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THE GEARING LINE' LIMITS OF THE CYCLOID GEAR WITH ROLLER TEETH

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¶A***bstract:*** *From dynamic point of view, at the cycloid gear transmission is not rational to utilize the entire length of the cycloid profile of the satellite gear. The favorable interval of the cycloid profile of a tooth is limited by the tooth addendum' circle with r_a radius and the tooth dedendum' circle with r_f radius. The dynamic favorable interval represent ≈45% from entire cycloid profile length of a satelite gear tooth.*
Key words: *gearing line, pressure angle, reaction values.*

1. INTRODUCTION

The study of the gearing line' limits of the cycloid gear with roller teeth includes determinations on two types of cycloid reducer with roller teeth: **M1C-23** with gear ratio $\boldsymbol{i_r} = \boldsymbol{z_1}$ = 17, number of roller teeth $\boldsymbol{z_2}$ = 18, eccentricity $\boldsymbol{a}$ = 2,4 [mm], driving engine power $\boldsymbol{P}$ = 1,1 [kW], the driving engine' nominal speed $\boldsymbol{n_m}$ = 1500 [rot/min] and the roller tooth diameter $\boldsymbol{d_{bo}}$ = 10 [mm], respectively **RPR – 150** with gear ratio $\boldsymbol{i_r} = \boldsymbol{z_1}$ = 25, number of roller teeth, $\boldsymbol{z_2}$ = 26, eccentricity $\boldsymbol{a}$ = 2 [mm], driving engine power $\boldsymbol{P}$ = 1,5 [kW], the driving engine' nominal speed $\boldsymbol{n_m}$ = 1500 [rot/min] and the roller tooth diameter $\boldsymbol{d_{bo}}$ = 12 [mm].

The pressure angle between a roller tooth and the cycloid profile represents an essential indicator on the estimation process of the energetic flux transmission (forces). The satellite gear' cycloid profile optimization takes into account the variation of this angle during a cycle of gearing of a roller tooth. When the pressure angle is zero, or has small values, the energetic flux transmission is carry out in good condition.

Based on the transmission angle $\gamma(\varphi)$ variations' graphs, we conclude that the central interval of the cycloid profile of a tooth, between 13°-119°, is favorable to the transmission of the forces and the beginning/ending part of the cycloid profile of a tooth corresponds to some transmission angle unacceptable for the forces.

We conclude also that the entire gear line utilization, from φ=0° ... 180°, is not possible because we see the pressure angle γ tends to 90^0 on the extremity parts of the gearing line, witch proves that exists blocking situations.

The analyze of the R_{12}, R_{32}, R_{42} reactions' variation graph shows us that, theoretically, the reactions are infinite at the beginning/ending of the gearing of the sun gear roller tooth with the satelite gear cycloid profile The reactions decrease in the central part of the active profile and, practical, became constant on the interval φ=20°-120°.

By dynamical point of view, on the forces transmission is not rational to use the entire length of the cycloid profile. The cycloid profile' favorable interval to transmit the forces is delimitated by the tooth addendum circle with r_a radius and the tooth dedendum circle with r_f radius. In the paper, we present the mathematical expressions of the tooth addendum circle radius, respectively tooth dedendum circle radius. These circles limit the optimal cycloid profile considering the forces transmission favorable angle.

The dynamic favorable interval represent ≈45% from entire cycloid profile length of a satelite gear tooth. We obtain this shorten length of the gearing line by supplementary manufacturing process of the beginning / ending part of the tooth' active profile. To use only the dynamic favorable domain of the cycloid profile is possible by elimination of the beginning / ending part of the tooth' active profile through decreasing the tooth addendum circle' diameter, respectively through deepening the tooth dedendum profile.

2. THE PRESSURE ANGLE AND THE GEARING LINE

At the cycloid gear, the pressure angle γ of the roller tooth and the cycloid profile is formed between the normal direction of the CA segment in the gearing point of a roller tooth and the line AD, perpendicular to the centre line direction (fig.1), (1). When the pressure angle is zero, or has small values, the energetic flux transmission is carry out in good condition.

$$\gamma_i = \operatorname{arc\,tg} \frac{r_3 \cos\varphi_i - r_g}{r_3 \sin\varphi_i} \tag{1}$$

In the case of the cycloid gear with X-profile, the gearing line don't pass through the gearing pole (extrapole gearing), (fig. 1).

We consider the beginning of the gearing line in the contact point of the roller tooth with the tooth dedendum of the satellite gear. Is situated at $\rho_{min}=r_3-r_g-r_{bo}$ distance from the pole C.

The ending of the gearing line is in the contact point of the roller tooth with the tooth addendum of the satellite gear. Is situated at $\rho_{max} = r_3+r_g-r_{bo}$ distance from the pole C.

The gearing line's points are situated on the normal line of the cycloid profile, in the contact points of the roller tooth with the cycloid profile, at r_{bo} distance from the point B of the theoretical profile.

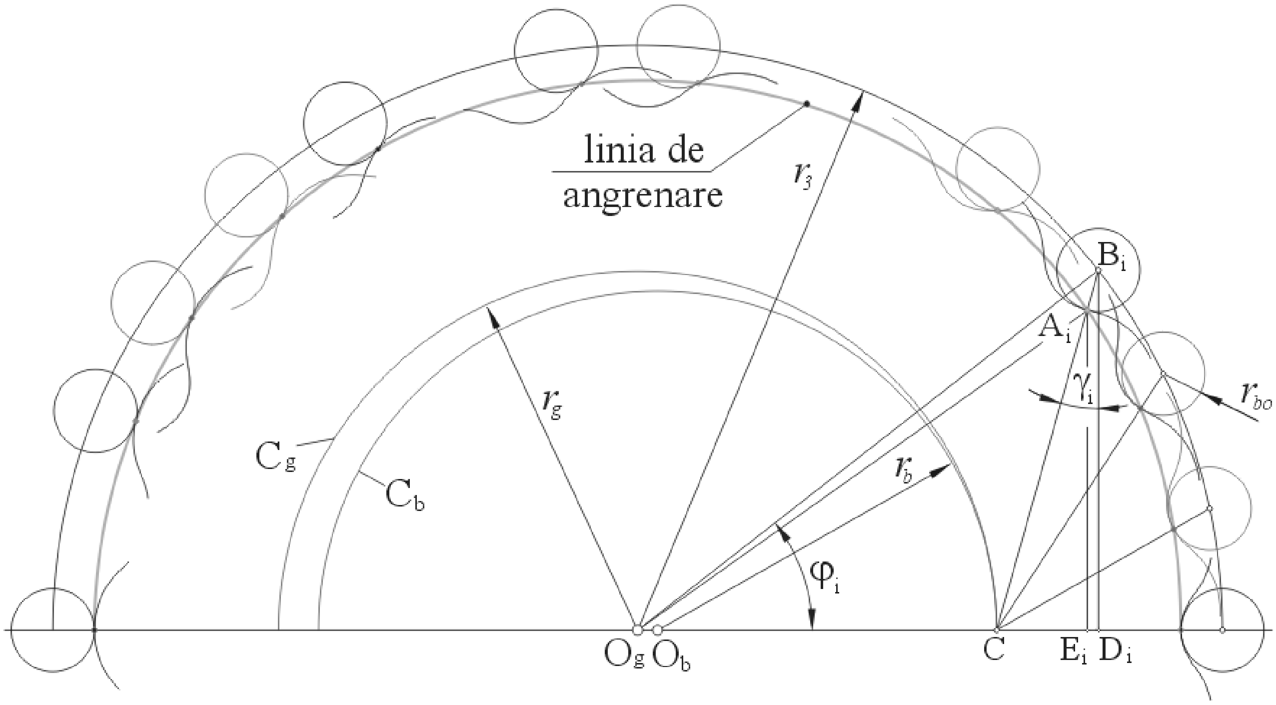


Fig. 1. *The pressure angle and the gearing line*

In the coordinate system fixated on the sun gear with roller teeth, the parametric equations of the gearing line points' coordinate are:

$$x_{Ai} = r_g + (\rho_{Bi} - r_{bo})\sin\gamma_i \tag{2}$$

$$y_{Ai} = (\rho_{Bi} - r_{bo})\cos\gamma_i \tag{3}$$

3. THE ANALYZE OF THE VARIATION OF THE REACTIONS WHO ACTS ON THE SATELLITE GEAR. THE ANALYZE OF THE PRESSURE ANGLE

Based on the determination relations of the reactions' values, we present the variation of the $R_{12}(\varphi)$, $R_{32}(\varphi)$ şi $R_{42}(\varphi)$ reactions who act on the satellite gear, calculated in MathCAD utilitarian program, for different position of the satellite gear expressed by the rotation angle of the centroides tangent point, φ,. (fig. 2, 4).

The variations of the pressure angle are presented in figures 3, 5.

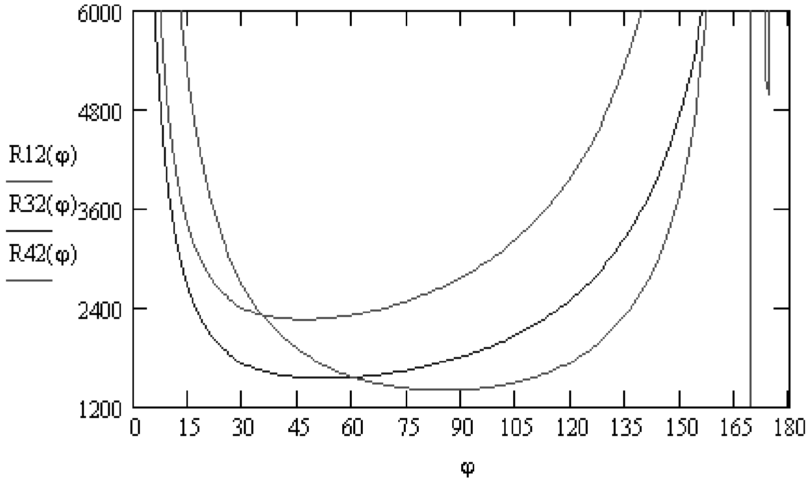
$z1 := 17 \quad rg = 43.2 \quad rb = 40.8 \quad a = 2.4 \quad ex = 1.63 \quad M1 = 739 \quad rbo = 5 \quad r3 = 70.398 \quad r4 = 45$

$$\varphi r(\varphi) := \varphi\cdot\frac{\pi}{180} \qquad \rho(\varphi) := rg\cdot\sqrt{ex^2 + 1 - 2ex\cdot\cos(\varphi r(\varphi))} - rbo \qquad \theta(\varphi) := \operatorname{asin}\left(\frac{r3\cdot\cos(\varphi r(\varphi)) - rg}{\rho(\varphi) + rbo}\right)$$

$$R32(\varphi) := \frac{10M1\cdot\sqrt{ex^2 + 1 - 2\cdot ex\cdot\cos(\varphi r(\varphi))}}{2\cdot a\cdot ex\cdot\sin(\varphi r(\varphi))} \qquad R42(\varphi) := \frac{10M1}{2a}\cdot\frac{rb}{r4\cdot\sin\left(\frac{rg}{rb}\cdot\varphi r(\varphi)\right)} \qquad R12x := \frac{10M1}{2a}$$

$R12x = 1539.368 \quad N$

$$R12(\varphi) := \frac{10M1}{2\cdot a}\cdot\sqrt{1 + \left(\frac{2\cdot a\cdot Fc}{10M1} + \frac{ex\cdot\cos(\varphi r(\varphi)) - 1}{ex\cdot\sin(\varphi r(\varphi))} - \frac{rb}{r4\cdot\sin\left(\frac{rg}{rb}\cdot\varphi r(\varphi)\right)}\right)^2} \qquad \varphi := 0..180$$

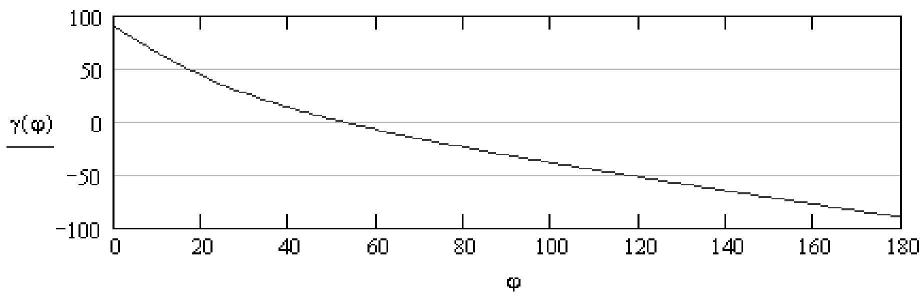


φ =	R12(φ) =	R32(φ) =	R42(φ) =
18	2982	2279	4274
36	2301	1622	2261
54	2271	1540	1661
72	2438	1616	1437
90	2773	1806	1402
108	3359	2145	1532
126	4443	2756	1921
144	6880	4031	3020
162	17299	7945	9475

Fig. 2. *The $R_{12}(\varphi)$, $R_{32}(\varphi)$ şi $R_{42}(\varphi)$ reactions graphs who act on the cycloid satellite gear corresponding to cicloyd reducer* ***M1C-23***

$z1 := 17 \quad rg = 43.2 \quad rb = 40.8 \quad a = 2.4 \quad ex = 1.63 \quad rbo = 5 \quad r3 = 70.398 \quad r4 = 45$

$$\varphi r(\varphi) := \varphi\cdot\frac{\pi}{180} \qquad \rho(\varphi) := rg\cdot\sqrt{ex^2 + 1 - 2ex\cdot\cos(\varphi)} - rbo \qquad \gamma(\varphi) := \operatorname{asin}\left(\frac{r3\cdot\cos(\varphi r(\varphi)) - rg}{\rho(\varphi r(\varphi)) + rbo}\right)\cdot\frac{180}{\pi} \qquad \varphi := 0..180$$



$\varphi := 1$

$\operatorname{root}(\gamma(\varphi), \varphi) = 52.145$

$\operatorname{root}(\gamma(\varphi) - 50, \varphi) = 16.768$

$\operatorname{root}(\gamma(\varphi) + 50, \varphi) = 116.768$

Fig. 3. *The variation of the pressure angle corresponding to* ***M1C-23***

z1 := 25 rg = 52 rb = 50 a = 2 ex = 1.413 M1 = 981 rbo = 5 r3 = 73.502 r4 = 44

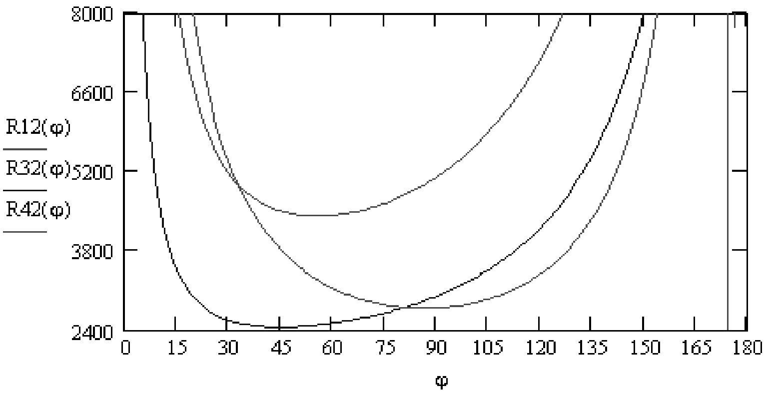
$$\varphi r(\varphi) := \varphi\cdot\frac{\pi}{180} \qquad \rho(\varphi) := rg\cdot\sqrt{ex^2 + 1 - 2ex\cdot\cos(\varphi r(\varphi))} - rbo \qquad \theta(\varphi) := \operatorname{asin}\left(\frac{r3\cdot\cos(\varphi r(\varphi)) - rg}{\rho(\varphi) + rbo}\right)$$

$$R32(\varphi) := \frac{10M1\cdot\sqrt{ex^2 + 1 - 2\cdot ex\cdot\cos(\varphi r(\varphi))}}{2\cdot a\cdot ex\cdot\sin(\varphi r(\varphi))} \qquad R42(\varphi) := \frac{10M1}{2a}\cdot\frac{rb}{r4\cdot\sin\left(\frac{rg}{rb}\cdot\varphi r(\varphi)\right)} \qquad R12x := \frac{10M1}{2a}$$

R12x = 2451.342 N

$$R12(\varphi) := \frac{10M1}{2\cdot a}\cdot\sqrt{1 + \left(\frac{2\cdot a\cdot Fc}{10M1} + \frac{ex\cdot\cos(\varphi r(\varphi)) - 1}{ex\cdot\sin(\varphi r(\varphi))} - \frac{rb}{r4\cdot\sin\left(\frac{rg}{rb}\cdot\varphi r(\varphi)\right)}\right)^2}$$

φ := 0 .. 180

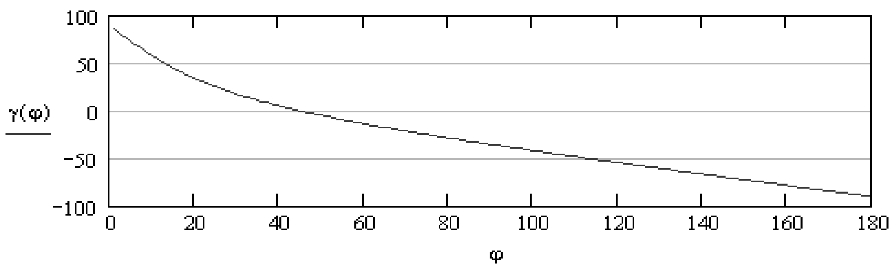


φ =	R12(φ) =	R32(φ) =	R42(φ) =
18	7140	3121	8679
36	4792	2488	4582
54	4418	2478	3354
72	4582	2658	2886
90	5110	3003	2791
108	6104	3588	3011
126	7963	4627	3693
144	12066	6783	5531
162	27174	13383	13948

Fig. 4. *The $R_{12}(\varphi)$, $R_{32}(\varphi)$ şi $R_{42}(\varphi)$ reactions graph who act on the cycloid satellite gear corresponding to cycloid reducer* **RPR – 150**

z1 := 17 rg = 52 rb = 50 a = 2 ex = 1.413 rbo = 5 r3 = 73.502 r4 = 44

$$\varphi r(\varphi) := \varphi\cdot\frac{\pi}{180} \qquad \rho(\varphi) := rg\cdot\sqrt{ex^2 + 1 - 2ex\cdot\cos(\varphi)} - rbo \qquad \gamma(\varphi) := \operatorname{asin}\left(\frac{r3\cdot\cos(\varphi r(\varphi)) - rg}{\rho(\varphi r(\varphi)) + rbo}\right)\cdot\frac{180}{\pi} \qquad \varphi := 0..180$$



φ := 1

root(γ(φ), φ) = 44.971

root(γ(φ) − 50, φ) = 12.951

root(γ(φ) + 50, φ) = 112.951

Fig. 5. *The variation of the pressure angle corresponding to* **RPR – 150**

We present the graphs of the *R_{12}, R_{32}, R_{42}* reactions' variations, figures 2 and 4, calculate in hypothesis I (each satellite gear gears with a roller tooth and a thumb -of the transversally coupling- on the entire length of the gearing line). Analyzing the reactions graphs, we conclude that, in the beginning/ending of the roller tooth gearing with the cycloid profile, theoretical the reactions are infinite and decrease on the central interval of the active profile, practical being constant on the interval φ=20°-120°.

The accepted values of the transmission angle we consider to be until 50° , and these accepted values are situated between the limits 12,951° and 112,951° of the angle φ for the first analyzed reducer, respectively 16,768° and 116,768° of the angle φ for the second analyzed reducer.

Analyzing the graphs of transmission angle variations γ(φ) (fig. 3 and 5), we consider that the central interval of the cycloid profile limited by 13°-119° is favorable to the forces transmission. The beginning / ending of the cycloid profile correspond to some transmission angles unacceptable for the forces.

The usage of the entire length of the gearing line, from φ=0° ... 180°, is not rational, because, at the ending part of the gearing line, the pressure angle increase to 90^0 (fig. 3 and 5) showing us the existing possibilities of blocking.

We conclude that, **from dynamic point of view, to transmit the forces is not rational to utilize the entire length of the cycloid profile**

From point of view of the transmission of the forces, the favorable interval of the cycloid profile is limited by the tooth addendum circle with r_a radius and the tooth dedendum circle with r_f. radius.

We present the determination relation of these circles' radius values (4.) Porțiunile profilului cicloidal avantajoase din punct de vedere al transmiterii forțelor sunt delimitate de cercul de cap de raza r_a şi cercul de picior r_f.

$$r_{a,f} = \sqrt{x^2_{A_{a,f}} + y^2_{A_{a,f}}} \tag{4}$$

where: x_{Aa} and y_{Aa} , respectively x_{Af} and y_{Af} , are the coordinate of the gearing entrance point, respectively the coordinate of the gearing exit point of a tooth.

$$x_{A_{a,f}} = r_g - a + \left(\sqrt{r_3^2 + r_g^2 - 2r_3r_g\cos\varphi_{a,f}} - r_{bo}\right)\frac{r_3\cos\varphi_{a,f} - r_g}{\sqrt{r_3^2 + r_g^2 - 2r_3r_g\cos\varphi_{a,f}}} \tag{5}$$

$$y_{A_{a,f}} = \left(\sqrt{r_3^2 + r_g^2 - 2r_3r_g\cos\varphi_{a,f}} - r_{bo}\right)\frac{r_3\sin\varphi_{a,f}}{\sqrt{r_3^2 + r_g^2 - 2r_3r_g\cos\varphi_{a,f}}} \tag{6}$$

In figure 6 we present, with thick line, the effectively length of the gearing line between the limits φ_f and $\varphi_a+\varphi_f$, The active interval of the cycloid profile corresponds to **a-b** interval to a direction of rotation, respectively **c-d** to the inverse direction of rotation. The dynamic favorable interval represent ≈45% from entire cycloid profile length of a satelite gear tooth.

We obtain this shorten length of the gearing line by supplementary manufacturing process of the beginning / ending part of the tooth' active profile.

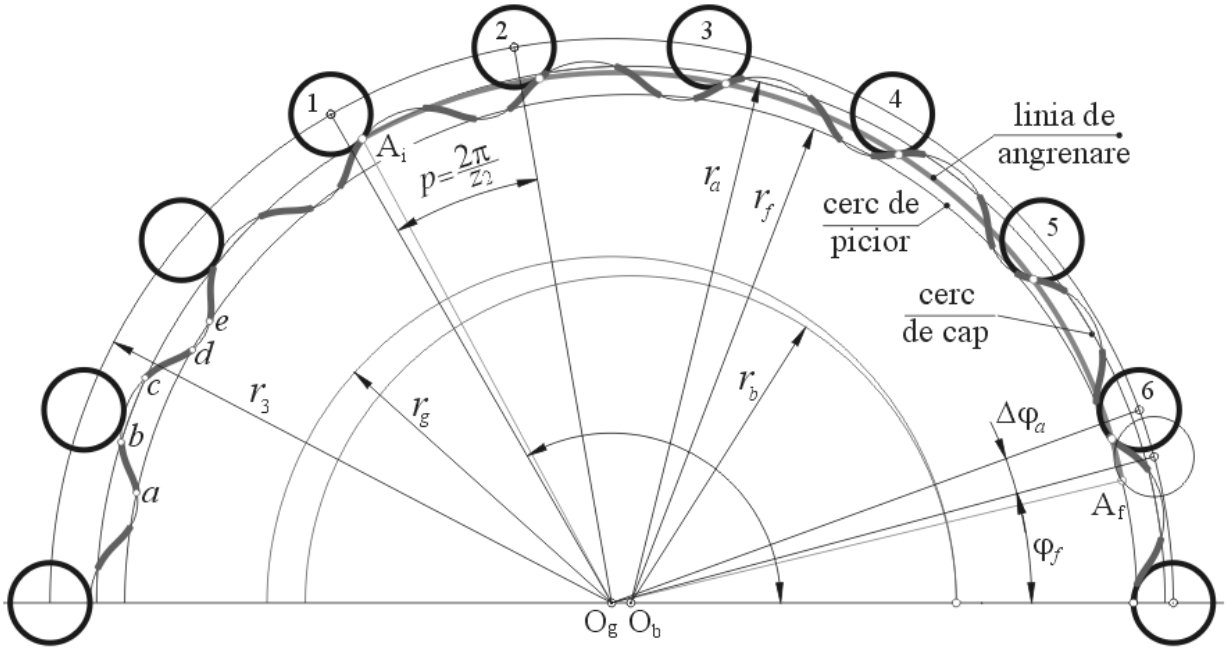


Fig. 6. *The limits of the gearing line*

To use only the dynamic favorable domain of the cycloid profile is possible by elimination of the beginning / ending part of the tooth' active profile through decreasing the tooth addendum circle' diameter corresponding to b-c interval, respectively through deepening the tooth dedendum profile corresponding to d-e interval.

RESULTS AND CONCLUSIONS

Analyzing the graphs of R_{12}, R_{32}, R_{42} reactions' variations, figures 2 and 4, calculate in hypothesis I (each satellite gear gears with a roller tooth and a thumb -of the transversally coupling- on the entire length of the gearing line), we conclude that, in the beginning/ending of the roller tooth gearing with the cycloid profile, theoretical the reactions are infinite and decrease on the central interval of the active profile, practical being constant on the interval φ=20°-120°.

Analyzing the graphs of transmission angle variations γ(φ), we consider that the central interval of the cycloid profile limited by 13°-119° is favorable to the forces transmission. The beginning / ending of the cycloid profile correspond to some transmission angles unacceptable for the forces.

The usage of the entire length of the gearing line, from φ=0° ... 180°, is not rational, because, at the ending part of the gearing line, the pressure angle increase to 90^0 showing us the existing possibilities of blocking.

We conclude that, **from dynamic point of view, to transmit the forces is not rational to utilize the entire length of the cycloid profile**

From point of view of the transmission of the forces, the favorable interval of the cycloid profile is limited by the tooth addendum circle with r_a radius and the tooth dedendum circle with r_f. radius.

REFERENCES

[1] Dăscălescu, A.: *Contribution to the Kinematics and Dynamics Studys of the Planetary Gears with Cycloid Toothing and Roller Teeth,* PhD Theses, Cluj-Napoca Technical University, 2005