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ABOUT KINEMATIC REDUCED MODEL OF LOWER LIMB

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Abstract:*The paper presents the complete and the reduced model of the lower limb.It can be modelled as an kinematic chain consisting of several links connected by revolute joints driven by actuators.For geometric and kinematic modelling of the reduced model with 5 dof it is used a DH –convention.*

Key words: *biokinematics, lower limb, reduced model.*

1. INTRODUCTION

In biomechanics, modelling deals with biosystems: cells, tissues, organisms. These have properties that other functional systems lack, in other words, the living matter has the capacity to grow, to reproduce, to resorbe itself. Living tissues can change their dimension and-sometimes-their mechanical properties, depending on external stress and because of some inner biochemical processes.

As functional anatomy, biomechanics is an exact science too. Human body's biomechanical behaviour and functional adaptation are extremely complex and hard to describe mathematically. However, so that the human movements could be studied, simplifying hypotheses are being formulated, making modelling and mathematical analysis easier to perform.

In prosthetics, the first requirements that must be followed are the kinematical ones. This is the reason why the kinematic model of the human limb to be prosthesised is extremely important.

2. KINEMATIC MODELLING OF OSTEOARTICULAR SYSTEM

The osteoarticular system of the humanlower limbs is the main system responsible for the human locomotion (or movements). For its kinematic modelling, a complex spatial mechanism with a large number of degrees of freedom must be considered [1],[2],[4].

From an anatomical structure point of view, the biomecanics of lower limb consists of rigid elements (bones) linked together by articulations and activated by muscles.

2.1 The lower limb’s joints

Articulations are machanical joints that can have one, two or three degrees of freedom [3].

The main lower limb articulations (Fig. 1) are:

- the hip joint, with three degrees of freedom (3 DOF), considered as a multiaxial ball-and-socket (spheroidal) joint. Its movements are: flexion/extension, adduction/abduction, inward/outward rotation and, a combination of all, circumduction;

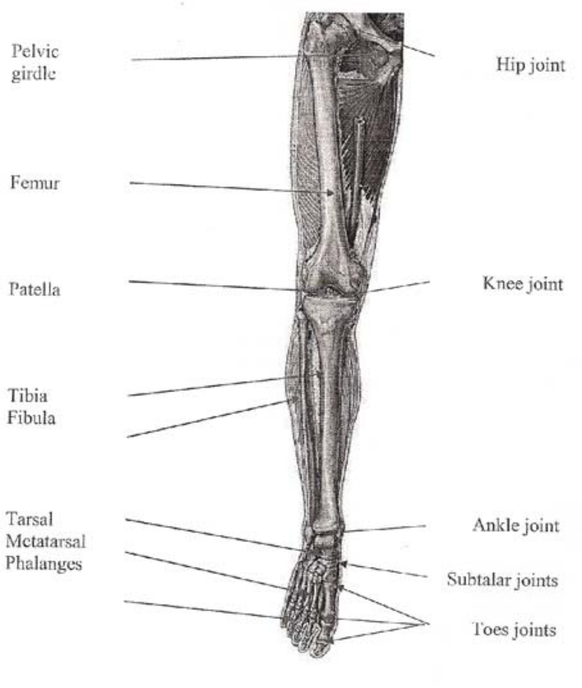


Fig. 1 Anatomical reprezentation of human lower limb

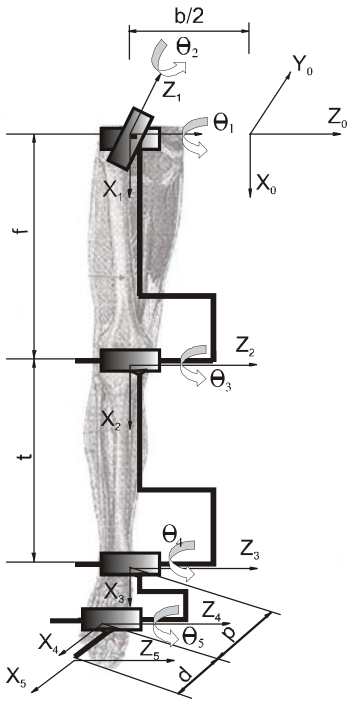


Fig. 2 Kinematic reduced model

- the knee joint is the human body's biggest joint. It has 3 DOF but through a mathematical simplification, these degrees can be reduced to two: flexion/exrension and inward/outward rotation. Its articulation can be considered to be an uniaxial modified hinge joint;
- the ankle joint have also 3 DOF. Two of its characteristics movements are worthy to be looked opon: dorsi/plantar flexion on the level of talocrural joint and inversion/eversion on the level of talotarsal joint.
- phalangeal joint with its main flexion/extension movements considered to be a monomobile one (1 DOF).

2.2 Kinematic complete model of the lower limb

All these combined movements can be modelled with the help of some simple revolute joints resulting in a human lower limb linkage with 10 DOF [1].

For the kinematic modelling the Denavitt-Hartenberg convention [4] was used. According to this method, the lower limb may be regarded as a rigid body linkage, each of the links having the motor coupling at one end, and a triorthogonal reference frame at the other one. The first of these reference frame is attached to the pelvic girdle, being considered the base (fixed) one. The other frames are placed in joints, at distances dictated by the anatomic dimensions. The change from one frame to another is made through some homogeneous transformation matrices that express each relative position and orientation.

The transfer matrix that connects the link between two adjacent elements, (i-1) and (i), is ${}^{i-1}T_i$ and has as a single time dependent variable -the joint variable q_i (t).

$$
{}^{i-1}T_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \cos\alpha_i & \sin\theta_i \sin\alpha_i & L_i \cos\theta_i \\ \sin\theta_i & \cos\theta_i \cos\alpha_i & -\cos\theta_i \sin\alpha_i & L_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}; \qquad i=1,\ldots,10 \qquad (1)
$$

θ_i -the rotation angle around the axis $O_{i-1}Z_{i-1}$; L_i -translation lengthways the axis $O_{i-1}Z_{i-1}$;

α_i -the rotation angle around the axis O_iX_i; d_i -translation lengthways the axis O_iX_i.

3. THE TRANSFER MATRICES OF KINEMATIC REDUCED MODEL

Given the complexity of the considered biomechanism and as not all the movements present a major importance for the process of locomotion, the model below is a simplified mode of the human lower limb, with only 5 DOF (Fig. 2). The q_i articulation variables are presented in the Table 1 according Fig. 1.

Table 1. Values of the geometric (DH) parameters

Joint	θ_i	α_i	d_i	L_i	$\sin\alpha_i$	$\cos\alpha_i$
1	q_1	90°	-b/2	0	1	0
2	q_2	-90°	0	f	-1	0
3	q_3	0°	0	t	0	1
4	q_4-90°	0°	0	p	0	1
5	q_5	0°	0	d	0	1

The general transformation matrix G_5 is obtained as product of the transfer matrices:

$$G_5 = {}^0T_1 \cdot {}^1T_2 \cdot {}^2T_3 \cdot {}^3T_4 \cdot {}^4T_5 \tag{2}$$

The following anatomical dimensions have been considered: b-sacrum length; f-femur length; t-tibia length; p-plantar (tarsal+metatarsal) length; d-phalanges length.

$${}^0T_1 = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & 0 \\ \sin\theta_1 & 0 & -\cos\theta_1 & 0 \\ 0 & 1 & 0 & -b/2 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad {}^1T_2 = \begin{bmatrix} \cos\theta_2 & 0 & -\sin\theta_2 & f\times\cos\theta_2 \\ \sin\theta_2 & 0 & \cos\theta_2 & f\times\sin\theta_2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

$${}^2T_3 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & t\times\cos\theta_3 \\ \sin\theta_3 & \cos\theta & 0 & t\times\sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad {}^3T_4 = \begin{bmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & p\times\cos\theta_4 \\ \sin\theta_4 & \cos\theta_4 & 0 & p\times\sin\theta_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

$${}^4T_5 = \begin{bmatrix} \cos\theta_5 & -\sin\theta_5 & 0 & d\times\cos\theta_5 \\ \sin\theta_5 & \cos\theta_5 & 0 & d\times\sin\theta_5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

The following notes have been made:

$$si = \sin\theta_i \qquad\qquad ci = \cos\theta_i \,. \tag{4}$$

After a simple computation can be obtaining G5.

Identifying, element by element, the components of matrix:

$$G_5 = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

n-normal unit vector; o-orientation vector; a-approach vector; p-position vector.

Numerical example

The position of the toes (the 5-th frame) relativ to fixed reference frame is expressed by p_x , p_y , p_z as it shown below:

px= G5(1,4)= ((c1c2c3-s1s3)c4+(-c1c2s3-s1c3)s4)d c5+(-(c1c2c3-s1s3)s4+(-c1c2s3-s1c3)c4) d s5+(c1c2c3-s1s3) p c4+(-c1c2s3-s1c3) p s4+c1c2 t c3-s1 t s3+c1 f c2.

py=G5(2,4)=((s1c2c3+c1s3)c4+(-s1c2s3+c1c3)s4)dc5+(-(s1c2c3+c1s3)s4+(-s1c2s3+c1c3)c4)d s5+(s1c2c3+c1s3)pc4+(-s1c2s3+c1c3)p s4+ s1c2 t c3+c1 t s3+s1 f c2.

pz=G5(3,4)=(s2c3c4-s2s3s4)dc5+(-s2c3s4-s2s3c4)ds5+s2c3pc4-s2s3ps4+s2tc3+fs2-1/2b.

(6)

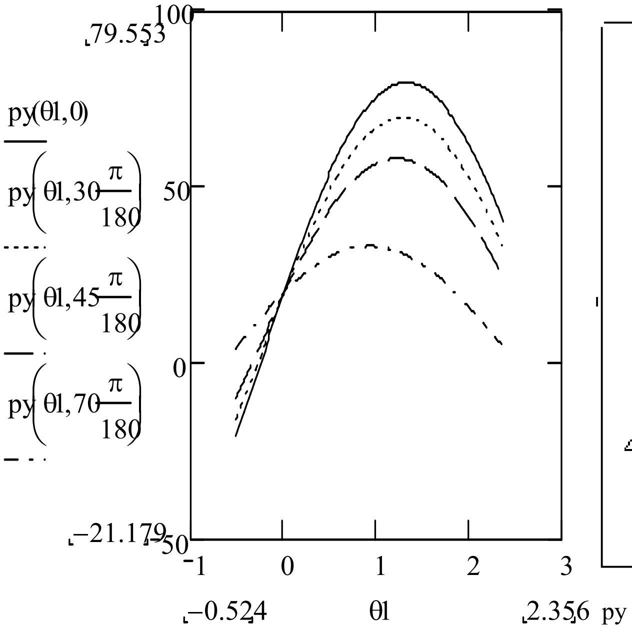
The articulation movements must comply with the following anatomical limitations:

$\theta_1\in[-30°,135°]$; $\theta_2\in[-30°,70°]$; $\theta_3\in[0°,150°]$; $\theta_4\in[-15°,70°]$; $\theta_5\in[0°,35°]$. (7)

The following anatomical dimensions have been considered, corresponding to a medium-sized person: b=40 mm; f=42 mm; t=35 mm; p=15 mm; d=5mm.

For the description of the positions of toes in sagittal plane on the mouving direction, it is taken in to account only the range of motion of the hip joint (θ_1, θ_2) while the other joints remaining blocked-Fig.3 and Fig.4.

It can be observed that p_y's range decreases at the same time with θ_2 's increase, which confirm that while the limb rises laterally the amplitude of flexion/extension decreases (from about 100 mm to about 30 mm).



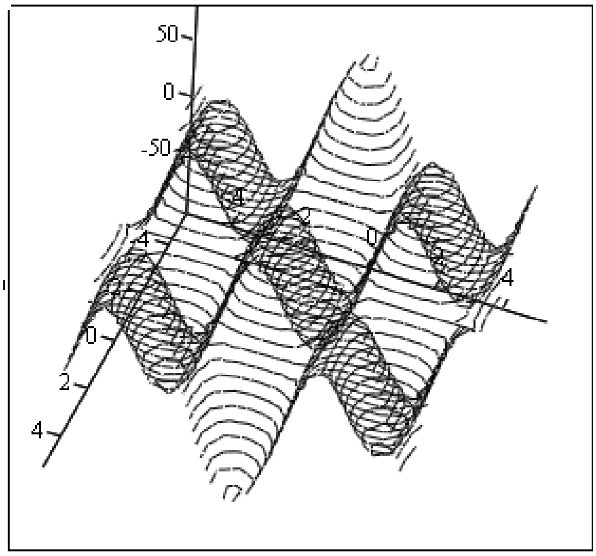


Fig.3 Representation of *py*-position of toes

Fig.4 The motion only in the hip joint

At the same time, the maximum of the p_y curve is obteined nearly θ_l=(1…1,3)rad, that is to say when the lower limb is flexed at 60°…75° in front. For θ_l=0 the size of the plant can be obteined (p+d=20 mm).

4. CONCLUSION

In conclusion, the range of flexion may be negatively influenced only when one of the hip joints is mobile.The movement of the human lower limbs is an extremely complex one, and for its study a series of simplifications must be considered.

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