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INTERNAL WORM GEARS WITH NO PERPENDICULAR AXIS

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ABSTRACT

The internal worm gear pairs are special ones, which are composed by a helical worm and an internal teeth worm wheel. The position of the axis can be parallel, perpendicular and in general case with angle between the axis in interval (0 – 90) degree. The paper presents this general case, the mathematical model, the simulation of connecting surfaces and some problems about manufacturing of the gear pair elements.

1. INTRODUCTION

A helical worm and an internal teeth worm wheel compose the internal worm gear pairs. At this gear pair the axis can have different positions: parallel, perpendicular or general.

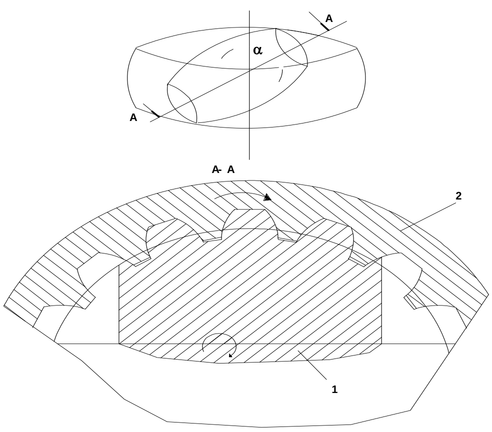


Figure 1. Internal worm gearing with any axes

Figure 1 presents the general case, which is the one when the angle between the worm's axe and that of the worm wheel encloses an angle between 0º and 90º [2], [3], [4].

Though mathematical modeling is more difficult, we can state that it is the most favorable case for an internal gearing, as worm bearing faces no problems and we can reach reasonable dimension, along with high efficiency.

At the same time, the driving elements can be fixed in the interior of the worm wheel so as to obtain a reduction of the necessary space also. The determining of the gearing field and the computer simulation of this type of gearing represent the research at present.

As we can observe from the facts above, at internal worm gearings it is not necessary for the angle between axes to be of 90º, even any angle between the axes being favorable, contrary to the cylindrical or globoid worm gearings, where the angle between the axes is of 90º.

2. THE MATHEMATICAL MODELING OF INTERNAL WORM GEAR PAIRS

The kinematical generation can be a vectorial mathematical model or an analytical mathematical model [1], [6].

In this paper we will present an analytical generation method in general case when the angle between the axes is between 0 and 90 degree [4]. This method is an analogy with the globoid worm gears [1].

The used coordinate systems in figure 2 are:

- $O_1x_1y_1z_1$ - the worm related reference system; the worm rotation axis is the Y_1 axis; the relative position of the technological reference system is given by the φ_1 parameter - the worm rotation angle;

- $O_0x_0y_0z_0$ - the functional reference system; it is the reference system considers to be fixed;

- $O_1^*x_1^*y_1^*z_1^*$ - a fix system, which is rotate to $O_0x_0y_0z_0$ system with γ = constant angle, where $\gamma = (0°, 90°)$;

- $O_2^*x_2^*y_2^*z_2^*$ - intermediary fix system which is translate to $O_0x_0y_0z_0$ system with the distance between the axis “a” on the direction of O_0z_0 ax;

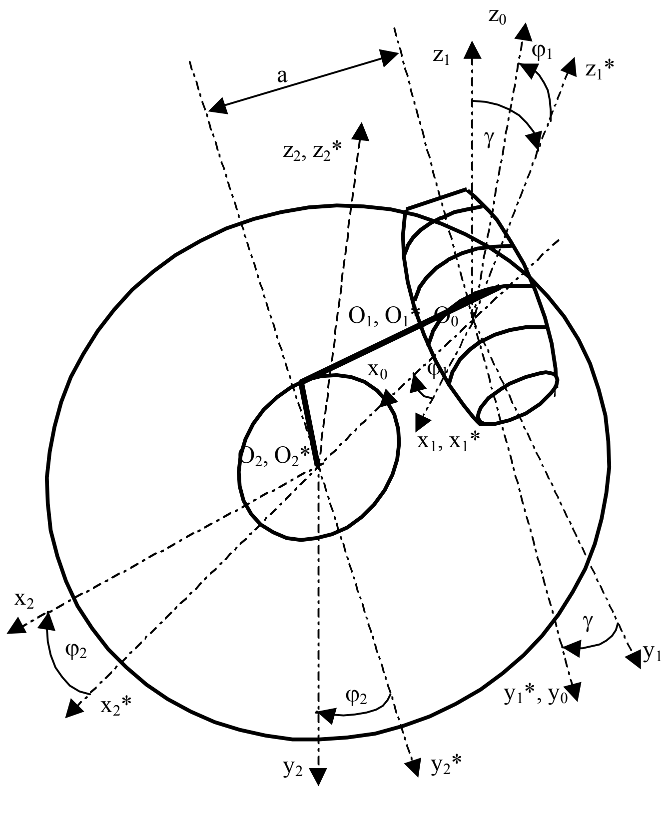


Figure 2. The model of internal worm gear pair

- $O_2x_2y_2z_2$ - the wheel related reference system; the wheel rotation axis Z_2 is parallel to Z_t and perpendicular on the paper plane; the relative position to the technological reference system is given by the φ_2 parameter-the wheel rotation angle;

The worm's flank is generated by "u" straight line, which is in the paper plane, also the division diameter of the wheel. The generate line is always tangent to the profile circle with "r" radius.

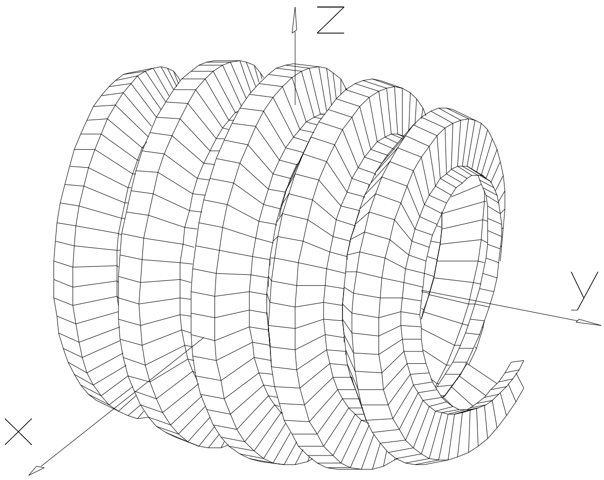


Figure 3. The model of helical worm

The coordinates of motion point from the worm after the transformed of generate line's coordinate from the worm wheels system into the worm system is the follow:

$$\begin{aligned}
x_1 &= -\cos\varphi_1\left[a - r_0\sin(\varphi_2 - \alpha_{ax}) + u\cos(\varphi_2 - \alpha_{ax})\right] \\
y_1 &= -\sin\gamma\sin\varphi_1\left[a - r_0\sin(\varphi_2 - \alpha_{ax}) + u\cos(\varphi_2 - \alpha_{ax})\right] + \\
&\quad + \cos\gamma\left[r_0\cos(\varphi_2 - \alpha_{ax}) + u\sin(\varphi_2 - \alpha_{ax})\right] \\
z_1 &= -\cos\gamma\sin\varphi_1\left[a - r_0\sin(\varphi_2 - \alpha_{ax}) + u\cos(\varphi_2 - \alpha_{ax})\right] - \\
&\quad - \sin\gamma\left[r_0\cos(\varphi_2 - \alpha_{ax}) + u\sin(\varphi_2 - \alpha_{ax})\right]
\end{aligned} \tag{1}$$

The generated worm is present in fig. 3.

In the follow, we present the gearing equation for this type of worm gear pair. As is known the gearing equation is:

$$n_{x1}v_{21x1} + n_{y1}v_{21y1} + n_{z1}v_{21z1} = 0 \tag{2}$$

The components of comun normal are:

$$\begin{cases}
n_{x1} = a_1 \cdot u + b_1 \\
a_1 = \cos\varphi_1\sin(i_{21}\varphi_1 - \alpha_{ax})\cos(i_{21}\varphi_1 - \alpha_{ax}) - i_{21}\sin\varphi_1 \\
b_1 = \cos\varphi_1\sin(i_{21}\varphi_1 - \alpha_{ax})\left[a - r_0\sin(i_{21}\varphi_1 - \alpha_{ax})\right] \\
n_{y1} = a_2 \cdot u + b_2 \\
a_2 = \left[\cos\gamma\cos(i_{21}\varphi_1 - \alpha_{ax}) + \sin\gamma\sin\varphi_1\sin(i_{21}\varphi_1 - \alpha_{ax})\right]\cos(i_{21}\varphi_1 - \alpha_{ax}) + i_{21}\sin\gamma\cos\varphi_1 \\
b_2 = \left[\cos\gamma\cos(i_{21}\varphi_1 - \alpha_{ax}) + \sin\gamma\sin\varphi_1\sin(i_{21}\varphi_1 - \alpha_{ax})\right]\cdot\left[a - r_0\sin(i_{21}\varphi_1 - \alpha_{ax})\right] \\
n_{z1} = a_3 \cdot u + b_3 \\
a_3 = \left[\cos\gamma\sin\varphi_1\sin(i_{21}\varphi_1 - \alpha_{ax}) - \sin\gamma\cos(i_{21}\varphi_1 - \alpha_{ax})\right]\cos(i_{21}\varphi_1 - \alpha_{ax}) + i_{21}\cos\gamma\cos\varphi_1 \\
b_3 = \left[\cos\gamma\sin\varphi_1\sin(i_{21}\varphi_1 - \alpha_{ax}) - \sin\gamma\cos(i_{21}\varphi_1 - \alpha_{ax})\right]\cdot\left[a - r_0\sin(i_{21}\varphi_1 - \alpha_{ax})\right]
\end{cases} \tag{3}$$

The components of the relative speed are:

$$\begin{cases}
v_{21x1} = c_1 \cdot u + d_1 \\
c_1 = \left[(i_{21}\sin\gamma - 1)\sin\gamma\sin\varphi_1\cos\mu_1 - i_{21}\cos^2\gamma\sin\varphi_1\right]\cos(i_{21}\varphi_1 - \alpha_{ax}) + \\
\quad + \left[(1 - i_{21}\sin\gamma)\cos\gamma\cos\mu_1 - i_{21}\sin\gamma\cos\gamma\right]\sin(i_{21}\varphi_1 - \alpha_{ax}) \\
d_1 = \left[(i_{21}\sin\gamma - 1)\sin\gamma\sin\varphi_1\cos\mu_1 - i_{21}\cos^2\gamma\sin\varphi_1\right]\cdot\left[a - r_0\sin(i_{21}\varphi_1 - \alpha_{ax})\right] + \\
\quad + \left[(1 - i_{21}\sin\gamma)\cos\gamma\cos\mu_1 - i_{21}\sin\gamma\cos\gamma\right] r_0\cos(i_{21}\varphi_1 - \alpha_{ax}) \\
v_{21y1} = c_2 \cdot u + d_2 \\
c_2 = (1 - i_{21}\sin\gamma)(\cos\varphi_1\cos\mu_1 - \cos\gamma\sin\varphi_1\sin\mu_1)\cos(i_{21}\varphi_1 - \alpha_{ax}) + \\
\quad + (i_{21}\sin\gamma - 1)\sin\gamma\sin\mu_1\sin(i_{21}\varphi_1 - \alpha_{ax}) \\
d_2 = (1 - i_{21}\sin\gamma)(\cos\varphi_1\cos\mu_1 - \cos\gamma\sin\varphi_1\sin\mu_1)\cdot\left[a - r_0\sin(i_{21}\varphi_1 - \alpha_{ax})\right] + \\
\quad + (i_{21}\sin\gamma - 1)\sin\gamma\sin\mu_1 r_0\cos(i_{21}\varphi_1 - \alpha_{ax}) - a\cos 2\mu_1 \\
v_{21z1} = c_3 \cdot u + d_3 \\
c_3 = \left[(1 - i_{21}\sin\gamma)\sin\gamma\sin\varphi_1\sin\mu_1 + i_{21}\cos\gamma\cos\varphi_1\right]\cos(i_{21}\varphi_1 - \alpha_{ax}) + \\
\quad + (i_{21}\sin\gamma - 1)\cos\gamma\sin\mu_1\sin(i_{21}\varphi_1 - \alpha_{ax}) \\
d_3 = \left[(1 - i_{21}\sin\gamma)\sin\gamma\sin\varphi_1\sin\mu_1 + i_{21}\cos\gamma\cos\varphi_1\right]\cdot\left[a - r_0\sin(i_{21}\varphi_1 - \alpha_{ax})\right] + \\
\quad + (i_{21}\sin\gamma - 1)\cos\gamma\sin\mu_1 r_0\cos(i_{21}\varphi_1 - \alpha_{ax})
\end{cases} \tag{4}$$

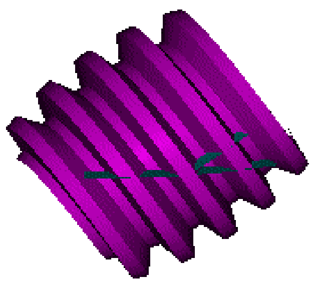
We can observe that the variable both in the normal components and the relative speed components is „u". The gearing equation will be a square equation in „u" [4]:

$$Mu^2 + Nu + P = 0 \tag{5}$$

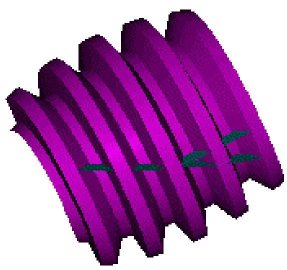
$$\begin{cases}
M = a_1 \cdot c_1 + a_2 \cdot c_2 + a_3 \cdot c_3 \\
N = a_1 \cdot d_1 + b_1 \cdot c_1 + a_2 \cdot d_2 + b_2 \cdot c_2 + a_3 \cdot d_3 + b_3 \cdot c_3 \\
P = b_1 \cdot d_1 + b_2 \cdot d_2 + b_3 \cdot d_3
\end{cases} \tag{6}$$

The discriminant of this equation is $\Delta = \sqrt{N^2 - 4MP} \geq 0$, therefore the equation has real roots.

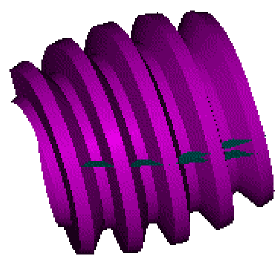
In figure 4 we presents the mating surfaces for different angles between the axes [4].



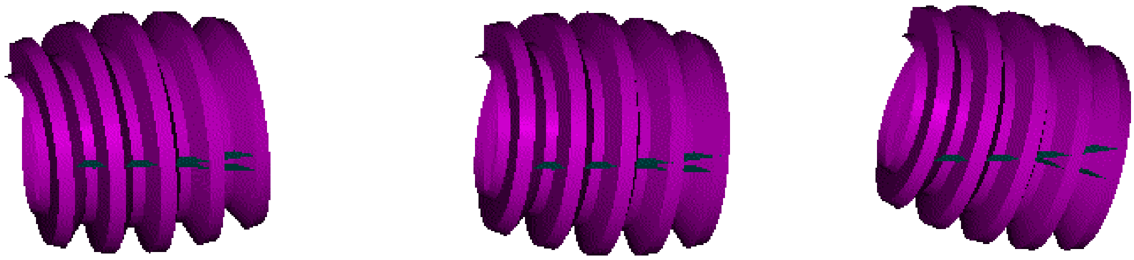
i*$_{21}$ *= 40, q = 14,
m = 10, γ ***= - 30***



i*$_{21}$ *= 40, q = 14,
m = 10, γ ***= - 20***



i*$_{21}$ *= 40, q = 14,
m = 10, γ ***= - 10***



$i_{21} = 40, q = 14,$	$i_{21} = 40, q = 14,$	$i_{21} = 40, q = 14,$
$m = 10, \gamma = 0$	$m = 10, \gamma = +10$	$m = 10, \gamma = +20$

Figure 4. The mating surfaces for internal worm gear pair for different angle between the axis

3. THE MANUFACTURE OF THE INTERNAL WORM GEAR PAIR ELEMENTS. SPECIAL DEVICES. HELICAL WORM HOB

One of the most important achievements of the research group was the execution of the prototype of the internal worm gearing elements. It was achieved on a universal teething grinding machine of the Donini type, which can cut teeth of to modules of m = 20 mm [3], [4].

Taking into account the advantages of the ruled profile generation, we considered both the helical worm and the helical worm hob with ruled profile. Figure 5 presents the technological scheme of manufacture the elements of this type of drive, and the special devices, which are necessary.

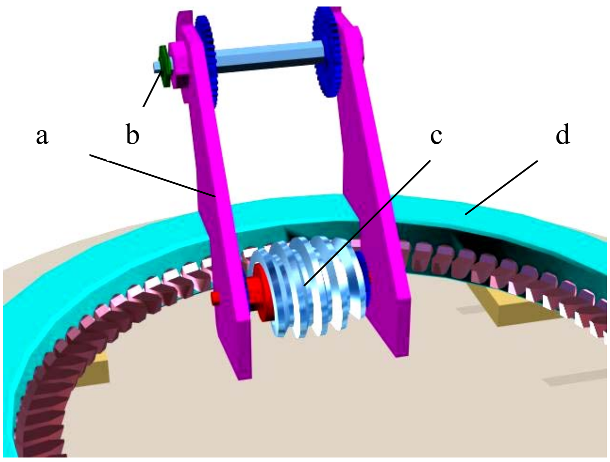


Figure 5. The execution scheme of the drive
a –arm device, b – tool arbor, c – the semi-manufactured worm, d – multi-cutter device

The helical worm precision is influenced by the fixing precision of the hob's teeth, by the precision of the tool's edges, by the precision of the devices used etc. Some of the errors can be avoided if the processing is divided into two operations: a roughing by the multi-knife device and a finish by a flying cutter.

In figure 6 we presents the helical worm – internal teeth worm wheel drive.

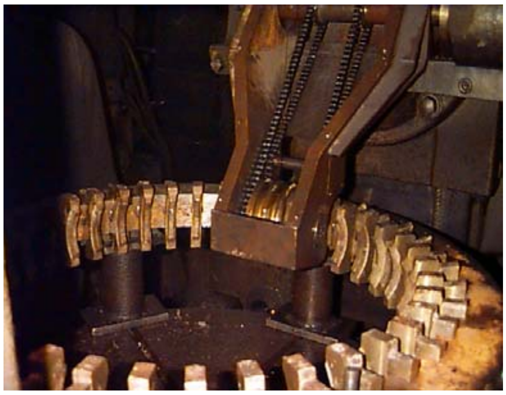


Figure 6. The helical worm – internal teeth worm wheel drive

CONCLUSIONS

The internal worm gearings are made up of an ellipsoid worm and an internal teething worm wheel. These gearings, especially those with perpendicular axes, can be named reverse globoid worm gearings or anti-globoid, as they have similar characteristics.

The angle between the worm axes and of the worm wheel can range between 0º and 90º. It is even advantageous for the angle to be other than 90º, thus not considering the problem of worm bearing.

For the time being we consider both the worm and the helical worm hob with ruled profiles, but theoretically their profile can be generated by any curve. For processing, we used a universal teeth-grinding machine of Donini type, with two special devices respectively. By there aid a processing, which we could call of anti Cone type, occurred.

The present research heads towards the determining of the gearing surfaces generally, towards the achievement of helical worm hob respectively.

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