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THE MATHEMATICAL MODEL FOR THE INTERNAL WORM GEARING WHIT CYLINDRICAL WORM

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***Abstract**: In this paper we present the mathematical model for an experimental driving whit small dimension and high transmission ratio. This driving is the internal worm gearing whit cylindrical worm.*
***Key words**: mathematical model, internal worm gearing whit cylindrical worm.*

1. INTRODUCTION

The internal worm gearing whit cylindrical worm it is an experimental driving on the researching.

This paper present the mathematical model for the internal worm gearing whit cylindrical worm. To obtain the mathematical model must be route the follow stage:

1. Assign the coordinate systems and the connections between them.
2. Determinate the transformations matrixes
3. Define the generator worm equation
4. Define the worm well mathematical equations.

2. THE COORDINATES SYSTEMS

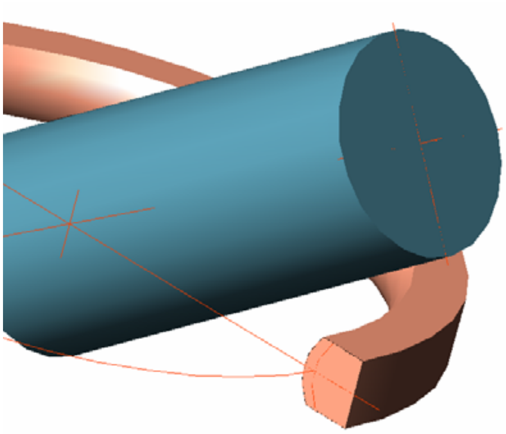
The coordinate systems for the mathematical model are presented in Picture 1. The coordinates systems are the follows:

- The $O_o x_o y_o z_o$ coordinate system – the functional reference system, considered fix;
- The $O_2 x_2 y_2 z_2$ coordinate system – the mobile reference system of the worm wheel;

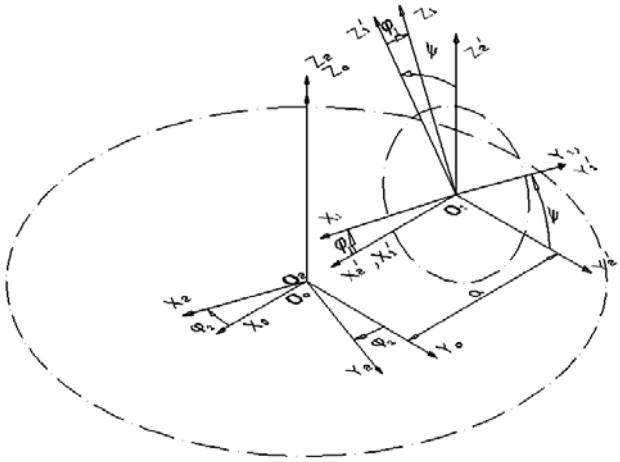
- The $O_2^{'}x_2^{'}y_2^{'}z_2^{'}$ coordinate system – the translated coordinate system whit the distance between the axis "a", on the direction of O_ox_o ax;

- The $O_1^{'}x_1^{'}y_1^{'}z_1^{'}$ coordinate system – the rotated coordinate system whit ψ=const angle.

- The $O_1x_1y_1z_1$ coordinate system – the mobile reference system of the worm.



Picture 1. The solid model of the internal worm gearing whit cylindrical worm



Picture 2. The coordinate systems for the mathematical model

The worm turn around $O_1'y_1'$ ax whit ω_1, and the worm wheel are turn around O_oz_o ax whit ω_2, angular velocity.

The transmission ratio are constant and is determinate by the equation:

$$i = i_{12} = \frac{1}{i_{21}} = \frac{\omega_1}{\omega_2} = \frac{\varphi_1}{\varphi_2} = \text{constant} \qquad \textbf{(1.)}$$

Where: ω_1, ω_2 – angular velocity for the worm and for the worm wheel;

φ_1, φ_2 – angular rotation for the worm and for the worm wheel.

The angular rotations are depending by the time. $\varphi_1 \cdot t = \varphi_2 \cdot i_{12} \cdot t$ **(2.)**

3. THE TRANSFORMATION MATRIX DETERMINATIONS

To define the worm it is two possibilities. If the surface of the worm it is not complicated (for example Archimedes worm) the mathematical equations define in $O_1^{'}x_1^{'}y_1^{'}z_1^{'}$ coordinate systems. ($T_{1'2}$ transformation) Else we define the generating curves of worm, which on moving the "P" point, are in the $O_1x_1y_1z_1$ mobile reference system of the worm. Thus the coordinates of the motion point from the coordinate system of the worm $O_1x_1y_1z_1$

must be transformed in to the coordinate system of the worm wheel $O_2x_2y_2z_2$. (T_{12} transformation)

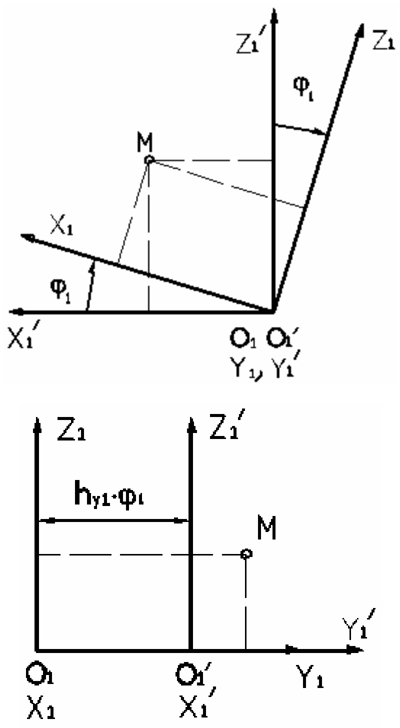
The transformations matrixes, which is the product of the elementary rotation and translation matrixes, are the following forms:

$$\mathbf{T_{12} = T_{1\,1'} \cdot T_{1'\,2'} \cdot T_{2'\,o} \cdot T_{0\,2}} \qquad (3.)$$

$$\mathbf{T_{1'\,2} = T_{1'\,2'} \cdot T_{2'\,o} \cdot T_{0\,2}} \qquad (4.)$$

The elementary transformations are a following:

Transformation $\mathbf{T_{1\,1'}}$



Picture 3.

$S_1 \rightarrow S_1'$

Rotation whit φ_1

$$\begin{cases} x_1' = x_1 \cdot \cos\varphi_1 + z_1 \cdot \sin\varphi_1 \\ y_1' = y_1 \\ z_1' = -x_1 \cdot \sin\varphi_1 + z_1 \cdot \cos\varphi_1 \end{cases}$$

$$T_{1\,1'\,rot} = \begin{bmatrix} \cos\varphi_1 & 0 & \sin\varphi_1 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\varphi_1 & 0 & \cos\varphi_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

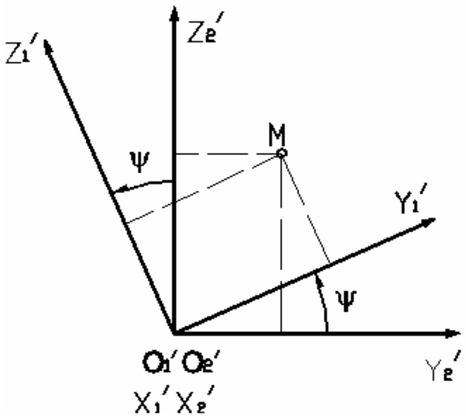
Translation

$$\begin{cases} x_1' = x_1 \\ y_1' = y_1 + h_{Y1} \cdot \varphi_1 \\ z_1' = z_1 \end{cases} \qquad T_{1\,1'\,trans} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & h_{Y1} \cdot \varphi_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$T_{1\,1'} = T_{1\,1'\,rot} \cdot T_{1\,1'\,trans}$

$$T_{1\,1'} = \begin{bmatrix} \cos\varphi_1 & 0 & \sin\varphi_1 & 0 \\ 0 & 1 & 0 & h_{Y1} \cdot \varphi_1 \\ -\sin\varphi_1 & 0 & \cos\varphi_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (5.)$$

Transformation $\mathbf{T_{1'\,2'}}$



Picture 4.

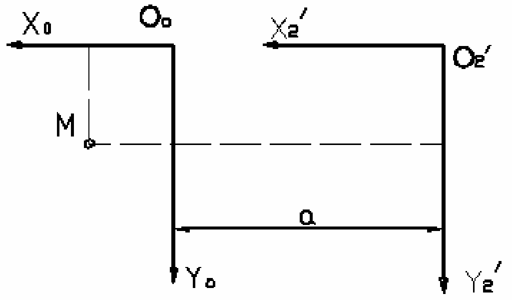
$S_1' \rightarrow S_2'$

Rotation whit ψ = const. $S2' = T_{1'2'} \cdot S_1'$

$$\begin{cases} x_2' = x_1' \\ y_2' = y_1' \cdot \cos\psi + z_1' \cdot \sin\psi \\ z_2' = -y_1' \cdot \sin\psi + z_1' \cdot \cos\psi \end{cases}$$

$$T_{1'2'} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\psi & \sin\psi & 0 \\ 0 & -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \textbf{(6.)}$$

Transformation $\mathbf{T_{2'o}}$



Picture 5.

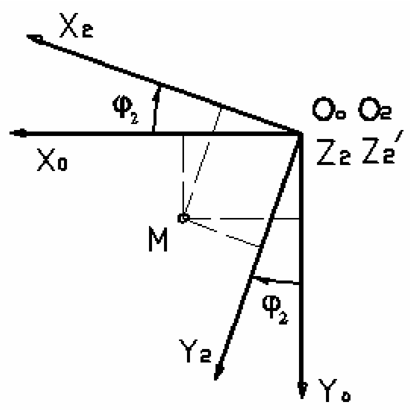
$S_2' \rightarrow S_o$

Translation whit distance between axis "a"

$S_o = T_{2'o} \cdot S_2'$

$$\begin{cases} x_o = x_2' + a \\ y_o = y_2' \\ z_o = z_2' \end{cases} \qquad T_{2'o} = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \textbf{(7.)}$$

Transformation $\mathbf{T_{0\,2}}$



Picture 6.

$S_o \rightarrow S_2$

Rotation whit φ_2 $S_2 = T_{o\,2} \cdot S_o$

$$\begin{cases} x_2 = x_o \cdot \cos\varphi_2 + y_o \cdot \sin\varphi_2 \\ y_2' = -x_o \cdot \sin\varphi_2 + y_o \cdot \cos\varphi_2 \\ z_2' = z_o \end{cases}$$

$$T_{o\,2} = \begin{bmatrix} \cos\varphi_2 & \sin\varphi_2 & 0 & 0 \\ -\sin\varphi_2 & \cos\varphi_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \textbf{(8.)}$$

Calculating the transformations matrixes T_{12} and $T_{1'2}$ the following result obtain:

$$\mathbf{T_{12} = T_{1\,1'} \cdot T_{1'\,2'} \cdot T_{2'\,o} \cdot T_{0\,2}}$$

$T_{12} =$

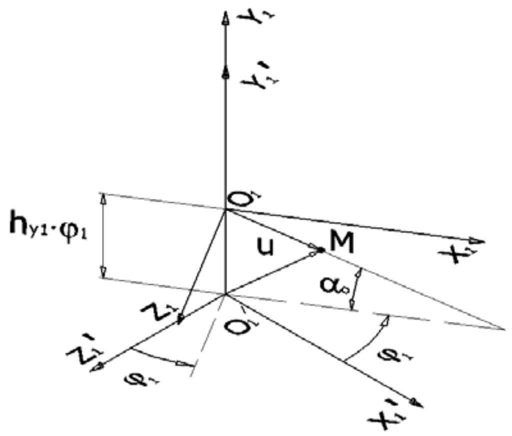
$$\begin{bmatrix} \cos\varphi_1 \cdot \cos\varphi_2 + \sin\varphi_1 \cdot \sin\psi \cdot \sin\varphi_2 & \cos\varphi_1 \cdot \sin\varphi_2 - \sin\varphi_1 \cdot \sin\psi \cdot \cos\varphi_2, & \sin\varphi_1 \cdot \cos\psi & \cos\varphi_1 \cdot a \\ -\cos\psi \cdot \sin\varphi_2 & \cos\psi \cdot \cos\varphi_2 & \sin\psi & hy1 \cdot \varphi_1 \\ -\sin\varphi_1 \cdot \cos\varphi_2 + \cos\varphi_1 \cdot \sin\psi \cdot \sin\varphi_2 & -\sin\varphi_1 \cdot \sin\varphi_2 - \cos\varphi_1 \cdot \sin\psi \cdot \cos\varphi_2 & \cos\varphi_1 \cdot \cos\psi & -\sin\varphi_1 \cdot a \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (9.)$$

$$\mathbf{T_{1'\,2} = T_{1'\,2'} \cdot T_{2'\,o} \cdot T_{0\,2}}$$

$$T_{1'2} = \begin{bmatrix} \cos\varphi_2 & \sin\varphi_2 & 0 & a \\ -\cos\psi \cdot \sin\varphi_2 & \cos\psi \cdot \cos\varphi_2 & \sin\psi & 0 \\ \sin\psi \cdot \sin\varphi_2 & -\sin\psi \cdot \cos\varphi_2 & \cos\psi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (10.)$$

4. EQUATION OF THE WORM

In case of the Archimedes worm, his equations determine in $O_1'x_1'y_1'z_1'$ coordinate system.



Picture 7.

The equation of the right surface of the worm in concordance whit the Picture 7:

$$\begin{cases} x' = u \cdot \cos\alpha_o \cdot \sin\varphi_1 \\ y' = h_{Y1} \cdot \varphi_1 - u \cdot \sin\alpha_o \\ z' = u \cdot \cos\alpha_o \cdot \cos\varphi_1 \end{cases} \text{ in matrix format } G = \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} \text{ or } G = \begin{bmatrix} u \cdot \cos\alpha_o \cdot \sin\varphi_1 \\ h_{Y1} \cdot \varphi_1 - u \cdot \sin\alpha_o \\ u \cdot \cos\alpha_o \cdot \cos\varphi_1 \\ 1 \end{bmatrix} \quad \mathbf{(11.)}$$

5. EQUATION OF THE WORM WHEEL

Perform the transformations of the equations from his coordinate system in the worm wheel coordinate system we obtain the equation of the worm wheel. The equation of transformation in vector form has a following:

$$\overline{r_2} = T'_{12} \cdot \overline{r'_1}\text{, and in matrix form } \begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{bmatrix} = T'_{12} \cdot \begin{bmatrix} x'_1 \\ y'_1 \\ z'_1 \\ 1 \end{bmatrix} \quad (12.)$$

Where: r_2, r_1 - the positional vectors of the "G" generator of the worm.

Replays the equations (10.) and (11.) in (12.) are calculable the mathematic equations of the worm wheel in the $O_2x_2y_2z_2$ coordinate system generated by the ZA type worm.

$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\varphi_2 & \sin\varphi_2 & 0 & a \\ -\cos\psi \cdot \sin\varphi_2 & \cos\psi \cdot \cos\varphi_2 & \sin\psi & 0 \\ \sin\psi \cdot \sin\varphi_2 & -\sin\psi \cdot \cos\varphi_2 & \cos\psi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} u \cdot \cos\alpha_o \cdot \sin\varphi_1 \\ h_{Y1} \cdot \varphi_1 - u \cdot \sin\alpha_o \\ u \cdot \cos\alpha_o \cdot \cos\varphi_1 \\ 1 \end{bmatrix} \quad (13.)$$

$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\varphi_2 \cdot u \cdot \cos\alpha_0 \cdot \sin\varphi_1 + \sin\varphi_2 \cdot (hy1 \cdot \varphi_1 - u \cdot \sin\alpha_0) + a \\ -\cos\psi \cdot \sin\varphi_2 \cdot u \cdot \cos\alpha_0 \cdot \sin\varphi_1 + \cos\psi \cdot \cos\varphi_2 \cdot (hy1 \cdot f1 - u \cdot \sin\alpha_0) + \sin\psi \cdot u \cdot \cos\alpha_0 \cdot \cos\varphi_1 \\ \sin\psi \cdot \sin\varphi_2 \cdot u \cdot \cos\alpha_0 \cdot \sin\varphi_1 - \sin\psi \cdot \cos\varphi_2 \cdot (hy1 \cdot \varphi_1 - u \cdot \sin\alpha_0) + \cos\psi \cdot u \cdot \cos\alpha_0 \cdot \cos\varphi_1 \\ 1 \end{bmatrix} \quad (14.)$$

The equations define the left surface of the internal worm wheel generated by the worm right surface whit the left hand thread.

To obtain the right surface of the internal worm wheel the α_o muss to modify in $-\alpha_o$

The equation for the right hand thread is obtained modified the h_{Y1} coefficient in $-h_{Y1}$

6. REFERENCE

1. Boloş, V. *Angrenaje melcate spiroide. Danturarea roţilor plane*. Editura Universităţii "Petru Maior", Tîrgu Mureş, 1999.
2. Drobni, I. *Korszerű csigahajtások*. Editura Tenzor Kft., Miskolc, 2001.
3. Dudas, I., Theory & Practice of Worm Gear Drives. Penton Press, London, 2000.
4. Litvin, F., L. *A fogaskerek kapcsolas elmelete*. Muszaki Konyvkiado, Budapest, 1972.
5. Liuksin, . Teoria suprafeţelor elicoidale. (traducere din limba rusă).
6. Pay, E., Ravai Nagy, S., Pay, G. *Mathematical model for internal worm gearing whit parallel axis*. MicroCAD 2005. International Scientific Conference, University of Miskolc, Section K, p.75-79, Miskolc, 2005.
7. Páy, G. *Belso csigás hajtások* (PhD thesis). University of Miskolc, Hungary, 2001.