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ALGORITHM FOR DETERMINING THE PARAMETERS OF THE GEAR RACK WITH ASYMMETRICAL TEETH

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Abstract: *This paper intends to establish the analytical relationships of an algorithm for calculating the parameters of the gear rack with asymmetrical flanks, with the aim of creating a design program, which would allow the creation of a gear rack for a gear with asymmetrical involute teeth, starting from the generalized parameters of the gear.*

The generator edge radius is a parameter that influences tension concentration at the tooth root, thus directly influencing its resistance. The gear rack tooth height must ensure the realization, without interference, of the two asymmetrical profiles.
Keywords: *spur gears, asymmetric profiles, involute teeth, rack-cutter.*

1. INTRODUCTION

Considering that one of the gears of a asymmetrical gear pair, for example gear 2 with center O_2, has the base circles infinitely large, its active and inactive profiles will degenerate into lines, tangential to the active and inactive gear 1 profiles respectively, the gear 2 rolling circle will also have an infinite radius and becomes a line tangential to the gear 1 rolling circle. (Fig.1). Gear 2 thus obtained is a gear rack with asymmetrical flanks [1].

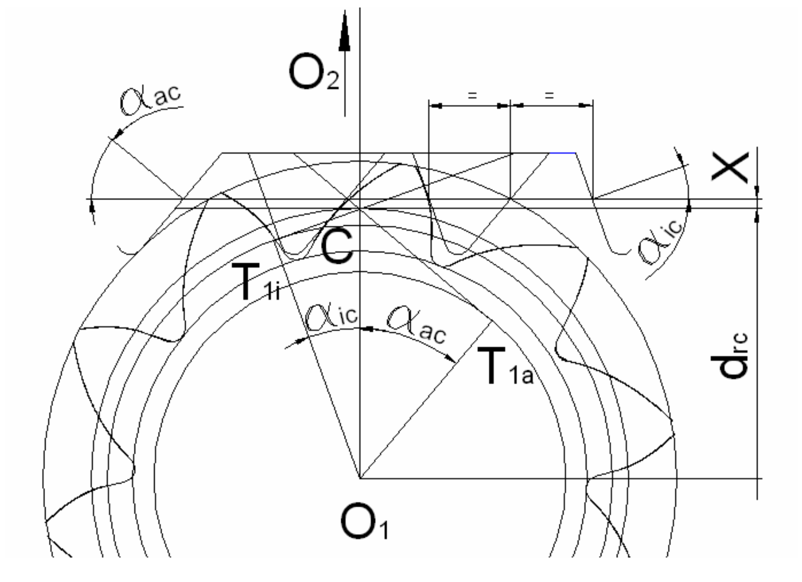

Fig. 1: Generating with the gear rack with asymmetrical flanks

While generating the symmetrical teeth the opposite profiles are in pairs symmetrical to a line perpendicular on the reference line [2].

While generating asymmetrical teeth, the opposite profiles generating the active and inactive flanks respectively, are no longer symmetrical, having different angles in relation to the gear rack's reference line [5].

The generating gear rack must ensure the formation of the asymmetrical involute flanks, without undercuting and with corresponding radial clearance between the root circle and the exterior circle of the conjugate gear [3].

The radius of the fillet of the generating gear rack's asymmetrical profiles influences tooth embedding section, the asymmetrical tooth resistance and rigidity .

2. THE TIP RADIUS OF THE GENERATING RACK CALCULATION

The diameter of rolling circle with the rack d_{rc} is [2]:

$$d_{rc} = \frac{d_{ba}}{\cos\alpha_{ac}} = \frac{d_{bi}}{\cos\alpha_{ic}} \tag{1}$$

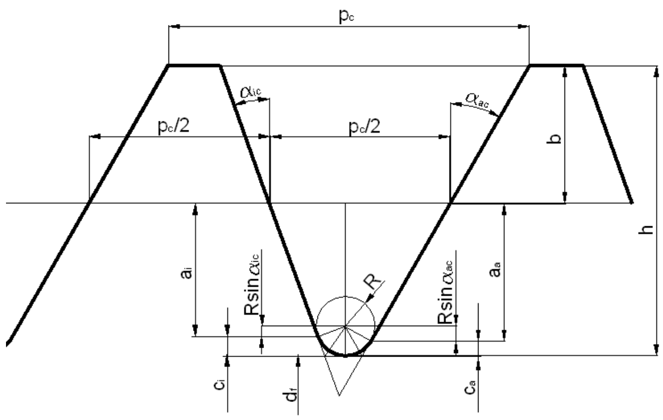

Fig. 2: The generating asymmetrical gear rack dimensions

The distance measured on the common normal line between two consecutive profiles of the gear's active and inactive flanks respectively, must be equal to the distance between the corresponding gear rack profiles, so the following relations must be carried out:

$$p_{ba} = p_r \bullet \cos\alpha_a = p_c \bullet \cos\alpha_{ac} \quad ; \quad p_{bi} = p_r \bullet \cos\alpha_i = p_c \bullet \cos\alpha_{ic}, \tag{2}$$

in wich:

- α_a, α_i are the presssure angles for active and inactive profile of the assimetric gear tooth;
- p_{ba}, p_{bi} are the pitches on the two different base circles (the base diameters d_{ba}, d_{bi});
- p_c is the generating rack pitch;
- p_r is the pitch on the operating pitch circle (the same for the active and inactive profiles).

Thus the gear rack's two flanks angle report α_{ac}/α_{ic} is given by the asymmetry coefficient k [4] of the gear teeth being generated:

$$k = \frac{\cos\alpha_i}{\cos\alpha_a}. \tag{3}$$

One of the two parameters is chosen and the other results from:

$$p_r = \frac{\cos\alpha_{ac}}{\cos\alpha_a} = \frac{\cos\alpha_{ic}}{\cos\alpha i} \quad ; \quad \cos\alpha_{ic} = k \bullet \cos\alpha_{ac} \quad ; \quad \cos\alpha_{ac} = \frac{\cos\alpha_{ic}}{k} \tag{4}$$

For a preliminary calculation the gear rack’s angle for the active flank α_{ac} can be chosen as equal to (or smaller than) the pressure angle on the active flank α_a, later to be modified as the gear rack’s dimensions are being optimized.

The gear rack’s angle for the inactive flank results:

$$\alpha_{ic} = \arccos\left(k \bullet \cos\alpha_a\right) \tag{5}$$

The rolling circle at generation [3]:, for the pinion and the gear, is obtained with:

$$d_{rc1} = \frac{z_1 \bullet p_c}{\pi} \quad ; \quad d_{rc2} = \frac{z_2 \bullet p_c}{\pi} \tag{6}$$

The diameter of the root circle d_f :

$$d_f = 2\left(a - \frac{d_{e\,rc}}{2} - c_{min}\right) \tag{7}$$

where $d_{e\,rc}$ is the exterior diameter of the conjugate gear, c_{min} is the minimum radial clearance, p_c is the pich on the roling circle at generation, equal to the gear rack’s pich.

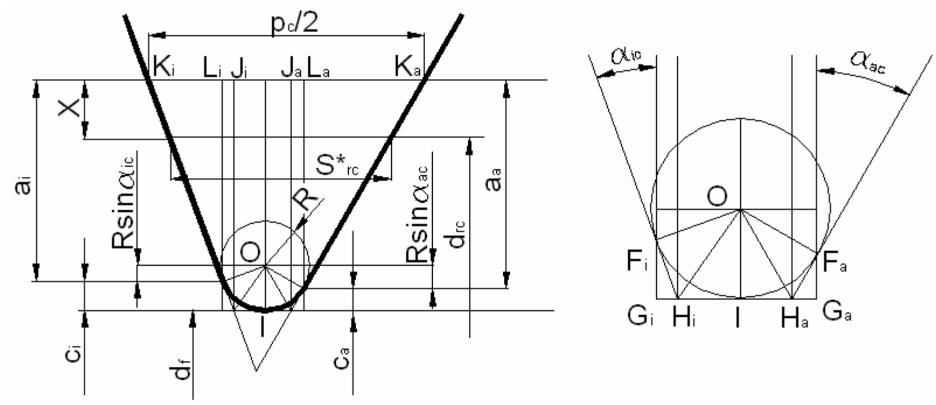

Fig. 3: Determination of the asymmetrical gear rack’s dimensions

The following result from figure 3:

$$c_a = R - R \bullet \sin\alpha_{ac} \quad ; \quad c_i = R - R \bullet \sin\alpha_{ic} \tag{8}$$

The gear rack’s tooth width on the rolling line is equal to the gash arch on the rolling circle at generation.

The gear’s tooth arc on the rolling circle at generation, with diameter d_{rc}, is:

$$S_{rc} = d_{rc}\left(inv\alpha_{va} + inv\alpha_{vi} - inv\alpha_{ac} - inv\alpha_{ic}\right)/2 \tag{9}$$

$$S^*_{rc} = p_c - S_{rc} = d_{rc}\left(2\pi / z + \text{inv}\alpha_{ac} + \text{inv}\alpha_{ic} - \text{inv}\alpha_{va} - \text{inv}\alpha_{vi}\right)/2 \qquad (10)$$

The shift of the gear rack result according to figure 3:

$$X \bullet \tan\alpha_{ac} + S^*_{rc} + X \bullet \tan\alpha_{ic} = \frac{p_c}{2} \quad ; \quad X = \frac{p_c/2 - S^*_{rc}}{\tan\alpha_{ac} + \tan\alpha_{ic}}. \qquad (11)$$

Considering the rack tooth rounding made up of one circular arc, its center is at the junction of the bisectors of the angles formed by the gear rack's flanks with the horizontal line corresponding to the minor diameter of the dressed gear. Thus one can deduce:

$$H_iI + IH_a = F_iH_i + F_aH_a\,; \qquad (12)$$

$$F_iH_i + F_aH_a = \frac{F_iG_i}{\cos\alpha_{ic}} + \frac{F_aG_a}{\cos\alpha_{ac}} = \frac{R - R\sin\alpha_{ic}}{\cos\alpha_{ic}} + \frac{R - R\sin\alpha_{ac}}{\cos\alpha_{ac}} =$$
$$R\left(\frac{1}{\cos\alpha_{ic}} + \frac{1}{\cos\alpha_{ic}} - \tan\alpha_{ic} - \tan\alpha_{ac}\right); \qquad (13)$$

$$H_iI + IH_a = J_iJ_a = \frac{p_c}{2} - K_iJ_i - J_aK_a = \frac{p_c}{2} - H_iL_i \bullet \tan\alpha_{ic} - H_aL_a \bullet \tan\alpha_{ac} =$$
$$\frac{p_c}{2} - \left(\frac{d_{rc}}{2} - \frac{d_f}{2} + X\right)\left(\tan\alpha_{ic} + \tan\alpha_{ac}\right). \qquad (14)$$

From (12), (13), (14) there results:

$$R = \frac{p_c/2 - \left(\tan\alpha_{ac} + \tan\alpha_{ic}\right)\left(d_{rc}/2 - d_f/2 + X\right)}{1/\cos\alpha_{ac} + 1/\cos\alpha_{ic} - \tan\alpha_{ac} - \tan\alpha_{ic}} \qquad (14)$$

3. DETERMINATION OF THE LIMITS IMPOSED UPON THE ADDENDUMS AND DEDENDUMS OF THE GEAR RACK'S TOOTH

Dimensions a_a and a_i defined in figure 3 result from:

$$CO = a_i - R \bullet \sin\alpha_i - X = a_a - R \bullet \sin\alpha_a - X$$

$$a_a - a_i = R \bullet \sin\alpha_{ac} - R \bullet \sin\alpha_{ic} \qquad (15)$$

$$\frac{p_c}{2} = K_iL_i + K_aL_a + L_iL_a = a_i \bullet \tan\alpha_{ic} + a_a \bullet \tan\alpha_{ac} + R \bullet \cos\alpha_{ic} + R \bullet \cos\alpha_{ic}$$

$$a_a \bullet \tan\alpha_{ac} + a_i \bullet \tan\alpha_{ic} = \frac{p_c}{2} - R \bullet \cos\alpha_{ac} - R \bullet \cos\alpha_{ic} \qquad (16)$$

If one consecutively multiplies equation (15) with $\tan\alpha_{ic}$ and with $\left(-\tan\alpha_{ac}\right)$ and then one adds to (16) one obtains:

$$a_a = \frac{\frac{p_c}{2} - \frac{R}{\cos\alpha_{ic}} \bullet \left(1+\cos\left(\alpha_{ic}+\alpha_{ac}\right)\right)}{\tan\alpha_{ac}+\tan\alpha_{ic}}; \tag{17}$$

$$a_i = \frac{\frac{p_c}{2} - \frac{R}{\cos\alpha_{ac}} \left(1+\cos\left(\alpha_{ac}+\alpha_{ic}\right)\right)}{\tan\alpha_{ac}+\tan\alpha_{ic}}. \tag{18}$$

From figures 5 and 6, in which there are presented the lines of action with the gear rack for the active and inactive flank respectively [1], one finds the limits of the dimensions a_a, b_a, a_i, b_i.

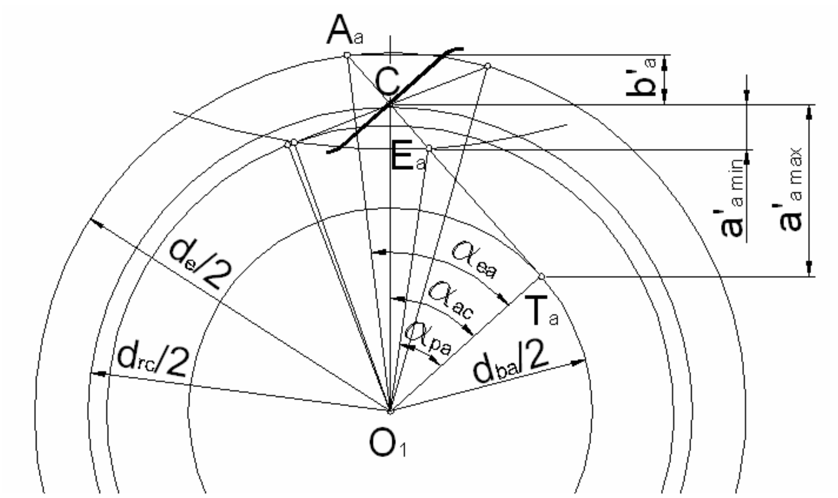

Fig. 5: Line of action with the gear rack for the active flank

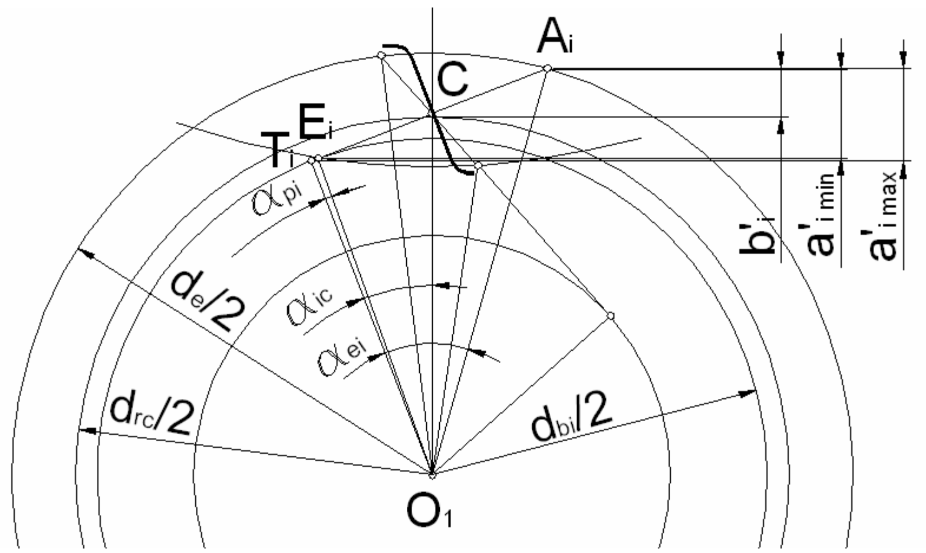

Fig. 6: The line of action with the gear rack for the inactive flank

Let's note $a_a - X = a'_a$; $b_a + X = b'_a$.

In order to realize the entire involute profile of the active flank it is necessary to:

$$a_a \geq a'_{a\,min} + X = \frac{d_{ba}}{2}\left(\tan\alpha_{ac} - \tan\alpha_{pa}\right)\bullet\sin\alpha_{ac} + X; \tag{19}$$

$$b_a \geq b'_a - X = \frac{d_{ba}}{2}\left(\tan\alpha_{ea} - \tan\alpha_{ac}\right)\bullet\sin\alpha_{ac} - X\ . \tag{20}$$

where α_{ea}, α_{pa} are the profile angles on the outside diameter and in the bottom point of the active profile.

Let's note $a_i - X = a'_i$; $b_i + X = b'_i$.

In order to realize the entire involute profile of the inactive flank one must respect the following conditions:

$$a_i \geq a'_{i\,min} + X = \frac{d_{bi}}{2}\left(\tan\alpha_{ic} - \tan\alpha_{pi}\right)\bullet\sin\alpha_{ic} + X \tag{21}$$

$$b_i \geq b'_i - X = \frac{d_{bi}}{2}\left(\tan\alpha_{ei} - \tan\alpha_{ic}\right)\bullet\sin\alpha_{ic} - X \tag{22}$$

where α_{ei}, α_{pi} are the profile angles on the outside diameter and in the bottom point of the inactive profile.

In order to avoid the undercuting of the teeth, the following conditions must be complied with:

$$a_a \leq a'_{a\,max} + X = \frac{d_{ba}}{2}\bullet\tan\alpha_{ac}\bullet\sin\alpha_{ac} + X \tag{23}$$

$$a_i \leq a'_{i\,max} + X = \frac{d_{bi}}{2}\bullet\tan\alpha_{ic}\bullet\sin\alpha_{ic} + X \tag{24}$$

4. CONCLUSION

The generating gear rack flank angles and the radius of the rack-cuter fillet, can be optimized with the help of the genetic algorithms, the restrictions being given by the limits imposed upon the tooth height. The purpose of this optimization is the reduction of the stress at the tooth base and the insurance of the necessary rigidity under load.Computerized design of the gear rack with asymmetrical teeth, allows the reduction of costs required for the preparation of the production of involute spur gear with asymmetrical teeth, wich are designed as nonstandard ones.

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