**THE INTERNATIONAL CONFERENCE OF THE CARPATHIAN EURO-REGION SPECIALISTS IN INDUSTRIAL SYSTEMS**
**6th edition**

# DEFINING TRANSMISSION FUNCTIONS AND PROPER FREQUENCIES OF DRILLING STRINGS

***V. Moisyshyn[1], Z. Kulynyn[2],***
*[1]Professor PhD. Eng., [2]Eng.*
*at Ivano-Frankivsk National Technical University of Oil and Gas,*
*Ukraine, Ivano-Frankivsk, 15, Karpatska str.*

**Abstract***: In this article the methodology of defining proper frequencies; transmission functions and coefficients of drilling strings dynamics with vibroprotective and back-up centered devices is being offered.*
**Key words:** *proper frequencies, drilling string transmission functions, dynamics coefficient, vibroprotective device, back-up centered device.*

While studying the oscillations of drilling mechanical system the drilling string is being modelled by a step-up rod with parameters dispensed lengthwise. The movement of the rod can be described by an operator equation taken from the theory of elasticity.

$$M\left\{\ddot{U}\right\}+L\left\{\dot{U}\right\}+K\left\{U\right\}=\left\{F\right\}, \tag{1}$$

where *M, K, L* – respectively inertial, dissipative and elastic matrix operators, $\left\{U\right\}$ – removal, $\left\{\dot{U}\right\}$ – rapidity, $\left\{\ddot{U}\right\}$ – acceleration, { F} – strengths. For the case of longitudinal oscillations let's take operators *K, M, L* like

$$M=\rho\,,\;L=h\,,\;K=-\frac{\partial}{\partial x}\left(AE\frac{\partial}{\partial x}\right),$$

where $\rho$ – weight of a string pipe unit,

$A$ – cross-section area,

$E$ – coefficient of elasticity,

$h$– coefficient of rigid friction during the drilling mud and the pipe interaction,

$x$ – moving reference.

While modeling let's consider the calculated scheme shown on Fig. 1. It includes the tackle system 1, shown by the weight of moving parts $m_0$ and stiffness $k_0$, the section of drilling 2 and heavy drilling 3,5,7 pipes, vibroprotective device (VPD) 4 with stiffness $k_m$, back-up centered device (BCD) 6 with the weight $m_n$ and the friction coefficient $\chi_n$, roller bit 8 and the drilling rock 9 with stiffness $k_l$. Let's connect current references $x_k\left(k=\overline{1,l}\right)$ with the upper end-walls of respective strings sections and direct down to the bit.

Let's set boundary conditions:

– for all boreholes

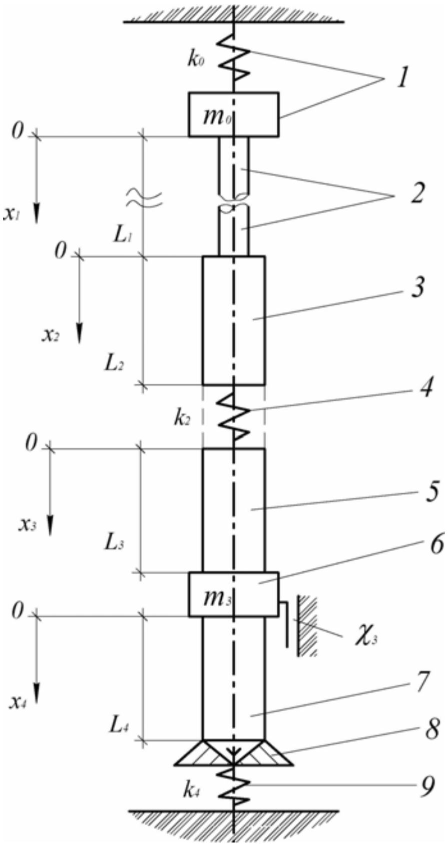


Fig. 1 – Calculation scheme

$$x = 0 \qquad A_1 E_1 \frac{\partial U_1}{\partial x_1} = m_0 \frac{A_1 E_1}{\rho_1} \frac{\partial^2 U_1}{\partial x_1^2} + k_0 U_1$$

– at joining sections without intermediate members

$$\left.\begin{aligned} x_r &= L_r, \\ x_{r+1} &= 0 \end{aligned}\right\}; \; U_r = U_{r+1} \; ; \; A_r E_r \frac{\partial U_r}{\partial x_r} = A_{r+1} E_{r+1} \frac{\partial U_{r+1}}{\partial x_{r+1}} ;$$

– at joining sections via elastic element (VPD)

$$\left.\begin{aligned} x_m &= L_m, \\ x_{m+1} &= 0 \end{aligned}\right\} \qquad k_m \left(U_m - U_{m+1}\right) = A_m E_m \frac{\partial U_m}{\partial x_m}, \; A_m E_m \frac{\partial U_m}{\partial x_m} = A_{m+1} E_{m+1} \frac{\partial U_{m+1}}{\partial x_{m+1}} ;$$

– at joining sections via (BCD)

$$\left.\begin{aligned} x_n &= L_n, \\ x_{n+1} &= 0 \end{aligned}\right\} \qquad U_n = U_{n+1}, \; A_{n+1} E_{n+1} \frac{\partial U_{n+1}}{\partial x_{n+1}} - A_n E_n \frac{\partial U_n}{\partial x_n} = m_n \frac{A_{n+1} E_{n+1}}{\rho_{n+1}} \frac{\partial^2 U_{n+1}}{\partial x_{n+1}^2} ;$$

– at bit

$$x_l = L_l \qquad A_l E_l \frac{\partial U_l}{\partial x_l} - k_l U_l = 0 .$$

If the underbody of the arrangement is free or fastened, the last condition should be changed into

$$x_l = L_l \qquad A_l E_l \frac{\partial U_l}{\partial x_l} = 0 \qquad \text{or} \qquad x_l = L_l \qquad U_l = 0 .$$

While solving the mathematical physics problem for four-section drilling string by developing the equation (1) in a row by their fundamental functions $X_{jk}\left(x_k\right)$

$\left(k=\overline{1,4}; r=1, m=2, n=3, l=4\right)$ we find proper frequencies $p_j$ form the condition

$$\begin{vmatrix} a_{11} & a_{12} & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & -1 & 0 & 0 & 0 & 0 & 0 \\ a_{31} & a_{32} & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{43} & a_{44} & 1 & 0 & 0 & 0 \\ 0 & 0 & a_{53} & a_{54} & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{65} & a_{66} & -1 & 0 \\ 0 & 0 & 0 & 0 & a_{75} & a_{76} & a_{77} & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & a_{87} & a_{88} \end{vmatrix} = 0, \qquad (2)$$

where $S_k = \sin\left(\frac{p}{a}L_k\right)$, $K_k = \cos\left(\frac{p}{a}L_k\right)$,

$a$ – the rate of elastic waves propagation in the pipes material,

$$a_{11} = \frac{m_0 p^2 - k_0}{A_1 E_1},\ a_{12} = \frac{p}{a},\ a_{21} = K_1,\ a_{22} = S_1,\ a_{31} = -\frac{A_1 E_1}{A_2 E_2} S_1,\ a_{32} = \frac{A_1 E_1}{A_2 E_2} K_1,$$

$$a_{43} = -\frac{A_2 E_2}{k_a}\frac{p}{a} S_2 - K_2,\ a_{44} = \frac{A_2 E_2}{k_a}\frac{p}{a} K_2 - S_2,\ a_{53} = -\frac{A_2 E_2}{A_3 E_3} S_2,\ a_{53} = \frac{A_2 E_2}{A_3 E_3} K_2,$$

$$a_{65} = K_3,\ a_{66} = S_3,\ a_{75} = \frac{A_3 E_3}{A_4 E_4} S_3,\ a_{76} = -\frac{A_3 E_3}{A_4 E_4} K_3,\ a_{77} = \frac{m_3}{\rho_4}\frac{p}{a},$$

$$a_{87} = K_4 + \frac{A_4 E_4}{k_4}\frac{p}{a} S_4,\ a_{88} = S_4 - \frac{A_4 E_4}{k_4}\frac{p}{a} K_4.$$

One of the most important dynamic characteristics of the drilling string during lengthwise oscillations are the transmitting functions of an arrangement which mean the proportion of the axial removal (or strength) range at the mouth of a borehole to the respective amplitude in $x_k$ cross-section of an arrangement. After rewriting boundary conditions at the bit

$$x = L_l \quad U_l = b \cdot \sin pt,$$

the dynamic constituent of removal is given as follows

$$U_k\left(x_k, t\right) = \left(C_k \cos\frac{p x_k}{a} + D_k \sin\frac{p x_k}{a}\right)\sin pt,\ k = \overline{1,4}, \qquad (3)$$

where $p$ – forced oscillations frequency.

The constants $C_k$, $D_k$ $\left(k=\overline{1,4}\right)$ are found from the system

$$\left.\begin{aligned}
& A_1 E_1 D_1 \frac{p}{a} = C_1\left(k_0 - m_0 \frac{A_1 E_1}{\rho_1}\frac{p^2}{a^2}\right), \\
& C_1 K_1 + D_1 S_1 = C_2, \\
& A_1 E_1\left(D_1 K_1 - C_1 S_1\right) = A_2 E_2 D_2, \\
& k_2\left(C_2 K_2 + D_2 S_2 - C_3\right) = A_2 E_2 \frac{p}{a}\left(D_2 K_2 - C_2 S_2\right), \\
& A_2 E_2\left(D_2 K_2 - C_2 S_2\right) = A_3 E_3 D_3, \\
& C_3 K_3 + D_3 S_3 = C_4, \\
& A_4 E_4 D_4 - A_3 E_3\left(D_3 K_3 - C_3 S_3\right) = -m_3 \frac{A_4 E_4}{\rho_4}\frac{p}{a} C_4, \\
& C_4 K_4 + D_4 S_4 = b,
\end{aligned}\right\}$$

acquired as a result of substitution (3) in boundary conditions. One of the constants is previously considered as the given quantity (e.g. $C_1 = 1$).

The transmission functions of the drilling string for the removal $\Phi^U(x_k)$ and the strength $\Phi^F(x_k)$ are defined by the formulae

$$\Phi^U(x_k) = \frac{C_1}{C_k \cos\frac{px_k}{a} + D_k \sin\frac{px_k}{a}}, \tag{4}$$

$$\Phi^F(x_k) = \frac{A_1 E_1 D_1}{A_k E_k \frac{p}{a}\left(D_k \cos\frac{px_k}{a} - C_k \sin\frac{px_k}{a}\right)}. \tag{5}$$

The dynamic constituents of axial strength and the stiffness in $x_k$ cross-section are connected with the transmission function $\Phi^F(x_k)$

$$F_k(x_k, t) = \frac{A_1 E_1 D_1}{\Phi^F(x_k)} \sin pt\,,\ \sigma_k(x_k, t) = \frac{A_1 E_1 D_1}{\Phi^F(x_k) A_k} \sin pt\,.$$

The dynamic coefficient of the drilling string is given as follows

$$K_d = \frac{A_1 E_1 D_1}{\Phi^F(L_4)(P_l + p_0 S_0)},$$

where $P_l + p_0 S_0$ – is a static compound of the axial load at the hollow subject to the pressure loss at the bit.

For defining the numeric notions $p_j$ the enumerative technique has been chosen. This chose has been stipulated by a great number of radicals placed close together. For solution of the equation (2) we work out a program realising the Jordan – Gauss strategy reduces the matrix to diagonal kind. As a result the coefficient is calculated as a sum of diagonal elements.

Starting data for the three arrangements are given in Table 1. Other basic values are accepted as: $m_0$= 8650 $kg$, $k_0 = 53 \cdot 10^6\, N/m$, $E = 2{,}06 \cdot 10^{11}\, N/m$. The stiffness of buffer $k_a = 20 \cdot 10^6\, N/m$, $a = 5100\, m/s$, plastic viscosity of the drilling mud is $\eta_p = 0{,}02 Pa \cdot s$.

*Table 1* – Drilling strings arrangements

| **Borehole number** | **Arrangements** |
|---|---|
| 1 | Bit III215,9 C3; HDS 146-112m; DS 127x9,2 – 1800m. |
| 2 | Bit III295,3 MC3; HDS 203-160m; DS 127x9,2 – 1500m. |
| 3 | Bit III295,9 C; HDS 229-112m; DS 140x10 – 1200m. |

The results of calculations are given in Tables 2-4. As it may be seen the engagement of VBD into drilling string arrangement leads to the increase of numeric notions of proper frequencies. This increase becomes more substantial with the increase of the ordinal number of j frequency. The following conformity can also be traced: the less is the stiffness $k_a$ of the buffer springing element, the more are the numeric notions of respective proper frequencies. The $p_j$ values also increase with the removal the VBD from the bit, while this increase becomes noticeable at each increase of the frequency ordinal number.

*Table 2 – Proper frequencies of drilling strings lengthwise oscillations*

| **Borehole number** | **Frequencies, Hz** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ | $p_7$ | $p_8$ | $p_9$ | $p_{10}$ |
| **1** | 3,71 | 11,34 | 19,36 | 27,64 | 36,06 | 44,53 | 52,95 | 61,21 | 69,06 | 76,18 |
| **2** | 3,34 | 11,88 | 21,77 | 31,98 | 42,23 | 52,38 | 62,21 | 71,22 | 79,13 | 87,16 |
| **3** | 3,93 | 14,60 | 27,03 | 39,77 | 52,42 | 64,48 | 74,89 | 84,32 | 95,50 | 107,83 |

*Table 3* – The influence of VPD elastic element on proper frequencies of the drilling string lengthwise oscillations (borehole 2, $L_{HDS} = 96$ m, $L_a = 16$ m)

| **Stiffness of VPD $k_a$, N/m** | **Frequencies, Hz** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ | $p_7$ | $p_8$ | $p_9$ | $p_{10}$ |
| $5 \cdot 10^6$ | 3,86 | 12,57 | 22,33 | 32,46 | 42,69 | 52,84 | 62,69 | 71,75 | 79,76 | 88,26 |
| $20 \cdot 10^6$ | 3,86 | 12,55 | 22,30 | 32,42 | 42,63 | 52,78 | 62,65 | 71,71 | 79,73 | 88,22 |
| $40 \cdot 10^6$ | 3,86 | 12,55 | 22,29 | 32,41 | 42,63 | 52,78 | 62,64 | 71,69 | 79,71 | 88,20 |
| **without VPD** | 3,86 | 12,55 | 22,28 | 32,40 | 42,61 | 52,76 | 62,61 | 71,66 | 79,68 | 88,15 |

*Table 4* – The impact of the VPD seating on proper frequencies of the drilling string lengthwise oscillations (borehole 2, $L_{HDS} = 96$ m, $k_a = 20 \cdot 10^6$ N/m)

| **The VPD seating $L_a$, m** | **Frequencies, Hz** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ | $p_7$ | $p_8$ | $p_9$ | $p_{10}$ |
| 0 | 3,86 | 12,57 | 22,28 | 32,40 | 42,61 | 52,76 | 62,61 | 71,66 | 79,68 | 88,15 |
| 16 | 3,86 | 12,55 | 22,30 | 32,42 | 42,64 | 52,80 | 62,65 | 71,71 | 79,73 | 88,22 |
| 32 | 3,86 | 12,57 | 22,34 | 32,48 | 42,72 | 52,88 | 62,75 | 71,80 | 79,82 | 88,34 |
| 48 | 3,86 | 12,60 | 22,40 | 32,58 | 42,83 | 53,01 | 62,88 | 71,93 | 79,84 | 88,49 |
| 64 | 3,86 | 12,64 | 22,50 | 32,71 | 43,01 | 53,20 | 63,08 | 72,11 | 80,12 | 88,70 |
| 80 | 3,87 | 12,69 | 22,62 | 32,91 | 43,26 | 53,50 | 63,40 | 72,41 | 80,43 | 89,07 |
| 96 | 3,87 | 12,76 | 22,79 | 33,20 | 43,68 | 54,06 | 64,06 | 73,10 | 81,20 | 90,08 |

Calculations testify that the change in length of a weighted underbody, weight $m_0$ of moving elements of the tackle system and stiffness $k_l$ of the drilling rocks do not influence greatly proper frequencies of lengthwise oscillations. The difference in values $p_j \left(j = \overline{1,10}\right)$ does not exceed 1,2%. Proper frequencies are less sensitive to less stiff rocks. Thus, for example, the difference in values of base frequency at transfer from $50 \cdot 50^6$ to $100 \cdot 10^6$ *N/m* totals 0,24 – 0,27 %, and at transfer from $100 \cdot 10^6$ to $150 \cdot 10^6$ *N/m* the latter is 0,075 – 0,077 % only. At last it has been noted that considering the viscous external friction (for drilling pipes *h* changes within the boundaries of 0,4 – 7,6 *N/m · s/m*, for HDS – 1 – 50 *N/m · s/m*) leads to minor (less than 0,14 %) break-up of the base and damping frequencies.

The investigation results show that by means of fitting the stiffness $k_a$ and the seating of VPD the level of dynamic arrangement can be changed within quite broad boundaries. Thus, for the arrangement 2 at "ground" frequency at buffer stiffness change within the limits of $5 \cdot 10^6 - 50 \cdot 10^6\ N/m$ and at the VPD removal from 1 to 100 *m*, the coefficient $K_d$

comprised 0,12 – 0,35. The increase of a string length of HDS from 140 to 180 *m* let us enhance the dynamic behaviour of arrangement 2 in 1,2 – 1,5 times.

The obtained conclusions will be of a great use while studying free and forced oscillations of a drilling string, investigating transient processes and resonant modes of operation of the drilling mechanical system, solving the problems of eliminating the captures of drilling tools.

## BIBLIOGRAPHY


1. Мойсишин В.М. Основи механіки бурильної колони при поглибленні свердловин роторним способом: Дис. ... докт.техн. наук. – Івано-Франківськ, 1996, 498 с.
2. Василенко Н.В. Теория колебаний: Учебн. пособие. – К.: Вища школа, 1992, 430 с.