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# MATHEMATICAL MODEL FOR ESTABLISHING THE VIBRATING CONDITIONS SPECIFIC TO ELECTROMAGNETIC SIEVES 

Mircea Horgos ${ }^{1}$ Mihai Banica ${ }^{2}$, Dinu Stoicovici ${ }^{3}$<br>${ }^{1}$ Lecturer Eng., ${ }^{2,3}$ Lecturer Eng. Ph.D.<br>North University of Baia Mare


#### Abstract

The theoretic bases of some mathematical models for the sorting processes on vibrating sieves were established once the modern calculation equipment appeared. The present paper presents a simplified calculation model for vibrating conditions specific to the sieving of pulverized materials on electromagnetic sieves.


Key words: sorting process, electromagnetic sieves, false position method

## 1. ESTABLISHING THE JUMP CONDITIONS

Let us consider an electromagnetic vibrator, with sieves tilted to angle $\alpha$ in reference to the horizontal line of the place, with the mobile coordinate system axes $x O_{1} y$, solitary with the sieve (axis $O x$ oriented along the sieve) and axes $\xi O \eta$ of the fixed coordinate system, as shown in figure 1.


Figure 1. The fixed and mobile reference axes system

We shall approach the case of these conditions with a single jump of the grain of material at an oscillation period of the perturbing force. Let us further consider types of intervals symbolized with "I" for the moments when the material is on the sieve and move solitary with it, and symbolized with "II" those moments when the material is in mid-jump on the sieve. Thus, for the case in the above figure we shall have the differential equation of the movement in interval I as follows:

$$
\begin{equation*}
\ddot{\eta}+\frac{k_{\eta}}{m_{1}+m_{2}} \cdot \eta=\frac{F_{\eta}}{m_{1}+m_{2}} \cdot(1+\cos \omega t)-g \cdot \cos \alpha \tag{1}
\end{equation*}
$$

The following notations are made:

$$
\begin{align*}
& z=\frac{m_{1} \cdot \omega_{1}^{2}}{\mathrm{~F}_{\eta}} \cdot \eta ; \quad z^{\prime}=\frac{d z}{d \tau} ; \quad \tau=\omega t ; \quad \omega_{1}=\sqrt{\frac{k_{\eta}}{m_{1}}} ;  \tag{2}\\
& \mathrm{k}_{1}=\frac{\omega_{1}}{\omega} ; \quad \mathrm{k}_{\mathrm{m}}=\frac{\mathrm{m}_{2}}{\mathrm{~m}_{1}} ; \quad \mathrm{n}_{1}=\frac{\mathrm{m}_{1} \cdot g \cdot \cos \alpha}{\mathrm{~F}_{\eta}}
\end{align*}
$$

With these notations the above equations becomes:

$$
\begin{equation*}
z^{\prime \prime}+\frac{k_{1}^{2}}{1+k_{m}} \cdot z=\frac{k_{1}^{2}}{1+k_{m}} \cdot(1+\cos 2 \tau)-k_{1}^{2} \cdot n_{1} \tag{3}
\end{equation*}
$$

The solution of this equation is:

$$
\begin{equation*}
z_{1}=A_{1} \cdot \sin \left(\frac{k_{1}}{\sqrt{1+k_{m}}} \cdot \tau\right)+B_{1} \cdot \cos \left(\frac{k_{1}}{\sqrt{1+k_{m}}} \cdot \tau\right)-\frac{k_{1}{ }^{2}}{4+4 \cdot k_{m}-k_{1}{ }^{2}} \cos 2 \tau+1-n_{1}\left(1+k_{m}\right) \tag{4}
\end{equation*}
$$

The movement of the sieve after the material lifts at moment $\tau_{0}$ is given by equation:

$$
\begin{equation*}
\ddot{\eta}+\frac{k_{\eta}}{m_{1}} \cdot \eta=\frac{F_{\eta}}{m_{1}} \cdot(1+\cos 2 \omega t)-g \cdot \cos \alpha \tag{5}
\end{equation*}
$$

We again adopt the notations in (1.2), and we obtain the following equation:

$$
\begin{equation*}
z^{\prime \prime}+k_{1}^{2} \cdot z=k_{1}^{2} \cdot(1+\cos 2 \tau)-k_{1}^{2} \cdot n_{1} \tag{6}
\end{equation*}
$$

The solution of the equation is:

$$
\begin{equation*}
z_{\| I}=A_{\| I} \cdot \sin \left(k_{1} \cdot \tau\right)+B_{\|} \cdot \cos \left(k_{1} \cdot \tau\right)-\frac{k_{1}^{2}}{4-k_{1}^{2}} \cdot \cos 2 \tau+1-n_{1} \tag{7}
\end{equation*}
$$

## 2. ESTABLISHING THE MATHEMATICAL MODEL

In order to determine the oscillating movement equations in the specific field of pulverized materials sorting, we further determine the constants $A_{\|}, B_{\|}, A_{\|}, B_{\|}$which appear
in relations (4) and (7). Determining the value of these constants was made using the nomograms in [2]. The values of the constants extracted from these nomograms are given in accordance with the throw coefficient „, c " in the table bellow (Table 1).

Table 1: The values of coefficients $A_{I}, B_{I}, A_{I I}, B_{I I}$

| $\mathbf{C}$ | A1 | B1 | A2 | B2 |
| :--- | :--- | :--- | :--- | :--- |
| $\mathbf{2 . 5}$ | 0.042 | -0.4 | -0.05 | -0.168 |
| $\mathbf{2 . 5 5}$ | 0.05 | -0.425 | -0.04 | -0.173 |
| $\mathbf{2 . 6}$ | 0.06 | -0.44 | -0.035 | -0.175 |
| $\mathbf{2 . 6 5}$ | 0.08 | -0.45 | -0.032 | -0.185 |
| $\mathbf{2 . 7}$ | 0.09 | -0.465 | -0.025 | -0.19 |
| $\mathbf{2 . 7 5}$ | 0.11 | -0.475 | 0 | -0.195 |
| $\mathbf{2 . 8}$ | 0.12 | -0.49 | 0.03 | -0.2 |
| $\mathbf{2 . 8 5}$ | 0.14 | -0.5 | 0.04 | -0.205 |
| $\mathbf{2 . 9}$ | 0.143 | -0.515 | 0.042 | -0.21 |
| $\mathbf{2 . 9 5}$ | 0.147 | -0.525 | 0.048 | -0.215 |
| $\mathbf{3}$ | 0.15 | -0.545 | 0.05 | -0.22 |
| $\mathbf{3 . 0 5}$ | 0.154 | -0.555 | 0.053 | -0.215 |
| $\mathbf{3 . 1}$ | 0.156 | -0.558 | 0.055 | -0.215 |
| $\mathbf{3 . 1 5}$ | 0.159 | -0.575 | 0.058 | -0.21 |
| $\mathbf{3 . 2}$ | 0.16 | -0.585 | 0.059 | -0.21 |
| $\mathbf{3 . 2 5}$ | 0.162 | -0.61 | 0.06 | -0.21 |
| $\mathbf{3 . 3}$ | 0.165 | -0.620 | 0.068 | -0.215 |

Using relation

$$
\begin{align*}
& A_{I} \cdot \sin \left(\frac{k_{1}}{\sqrt{1+k_{m}}} \cdot \tau_{0}\right)+B_{I} \cdot \cos \left(\frac{k_{1}}{\sqrt{1+k_{m}}} \cdot \tau_{0}\right)-\frac{k_{1}^{2}}{4+4 k_{m}-k_{1}^{2}} \cdot \cos 2 \tau_{0}-n_{1} \cdot\left(1+k_{m}\right)= \\
& =A_{I I} \cdot \sin \left(k_{1} \cdot \tau_{0}\right)+B_{\| I} \cdot \cos \left(k_{1} \cdot \tau_{0}\right)-\frac{k_{1}^{2}}{4-k_{1}^{2}} \cdot \cos 2 \tau_{0}+1-n_{1} \tag{8}
\end{align*}
$$

we will determine the moment when the throw of the grain off the sieve takes place. The left side member of the equation represents the movement of the mobile frame with the material-to-be-sieved on it, and the right side member represents the movement of the mobile frame without the material-to-be-sieved on it. Relation (8) represents the space continuity condition at the moment the grain is thrown, that is to say that the position of the mobile frame
immediately before the grain lifts must coincide with its position immediately after the grain lifts.

Because equation (8) is a transcendental one, it can be solved through the so-called method of the "false position" with the help of a proper calculation algorithm. The algorithm is designed with the help of the programmer MatLab ([1], [3], [4]).

The false position method consists in using an equality between two derivative functions. In this case we use, as mentioned, relation (8). In this case the difference between the two functions results from equation (9), where the unknown is $\tau_{0}$ - the moment of the grain's jump off the sieve:

$$
\begin{align*}
g\left(\tau_{0}\right)= & A_{I} \cdot \sin \left(\frac{k_{1}}{\sqrt{1+k_{m}}} \cdot \tau_{0}\right)+B_{1} \cdot \cos \left(\frac{k_{1}}{\sqrt{1+k_{m}}} \cdot \tau_{0}\right)-\frac{k_{1}^{2}}{1+k_{m}-k_{1}^{2}} \cdot \sin \tau_{0}-n_{1} \cdot\left(1+k_{m}\right)- \\
& -A_{\| I} \cdot \sin \left(k_{1} \cdot \tau_{0}\right)+B_{\| I} \cdot \cos \left(k_{1} \cdot \tau_{0}\right)-\frac{k_{1}^{2}}{1-k_{1}^{2}} \cdot \sin \tau_{0}-n_{1}=0 \tag{9}
\end{align*}
$$

The solution can be determined by linear approximation between two given values of equation (9), as in figure 2 :


Figure 2. Scheme for exemplifying the false position method

The approximation is done in the interval generated by two values given to the functions within the equations $\left[\mathrm{g}\left(\tau_{0_{-} 1}\right), \mathrm{g}\left(\tau_{0_{-}}\right)\right.$]. Generating the values is done precisely through the programmer, and the root can be determined through linear approximation. After each iteration, the domain shrinks so that the new domain determined by the new values of [ $\left.g\left(\tau_{0_{-} 1}\right), g\left(\tau_{0 \_2}\right)\right]$ contains the root as well. The latter is obtained by replacing $g\left(\tau_{0 \_1}\right)$ or $g\left(\tau_{0 \_}\right)$, depending on their positive or negative sign with a value calculated with the formula:

$$
\begin{equation*}
\tau_{0 \_} \mathrm{S}=\tau_{0_{-} 1}+\left(\tau_{0_{-} 2}-\tau_{0_{-} 1}\right) \cdot\left(\frac{-g\left(\tau_{0 \_} 1\right)}{g\left(\tau_{0 \_} 2\right)-g\left(\tau_{0_{-}} 1\right)}\right) \tag{10}
\end{equation*}
$$

The calculation programmer for solutions $\tau_{0}$ of the transcendental equation (9) is the following:

```
        function vibrating_movement
        clc
        %constants
        a1= 0.042;
        b1=-0.4;
        a2=-0.05;
        b2=-0.168;
        k1=0.1;
        km=0.4;
        % solving the equation
        ind=0;
        for c=2.5:0.05:3.3;
        ind=ind+1;
        [tauo_fin] = rezolvare_ecuatie_1(a1,k1,km,b1,c,a2,b2);
    tauo_fin_(ind)=tauo_fin;
    %afisare rezultate finale
end
tauo_fin
disp('FINISH')
save ('matlabM')
function [tauo_fin] = rezolvare_ecuatie_1(a1,k1,km,b1,c,a2,b2)
%valoare presupusa
tauo_0=pi;
tauo_1=tauo_0;
tauo_2=tauo_0;
% stabilirea lui Z1 si Z2
contor 1=0;
while 1
    contor 1=contor1+1;
    tauo_2=tauo_2+0.1*tauo_0;
    Z1=a}1*\operatorname{sin}(\textrm{k}1*\mathrm{ *tauo_1/sqrt( }1+\textrm{km}))+\textrm{b}1*\operatorname{cos}(\textrm{k}1*\mathrm{ tauo_ 1/sqrt(1+km))-k1^2/(4+4*km-
k1^2)*\operatorname{cos}(2*(tauo_1))-(1+km)/c-(a2*sin(k1*tauo_1)+b2*}\operatorname{cos}(\textrm{k}1*\mathrm{ tauo_1)-k1^2/(4-k ^^2)*}\operatorname{cos}(2*(tauo_1))-1/c)
    Z2=a1*}\operatorname{sin}(\textrm{k}1*\mathrm{ tauo_2/sqrt(1+km))+b1*}\operatorname{cos}(\textrm{k}1*\mathrm{ tauo_2/sqrt(1+km))-
k1^2/(4+4*km+k1^2)*\operatorname{cos}(2*(tauo_2))-(1+km)/c-(a2*sin(k1*tauo_2)+b2*\operatorname{cos}(\textrm{k}1*\mathrm{ tauo_2)-k1^2/(4-}
k1^2)*}\operatorname{cos(2*(tauo_2))-1/c);
            if Z1*Z2}<
            break
            end
        end
        %metoda falsei poziii
        contor2=0;
        while 1
        contor2=contor2+1;
        Z1=a1*sin(k1*tauo_1/sqrt(1+km))+b1*\operatorname{cos}(\textrm{k}1*\mathrm{ tauo_ 1/sqrt(1+km))-k1^2/(4+4*km-}
k1^2)*}\operatorname{cos}(2*(tauo_1))-(1+km)/c-(a2*sin(k1*tauo_1)+b2*\operatorname{cos}(\textrm{k}1*\mathrm{ tauo_1)-k1^2/(4-k1^2)*}\operatorname{cos}(2*(tauo_1))-1/c)
            Z2=a1*sin(k1*tauo_2/sqrt(1+km))+b1*cos(k1*tauo_2/sqrt(1+km))-
k1^2/(4+4*km+k1^2)*\operatorname{cos}(2*(tauo_2))-(1+km)/c-(a2*sin(k1*tauo_2)+b2*\operatorname{cos}(\textrm{k}1*\mathrm{ tauo_2)-k1^2/(4-}
k1^2)*}\operatorname{cos}(2*(tauo_2))-1/c)
            if abs(Z1)<10^-12
        tauo_s=tauo_1;
        break
        end
```

```
    if abs(Z2)<10^-12
        tauo_s=tauo_2;
        break
    end
    tauo_s=tauo_1+(tauo_2-tauo_1)*(-Z1/(Z2-Z1));
```



```
k1^2)*}\operatorname{cos}(2*(tauo_s))-(1+km)//c-(a2*\operatorname{sin}(\textrm{k}1*\mathrm{ tauo_s)+b2*}\operatorname{cos}(\textrm{k}1*\mathrm{ tauo_s)-k1^2/(4-k1^2)*}\operatorname{cos}(2*(tauo_s))-1/c)
    if abs(Zs)<10^-12
        tauo_s=tauo_s;
        break
    end
    if Zs*Z1>0
        tauo_1=tauo_s;
    else
        tauo_2=tauo_s;
    end
end
%valoarea finala
tauo_fin=tauo_s;
```


## 3. CONCLUSION

With the help of this programmer we calculated the values of the lift moment of the material grain lift for values of the throw coefficient from $c=2,5$ to $c=3,3$. For each value given to the throw coefficient „c", the corresponding values of coefficients $A_{l}, B_{I}, A_{I I}, B_{\|}$ were given. The values thus obtained for the throw moment are provided in the table bellow (table 2):

Table 2. The values of the throw moments of the grain on the sieve.

| $\mathbf{c}$ | 2.50 | 2.55 | 2.60 | 2.65 | 2.70 | 2.75 | 2.80 | 2.85 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\tau_{\text {aruncare }}$ | $\mathbf{2 6 . 8 5 5}$ | $\mathbf{2 5 . 6 0 8}$ | $\mathbf{2 4 . 6 4 5}$ | $\mathbf{2 3 . 7 4 7}$ | $\mathbf{2 3 . 1 8 6}$ | $\mathbf{2 3 . 0 0 8}$ | $\mathbf{2 3 . 4 0 4}$ | $\mathbf{2 2 . 7 3 8}$ |

Table 2 (continuation)

| $\mathbf{c}$ | 2.90 | 2.95 | 3.00 | 3.05 | 3.10 | 3.15 | 3.20 | 3.25 | 3.30 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\tau_{\text {aruncare }}$ | $\mathbf{2 2 . 4 8 4}$ | $\mathbf{2 2 . 4 1 6}$ | $\mathbf{2 2 . 1 4 3}$ | $\mathbf{2 1 . 7 8 9}$ | $\mathbf{2 1 . 6 7 6}$ | $\mathbf{2 1 . 3 6 0}$ | $\mathbf{2 1 . 2 2 0}$ | $\mathbf{2 0 . 9 6 9}$ | $\mathbf{2 1 . 0 4 5}$ |

## 4. REFERENCES

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