \varpi t = \pm 1$.
2. In points where $\sin \varpi t = \pm 1$, $\cos \varpi t = 0$ and the denominator of the multiplier of the cosine factor at least three orders of magnitude greater than the numerator, so the cosine factor can be neglected.
3. t_{k0} is the beginning of the k[th] cycle, hence $t_{k0} = \frac{k}{f}$, where f is the frequency of the excitation.
4. The stress function (eq 1) has its local extremas at the end of the first and third quarter of the sine wave.

After applying all the simplifying considerations we can define the local extremas of the strain functions of the Kelvin-Voight element and one Maxwell fiber in the k[th] cycle:

$$\begin{aligned} \varepsilon_{K\max} &= \frac{\sigma_0}{E_0} + c_K e^{-\frac{E_0(k+0.25)}{\eta_0 f}} + \frac{\sigma_A E_0}{E_0^2 + \eta_0^2 \varpi^2} \\ \varepsilon_{K\min} &= \frac{\sigma_0}{E_0} + c_K e^{-\frac{E_0(k+0.75)}{\eta_0 f}} - \frac{\sigma_A E_0}{E_0^2 + \eta_0^2 \varpi^2} \end{aligned} \quad \textbf{(11)}$$

$$\begin{aligned} \varepsilon_{Mi\max} &= c_M + \frac{\sigma_{0i}(k+0.25)}{\eta_i f} + \frac{\sigma_{Ai}}{E_i} \\ \varepsilon_{Mi\min} &= c_M + \frac{\sigma_{0i}(k+0.75)}{\eta_i f} - \frac{\sigma_{Ai}}{E_i} \end{aligned} \quad \textbf{(12)}$$

With equations 11 and 12 the maximal and minimal strain response of the Burgers model and the modulus factor (eq. 13) can be calculated in the k[th] cycle. By using the modulus factor as damage parameter we can compare the damage mechanism of several different types of materials.

$$\kappa = \frac{\dfrac{\sigma_0 + \sigma_A}{\varepsilon_{K\max} + \dfrac{\sum_{i=1}^{n} \varepsilon_{Mi\max}}{n}} + \dfrac{\sigma_0 - \sigma_A}{\varepsilon_{K\min} + \dfrac{\sum_{i=1}^{n} \varepsilon_{Mi\min}}{n}}}{2E_{stat}} \qquad \textbf{(13)}$$

To determine the values of the parameters of the springs and dumpers we can use the results of the static tensile tests. The dumping-ratio of the Maxwell fibers is a function of the frequency, while the other parameters are not sensitive to excitation frequency.

Parameter	E_M	E_0	η_M	η_0
Dimension	*MPa*	*MPa*	*MPas*	*MPas*
Value	5064	6954.6	$16475 - 690f$	2194

Table 1 – Model parameters

By using the parameters shown in Table 1 we can fit the model's strain function to static tensile tests ($f = 0Hz$) with correlation $R^2 = 0.983$.

4 RESULTS AND CONCLUSIONS

For $f = 10Hz$ excitation frequency the dumping ratio of the Maxwell fibers is $\eta_M = 16475 - 690f = 9575MPas$.

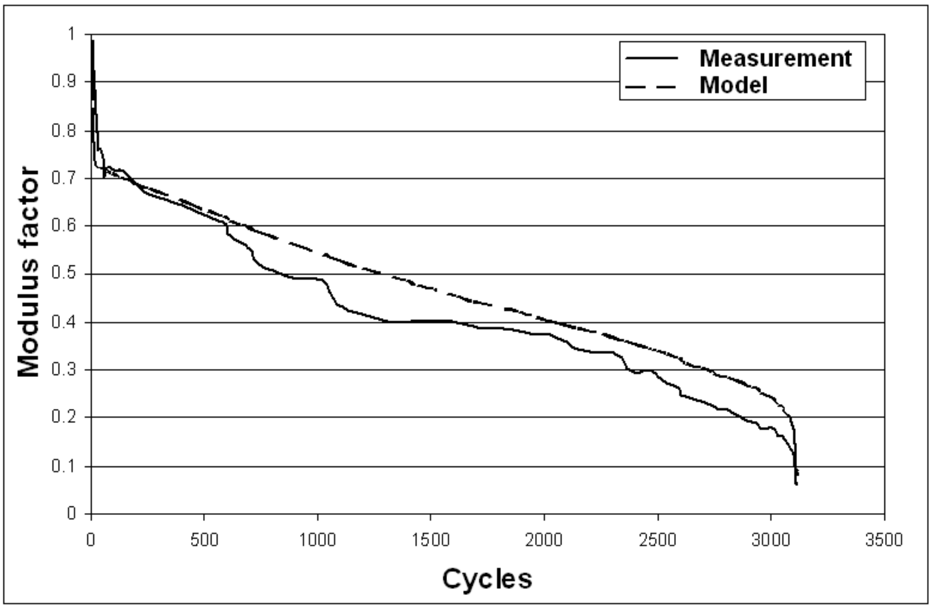

Figure 3 – Model results

We can apply the parameters defined above to equations 11 and 12 and by using equation 13 we can calculate the modulus factor in each cycle to simulate the damage process. If we assume 100 intact fibers at t=0, and that the material suffers its final damage at the 3150th cycle and damage accumulates linearly, we can plot the modulus factor as a function of cycles (Figure **3**). By comparing the model results with the measured data we can assert that our model can be used to describe the change in amplitude ratio during fatigue test. Although the stochastic effects are not considered in this model we are able to make qualitative analysis to simulate the material's response to different excitation functions. By changing the model's parameters we can study different materials or the effects of the manufacturing settings.

5. FURTHER RESEARCH

The damage mechanism can be described fairly well by this deterministic model, but in our model the fiber breakage is the function of time or number of cycles which does not represent the actual damage process. We have developed an auxiliary stochastic module (the supervision is in final phase) which is capable of determining whether a Maxwell fiber is torn or just yield. We have studied the fatigue process of non reinforced polypropylene specimens, but further laboratory tests are under process on nanoparticle-reinforced, carbon nanotube-reinforced and basalt fiber reinforced polyamide specimens.

Our further goal is to build up a knowledge center with data of several different materials and material parameters to use it as a design tool in the future.

5. REFERENCES

1. Pető I.N., Borbás L. – Experimental results of the change in amplitude ratio during fatigue test of non reinforced polypropylene specimens – International Conference "Advanced Composite Materials Engineering", COMAT 2006, ISBN-978-973-635-821-0, ISBN-973-635-821-8
2. Van Paepegem W., Degrieck J. – A new coupled approach of residual stiffness and strength for fatigue of fiber-reinforced composites, International Journal of Fatigue 24, 2002
3. Shokrieh M.M., Lessard L.B. – Progressive fatigue damage modeling of composite materials, Part I: Modeling, Journalof Composite Materials 34, 2000
4. Vas L.M., Rácz Zs. – Modeling and testing the fracture process of impregnated carbon-fiber roving specimens during bending, Part I: Fiber bundle model, Journal of Composite Materials 38, 2004
5. Vas L.M., Rácz Zs. – Modeling and testing the fracture process of impregnated carbon-fiber roving specimens during bending, Part II: Experimental Studies, Journal of Composite Materials 38, 2004
6. Vas, L.M. – Császi, F. (1993). Use of Composite-Bundle Theory to Predict Tensile Properties of Yarns. Journal of the Textile Institute Vol.84. No.3. 448-463
7. Echtermeyer A.T., Engh B., Buene L. – Lifetime and Young's modulus changes of glass/phenolic and glass/polyester composites under fatigue, Composites, 1995
8. Bodor G., Vas L.M. -(2000). Structure and Properties of Polymers. (in Hungarian). Műegyetemi Kiadó, Budapest.
9. Hertzberg, R.W. (1989). Deformation and Fracture Mechanics of Engineering Materials. 3rd edition. John Wiley & Sons, Inc., New York.
10. Retting, W. - (1992). Mechanik der Kunststoffe. Carl Hanser Verlag, München