7th INTERNATIONAL MULTIDISCIPLINARY CONFERENCE
Baia Mare, Romania, May 17-18, 2007
ISSN-1224-3264

A DESIGN METHOD FOR THE TRANSMISSIONS WITH ASYMMETRICAL SPUR GEARS

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¶A***bstract:*** *The paper presents a design method for spur gears with asymmetrical involute teeth. The initial phase, which is necessary for the study of these special gears, is the elaboration of a design programme for the asymmetrical gearing and for the asimetrical gear rack. These fact,makes possible the calculatation of the geometrical parameters for a great number of gearings, with the same initial data but with different design variant, which can be chosen by the designing engineer user of the program. The simulation and the determination of gearing performances, based on the calculated parameters, ensure the informations which one can establish the optimal variant.*

Key words: *Direct gear design, spur gears, asymmetrical involute teeth, asymmetric gear rack*

1. INTRODUCTION

Involute gears, having involute curves as teeth profiles, are the most frequent because of some advantages which are already known.. Designing gears with asymmetrical involute teeth is aimed at improving the performances of active flanks [4]. From these,it can result an increase in the loading capacity, an increase of the efficiency. It also can be obtained the transmission error, specific slidings, dimensiones, weight, noise and vibration reduction.

At the gearing formed with spur gears having asymmetrical teeth profiles, the performances of the transmission, from kinematic and also from dynamic point of view, modify with the change of the direction of the rotation. Because of the complex geometry, the design and processing cost for these special gearing are biger, as related the classical one, but are justifyed by the quality of the transmission.

The design method establised for the ordinary gears, based on the standard gear rack, can't be aplyed for the asymmetrical gears.

The developed method to determine the geometrical parameters of the asymmetrical gears [3], presented in this paper, use the „direct design" of the spur gears [5]. This method offer the possibility of determining first the parameters of the gears (fig. 1), folowed by the determination of the asymmetric gear rack's parameters (fig.2), based on those of the gears. Using direct gear design one can obtain the best performances for particular application [1].

An involute tooth with asymmetrical profiles is limited by two circular arcs, on the addendum circle and on the dedendum circle, and lateral by two asymetrical involute arcs and two fillet curves of the involute arcs with the root circle. The profile called „direct profile" belong to the pozitive side of an involute with the base circle diameter d_{bd} - the direct profile base circle diameter. The profile called „inverted profile" belong to the negative side of an involute with the base circle diameter d_{bi} - the inverted profile base circle diameter. The intersection point of the two involutes, with different base circles, caled tip of the tooth, indicate the maximum posible diameter of the toothed gear . This diameter, with the symbol d_v, which is coresponding with the tooth thicness on the addendum circle which is equal to zero, is caled the tip circle diameter [2].

In the followings the direct profile elements are attributed the index "d" and the inverted profile elements are attributed the index "i".

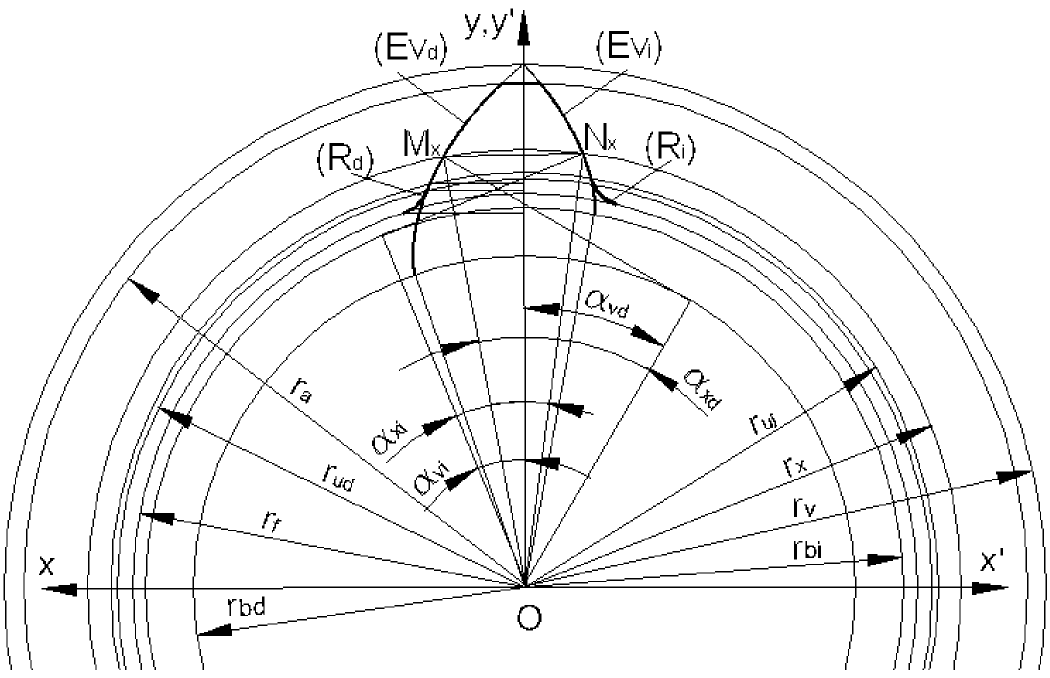

Fig. 1 *The asymmetrical involute tooth geometrical parameters*

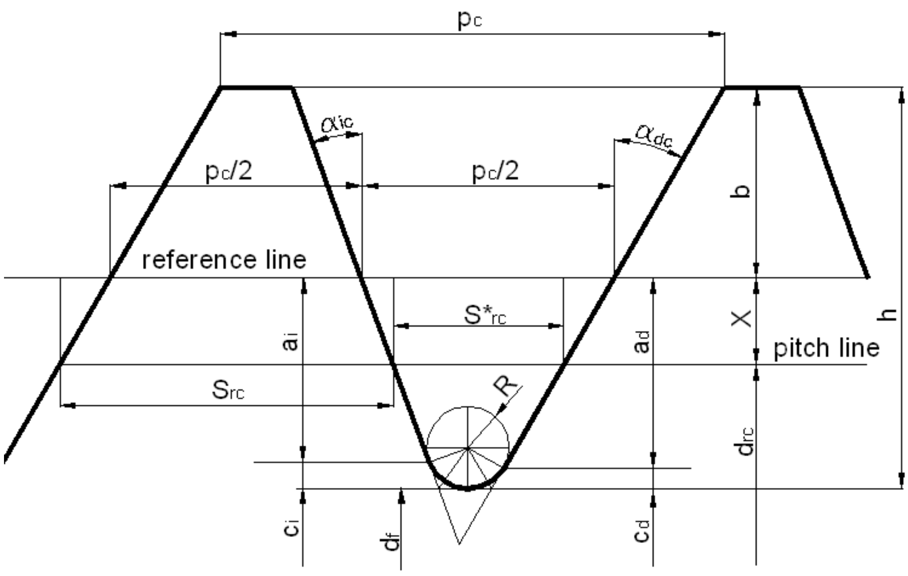

Fig. 2 *The asymmetrical gear rack dimensions*

2. THE CALCULATION PROCEDURE OF THE ASYMMETRIC GEARS GEOMETRICAL PARAMETERS

Based on a extended bibliographic research and after the elaboration of a large study about fundamental theorethical aspects about asymmetric gears [3] became posible to write, as aplication in MATLAB, a packet of *Software for designing, modelling and analysing the geometrical and functional parameters of gearing formed of asymmetric teeth spur gears*.

Table 1 show the algorithm used, by the authores of the paper, for the routine in which are calculated the dimensions of the gears and than the dimensions of the gear racks.

For a given center distance (a) , number of teeth (z_1, z_2) as initial data, the designing engineer choose, as designing variables, the pressure angle for the direct profile (α_{wd}) and the pressure angle for the inverted profile. Implicitly is also chosen the cofficient of assymetry of the tooth.

Another designing variable is the coefficient „f", which is called coefficient of modification of the gear rack angle. This is used, in the calculation programm, to determine the angle of the rack profile for generating the direct profile of the tooth (α_{dc}), beetwen the limits of this angle:

$$\alpha_{dc\,min} = \arccos(1/k)\,;\ \alpha_{dc\,max} = \alpha_{wd}\,, \tag{1}$$

$$\alpha_{dc} = \alpha_{wd} - \frac{f \cdot \left(\alpha_{dc\,max} - \alpha_{dc\,min}\right)}{\alpha_{gwd}}, \quad (2)$$

were α_{gwd} is the pressure angle on the direct profile expressed in degrees.

The value of coefficient „f" show how much the angle of the direct gear rack profile (α_{dc}) is different from the pressure angle of the direct profile of the gearing (α_{wd}) The shift rack of the pinion and the shift rack of the gear is determined by the α_{dc} angle of the rack. So is also established the ratio of the pinion tooth thickness on the pitch circle and gear tooth thickness on the pitch circle (S_{w1}/S_{w2}).

Table 1 The phases of geometrical designing of the asymmetric gears

Nr.	*Parameter*	*Symbol*
A. Initial data		
1.	Number of teeth	z_1, z_2
2.	Center distance	a
B. Designing variable (chosen by the designing engineer)		
3.	Pressure angle for the direct profile	α_{wd}
4.	Pressure angle for the inverted profile	α_{wi}
5.	Number of the gear racks (generating pinion and gear with unique or different rack)	n_{cr}
6.	The modification coefficient of the gear rack angle	f
7.	Coefficient of asymmetry of the tooth	k
8.	Generating rack profile angles	α_{dc}, α_{ic}
C. Geometrical parameters of the asymmetric gears		
9.	Coefficient of the tooth thicknes on addendum circle, for the pinion and for the gear	m_{o1}, m_{o2}
10.	Circular pitch on the pitch circle	p_w
11.	Diameter of the pich circle, for the pinion and for the gear	d_{w1}, d_{w2}
12.	Sum of the shift rack of the pinion and the shift rack of the gear	$X_1 + X_2$
13.	Shift rack of the pinion, shift rack of the gear	x_1, x_2

14.	Tooth thicknes on pitch circle, for the pinion and for the gear	S_{w1}, S_{w2}
15.	Profile angles on the tip circle of the pinion tooth	α_{v1d}, α_{v1i}
16.	Profile angles on the tip circle of the gear tooth	α_{v2d}, α_{v2i}
17.	Diameters of the base circles, for direct and inverted profile, for the pinion and for the gear	d_{b1d}, d_{b1i}, d_{b2d}, d_{b2i}
18.	Tooth thicknes on outside circle, for the pinion and gear	S_{a1}, S_{a2}
19.	Profile angles on the outside circle for the pinion	α_{a1d}, α_{a1i}
20.	Profile angles on the outside circle for the gear	α_{a2d}, α_{a2i}
21.	Diameters of the outside circles of the pinion and of the gear	d_{a1}, d_{a2}
22.	Contact ratio for the direct profile	$\varepsilon_{\alpha d}$
23.	Contact ratio for the inverted profile	$\varepsilon_{\alpha i}$
24.	Profile angles for the first contact point from the bottom of the tooth, for the direct and inverted profiles, for the pinion and the gear	α_{p1d}, α_{p1i}, α_{p2d}, α_{p2i}
D. Geometrical parameters of the generating gear racks		
25	Generating rack pitch	p_c
26.	Diameters of the generating pitch circles	d_{rc1}, d_{rc2}
27.	Minimum radial clearance	c_{min}
28.	Diameters of the dedendum circles of the pinion and of the gear	d_{f1}, d_{f2}
29.	Tip gear racks radius – for generating with different racks	$R_1 \neq R_2$
30.	Tip gear rack radius – for generating with one singular rack	$R_1 = R_2$
31.	The generating rack tooth addendums, the generating rack tooth dedendums and their limits imposed by the conditiones to avoid the interferance and for generating the entire profiles.	a_{d1}, $a_{d1\,min}$, $a_{d1\,max}$ a_{i1}, $a_{i1\,min}$, $a_{i1\,max}$ $b_{d1\,min}$, $b_{i1\,min}$ a_{d2}, $a_{d2\,min}$, $a_{d2\,max}$ a_{i2}, $a_{i2\,min}$, $a_{i2\,max}$ $b_{d2\,min}$, $b_{i2\,min}$

32.	The profile angles coresponding to the points of begining of useful profiles (the begining of involute profiles).	α_{u1d}, α_{u1d}, α_{u2d}, α_{u2d}
33.	Checking the achievement of the conditions imposed by the correct meshing , avoiding the interference, generating the entire profile, for the direct and also for the inverted profiles.	
34.	If the checking relations are not satisfyed the calculation is made again for a new value of the shift rack x_1 (from the 13-th row of the table).	

The following sequence from the designing program:

```
    if    (1<epsilon_a<2    &    1<epsilon_i<2    &    alfa_p1a>alfa_u1a    &
alfa_p1i>alfa_u1i & alfa_u1a>0 & alfa_u1i>0 & 0<a_1a_min<a_1a<a_1a_max &
0<a_1i_min<a_1i<a_1i_max   &   alfa_p2a>alfa_u2a   &   alfa_p2i>alfa_u2i   &
alfa_u2a>0      &      alfa_u2i>0      &      0<a_2a_min<a_2a<a_2a_max      &
0<a_2i_min<a_2i<a_2i_max   &   b_1a_min>0   &   b_1i_min>0   &   b_2a_min>0   &
b_2i_min>0&R_1>0&R_2>0)
    break
```

in which the calculated values of the parameters are checking, show that these parameters must satisfy a number bigger than twenty conditions to be accepted and used in the next routine .

To determine the geometrical parameters for asymmetrical gear, must be solved two ecuations systems (Table 1, 15-th and 16-th rows) with the form:

$$\begin{cases} \cos\alpha = F_1 \cdot \cos\beta \\ \mathrm{inv}\alpha + \mathrm{inv}\beta = F_2 \end{cases}, \tag{3}$$

where F_1 and F_2 are numerical values that are known at the respective step of the calculation.

The solution of the system can be obtained with $\beta = \arccos\left(\left(\cos\alpha\right)/F_1\right)$ by solving, with a calculation programme, the transcedental ecuation with a singular unknown:

$$\mathrm{inv}\alpha + \mathrm{inv}\left(\arccos\left(\left(\cos\alpha\right)/F_1\right)\right) - F_2 = 0\,. \tag{4}$$

The same method can be used for solving the ecuations systems from which results the parameters from the19-th and 20-th rows of the table 1, like:

$$\begin{cases} \cos\alpha = F_3 \cdot \cos\beta \\ \mathrm{inv}\alpha + \mathrm{inv}\beta + F_5 \cdot \cos\beta = F_4 \end{cases}. \tag{5}$$

The ecuation that must be solved for deternining the solutions is:

$$\mathrm{inv}\alpha + \mathrm{inv}\left(\arccos\left(\left(\cos\alpha\right)/F_3\right)\right) + F_5 \cdot \left(\cos\alpha\right)/F_3 - F_4 = 0\,. \qquad (6)$$

3. CALCULATION EXAMPLE

For evaluating the difference between the parameters obtained for the same initial data, but with different designing variables (one or two gear racks, different values for the asymmetry coefficient, different values for the „f" coefficient) has been calculated the parameters for the cases presented in table 3:

Tabele.3 Starting data for different designing variant

Gearing	1.	2.	3.	4.	5.	6.
Initial data	$z_1 = 16$; $z_2 = 57$; $a = 120$ [mm]					
Designing variable						
α_{wd} [grade]	40	40	30	30	30	23
α_{wi} [grade]	20	20	20	20	30	23
k (direct)	1.2267	1.2267	1.0851	1.0851	1	1
α_{wd} [rad]	0.6981	0.6981	0.5236	0.5236	0.5236	0.4014
α_{dc} [rad]	0.6981	0.6881	0.5236	0.4819	0.4363	0.4014
n_{cr}	1	2	2	2	2	1

Table.4 Resulting data for different designing variant (but a part)

The profile angles on the direct/inverted profile of the pinion [rad]						
α_{u1d}	0.5421	0.5480	0.2960	0.3237	0.1309	0.0009
α_{p1d}	0.5770	0.5825	0.3406	0.3703	0.2163	0.0685
α_{a1d}	0.8480	0.8505	0.7580	0.7679	0.6685	0.6746
α_{v1d}	0.8576	0.8599	0.7715	0.7807	0.6874	0.6930
α_{u1i}	0.0046	0.0010	0.0029	0.0076	0.1309	0.0009
α_{p1i}	0.0943	0.1047	0.0785	0.1198	0.2163	0.0685
α_{a1i}	0.6242	0.6281	0.6633	0.6752	0.6685	0.6746
α_{v1i}	0.6393	0.6429	0.6795	0.6906	0.6874	0.6930

Pinion diameters [mm]						
d_{f1}	46.4023	46.6199	46.0842	46.9393	42.7657	43.9839
d_{b1d}	40.2960	40.2960	45.5553	45.5553	45.5553	48.4211
d_{u1d}	47.0415	47.2081	47.6268	48.0503	45.9484	48.4211
d_{p1d}	48.0814	48.2522	48.3316	48.8685	46.6426	48.5350
d_{b1i}	49.4304	49.4304	49.4304	49.4304	45.5553	48.4211
d_{u1i}	49.4309	49.4304	49.4306	49.4318	45.9484	48.4211
d_{p1i}	49.6512	49.7025	49.5832	49.7873	46.6426	48.5350
d_{rc1}	52.6027	52.1654	52.6027	51.4110	50.2647	52.6027
d_{w1}	52.6027	52.6027	52.6027	52.6027	52.6027	52.6027
d_{a1}	60.9191	61.0875	62.7299	63.3261	58.0503	62.0016

4. CONCLUSION

The asymmetric gears designing method, presented in this paper, use as initial data the number of teeth and the center distance, and as designing variable the different pressure angles, the number of the racks and the modification coefficient of the gear rack angle.

The results, obtained for different designing variant,emphasize the modification of the geometrical parameters, implicitly of the functional parameters, by chosing the designing variables mentioned before. It is necesary to establishe those variables, in relation with the designed transmission beneficiary requirements (increase of the efficiency, reduction of the bending stress, reduction of the contact stress, minimum transmission error).

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