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CONTRIBUTION TO DISCRETE STRUCTURAL OPTIMISATION

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Abstract: *The goal of our paper is to present the application of the new discrete optimisation algorithms to structural mass minimissing subjected to the prescribed fatigue damage or life. This study considers the structural analysis (mainly shell structures) by finite element method, multiaxial rainflow counting, fatigue damage prediction and sizing optimisation process. It will be analysed a mechanical systems under random excitation in time domain. The new optimising approach will be implemented into solution program OPTIM_FAT.M (in Matlab).*

Key words: *optimising process, finite element method, multiaxial rainflow analysis.*

1. INTRODUCTION

A detailed review of discrete structural optimisation has been presented by Arora and Huang. Forty years ago, the discrete optimal design of trusses subjected to stress and displacement constraints was formulated as a linear programming or a mixed integer-continuous variable problem using the Gomory's method. Huang and Arora suggested interesting strategies that combine continuous variable optimisation methods with a genetic algorithm. Discrete optimal weight design of geometrically non-linear truss structures subjected to stress and displacement constraints was presented by Csébfalvi. The discrete optimisation process is characterised by following advantages: technically applied results, simplicity of optimising algorithms, advanced effect of the comparator methods, effectiveness for unconventional optimising approaches (fully stress design, etc.), effectiveness in the case of the very complicated objective function and constraint conditions.

2. FORMULATION OF THE OPTIMISING PROCESS

The optimising problem of the mass minimising subjected to the prescribed fatigue damage or life is a topical. For a structure of multiple elements, the optimisation problem of discrete variables can be stated mathematically as follows

$$F(\mathbf{x}) = \sum_{i=1}^{n} \rho_i \cdot l_i \cdot X_i \to \min . \tag{1}$$

Subjected to

$$D_{\max}(\mathbf{x}) - D_P \le 0, \quad \text{or} \quad T_{\min}(\mathbf{x}) - T_P \ge 0, \qquad i = 1,2,\ldots m, \tag{2}$$

there n is number of the elements, D_P is the prescribed cumulative damage, T_P is the prescribed fatigue life in hours, $D_{\max}$ is the calculated extreme value of the cumulative damage, $T_{\min}$ is the calculated extreme fatigue life in hours. Let’s form a new penalised objective function

$$\bar{F}(\mathbf{x}) = \sum_{i=1}^{n} \rho_i \cdot l_i \cdot X_i + \lambda \rightarrow \min ., \tag{3}$$

where the penalty function λ is following

$$\begin{aligned} &\lambda_i = 0 && \text{if} \quad D_{\max}(\mathbf{x}) - D_P \le 0 \quad \text{or} \quad T_{\min}(\mathbf{x}) - T_P \ge 0, \\ &\lambda_i = 10^k \ (\mathrm{k} = 4 - 9) && \text{if} \quad D_{\max}(\mathbf{x}) - D_P > 0 \quad \text{or} \quad T_{\min}(\mathbf{x}) - T_P < 0. \end{aligned} \tag{4}$$

The penalised objective function $\bar{F}(\mathbf{x})$ is solved by the presented new algorithm.

3. ALGORITHMIZATION OF THE OPTIMISING PROCESS

The effective calculation algorithm for the optimization of cross-section parameters of FE models will be presented on principle of discrete optimisation. Especially cross-section characteristics of truss and beam elements (cross-section area, moment of inertia, etc.) or thickness of membrane, plate and shell elements will be considered.

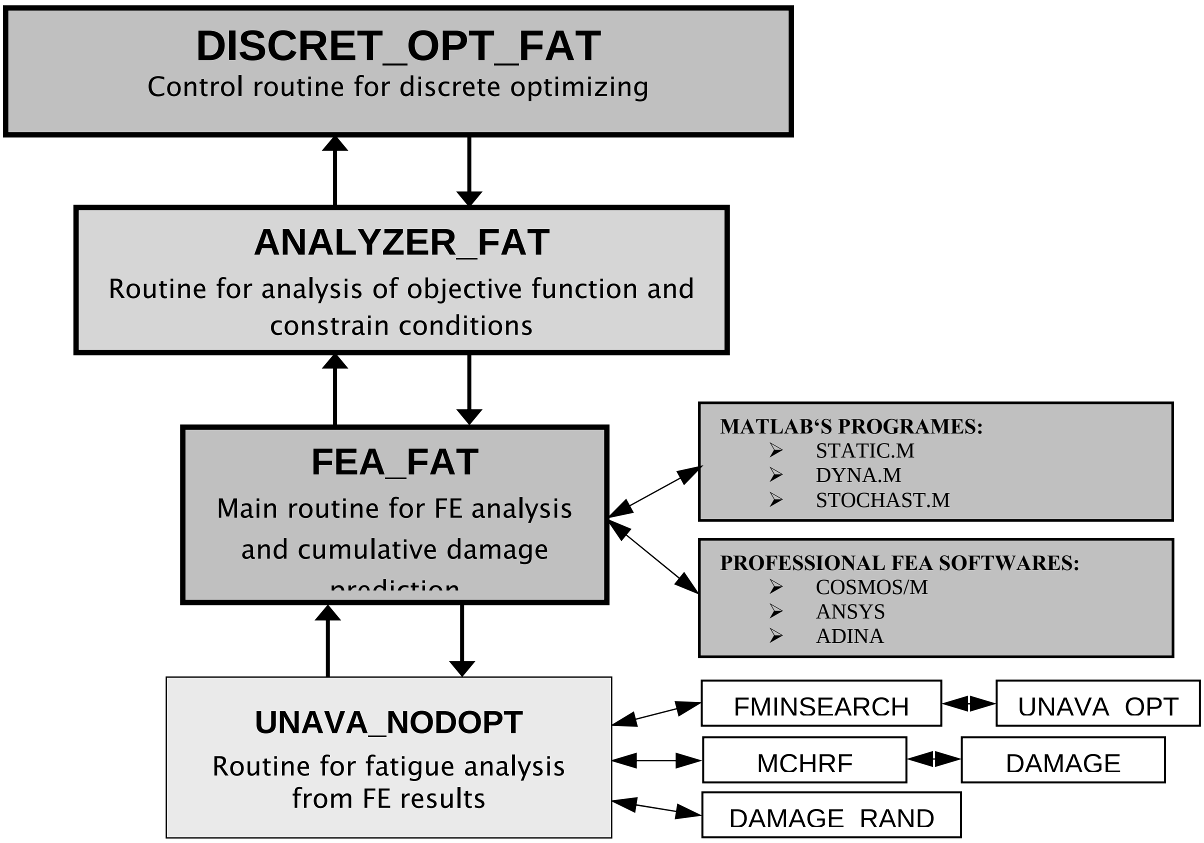

Fig. 1 *Structure of advanced program system*

By using of these finite elements the computing models of various frame structures such as transport and operating devices can be created. As the shell elements application is widely used in the engineering, they will be applied in the following. The certain particularity of these elements is also the fact that from the stress view it is the case of multiaxial tightness, what during the fatigue life computing in case of disproportionateness of the stress leads to the already mentioned multiaxial rainflow decomposition of stress response problem. This complicated computing process will be inbuilt and used in the presented computational program (See on fig. 1)

4. CUMALATIVE DAMAGE PREDICTION

To calculate the structural mass is a simple problem but the fatigue damage depends on FE analysis, identification of a "damage" critical point and multi-axial fatigue prediction. Let's focus on the cumulative damage counting by using multi-axial rainflow decomposition of the stress response.

Generally, we can employ two fundamental approaches to the multi-axial rainflow counting: Integral Approach (IA) and Critical Plane Approach (CPA).

The fundamental idea of an Integral Approach (Linear combination of stress tensor members version) is to count raniflow cycles on all linear combinations $\sigma_{MRF}(t)$ of the random stress vector components. Practically, when the stress state is biaxial (e.g. thin shell finite element), the stress components can by written under the form of three dimension vector $\boldsymbol{\sigma}=[\sigma_x, \sigma_y, \tau_{xy}]^T$. A set of linear combinations

$$\sigma_{MRF}(t) = c_1 \cdot \sigma_x(t) + c_2 \cdot \sigma_y(t) + c_3 \cdot \tau_{xy}(t) \tag{5}$$

can be chosen for values of c_i such as $c_1^2 + c_2^2 + c_3^2 = 1$ thus defining a sphere. The goal is to find extreme value of the estimated damage or life for vector **c**. Subject to the normality condition of **c**, the optimising variables vector is normalised by relation

$$\mathbf{c} = \frac{\mathbf{c}'}{\sqrt{\mathbf{c}'^T \cdot \mathbf{c}'}} \, . \tag{6}$$

Our aim is to find the optimum vector $\mathbf{c}_{opt}$ for the objective function

$$F = \max(D(\mathbf{c})) = \max\left(\sum_{i=1}^{nc} \frac{n_i}{N_i(\mathbf{c})}\right) \quad \text{or}$$

$$F = \min(T(\mathbf{c})) = \min\left(\frac{t}{3600 \cdot D(\mathbf{c})}\right) \quad [\text{hour}] \, , \tag{7}$$

where t is time interval of realisation, D is cumulative damage, T is fatigue life in hours, N_i is number of cycles to failure, n_i is number of cycles at one particular stress level. By rainflow decomposition of $\sigma_{MRF}(t,\mathbf{c})$, we obtain $N_i(\mathbf{c})$ and n_i. The searching process is realized by computational program FEA_FAT created in MATLAB. The program includes stress analyse by FEM and optimising procedure by MATLAB function FMINS.M.

5. FRAME DESIGNING OF THE TRACK MAINTENANCE MACHINE

The goal of this part is the optimum design of the chosen parameters of the track maintenance machine VKL 400. It's presented the dynamic model creation, the analysis of vertical and transversal stochastic vibrations and the process of dynamic strength dimension. The gist of the solution was the determination of the cumulative damage and designing of the cross sections of the main frame.

The objective function was defined by the equation. The design variables were the thicknesses of the sheets of the main vehicle structure. The power spectral density of the vertical unevenness of the track we obtained from the measuring on real track. In each optimizing step it was needed to solve stress response and the cumulative damage of the each element group.

table 1. Design variables

Parameter	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}
Value [mm]	30	20	25	20	35	35	15	25	15	25	25

Cross-section parameters, i.e. the thickness of the weld frame plates were parameterized (fig. 2a). Values used on the build of existing frame are applied in the initial analysis of course. List of these values is presented by tab. 1.

Computing model on the base of FEM in the system COSMOS/M (fig. 2b) was built-up from a virtual model created in PRO/Engineer (fig. 2a). Selected values describing physical properties of computing model were parameterized in order to their arbitrary changes. The goal of parameterization was to achieve the maximum variability of the model, which related mainly to verification and debugging of this model and consequently to the optimization process.

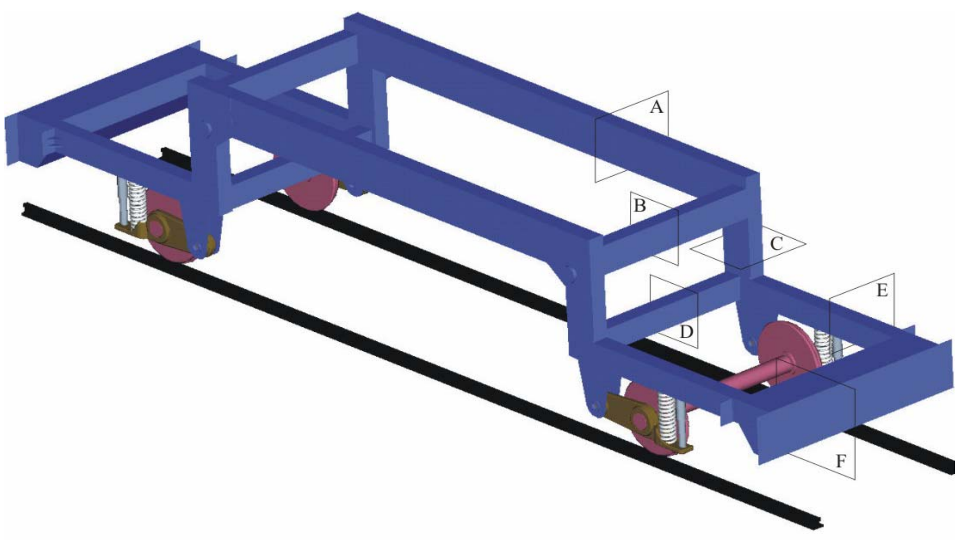

Fig. 2a V*irtual model of analysed vehicle frame with cross section*

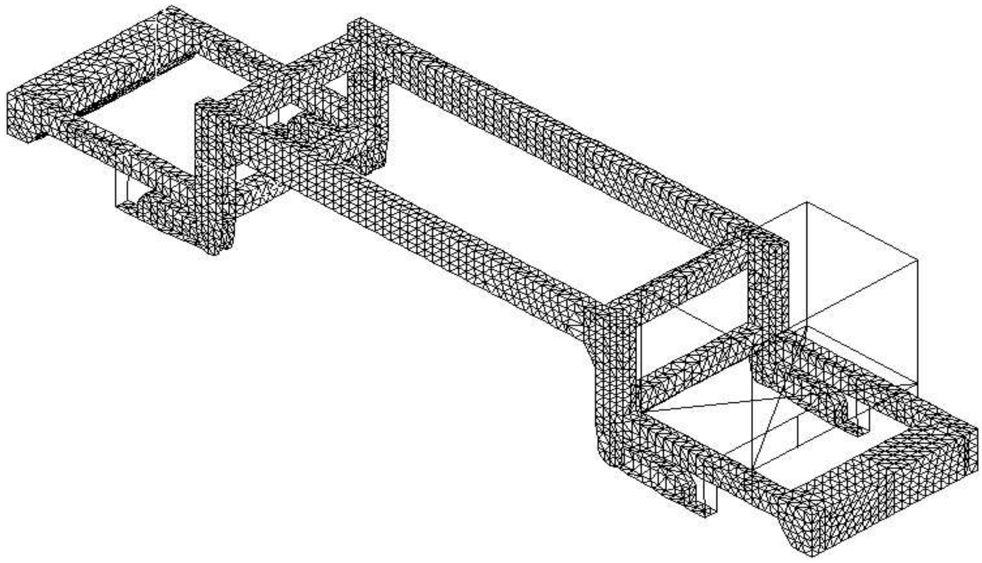

Fig. 2b *Finite element model of the frame in COSMOS/M*

The computational parameters are

- Young's modulus of elasticity $E=2.10^{11} Pa$,
- Poisson ratio $\mu=0,3$,
- Density $\rho=7800 kg/m^3$,
- Point of Woehler curve $N_A=10^3$ *cycles*, $\sigma_{Amax}=217\ MPa$,
- Fatigue limit $\sigma_C=68,7\ MPa$,
- Constant $k=0,8$,
- Exponent of Woehler's curve $m=5,2$,

Graphical presentation of the "working" Woehler curve reduced according to Corten-Dolan is on fig. 3. Program DISKRET_OPT_FAT is controlling the optimization process itself. It determines initial conditions and by running FEM analysis it obtains required results for the damage cumulation computation needed to determine the optimal cross-section properties with respect to measure of damage $D_p=0,6$.

Optimization process itself can be defined as follows:

- weight minimization of the frame structures,

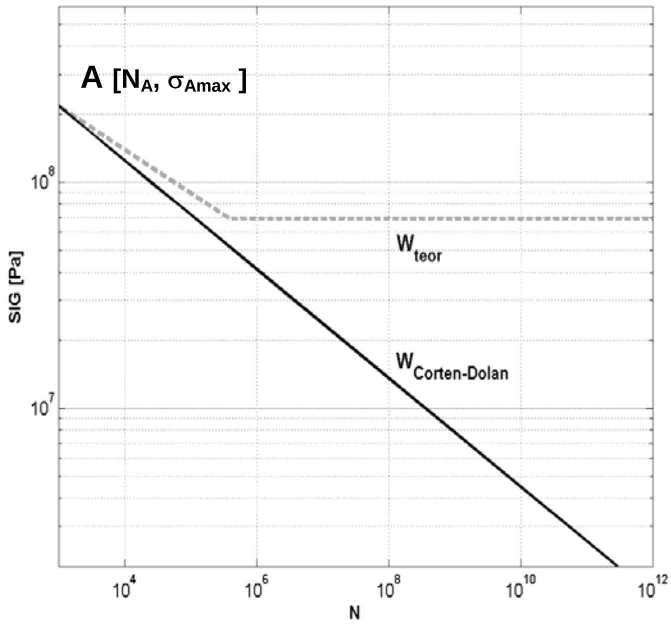

Fig. 3 *„Working" Woehler curve*

- regarding boundary condition that maximum damage of frame will be $D_p=0,6$
- 11 optimization variables from X_1 to X_{11} are defined.

Optimization variables can gain the discrete values listed in *tab. 2*. The values are standard thicknesses of chosen plates from catalogue of company Ferona.

table. 2 Sheets thicknesses using in optimizing process

Serial number X_i:	1	2	3	4	5	6	7	8	9	10
Thickness [mm]:	10	12	14	16	18	20	25	30	35	40

Follow operating conditions are assumed:

- movement 27.000 hours on velocity 40km/h,
- movement 18.000 hours on velocity 70km/h,
- movement 9.000 hours on velocity 100km/h.

The chosen results of the optimising process are presented on the figs. 4, 5 and 6.

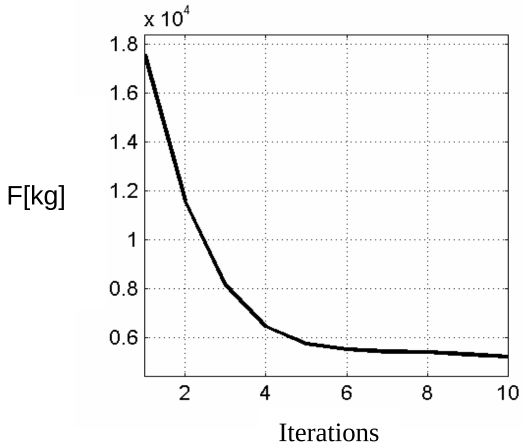

Fig. 4 *Reduction of the structural mass*

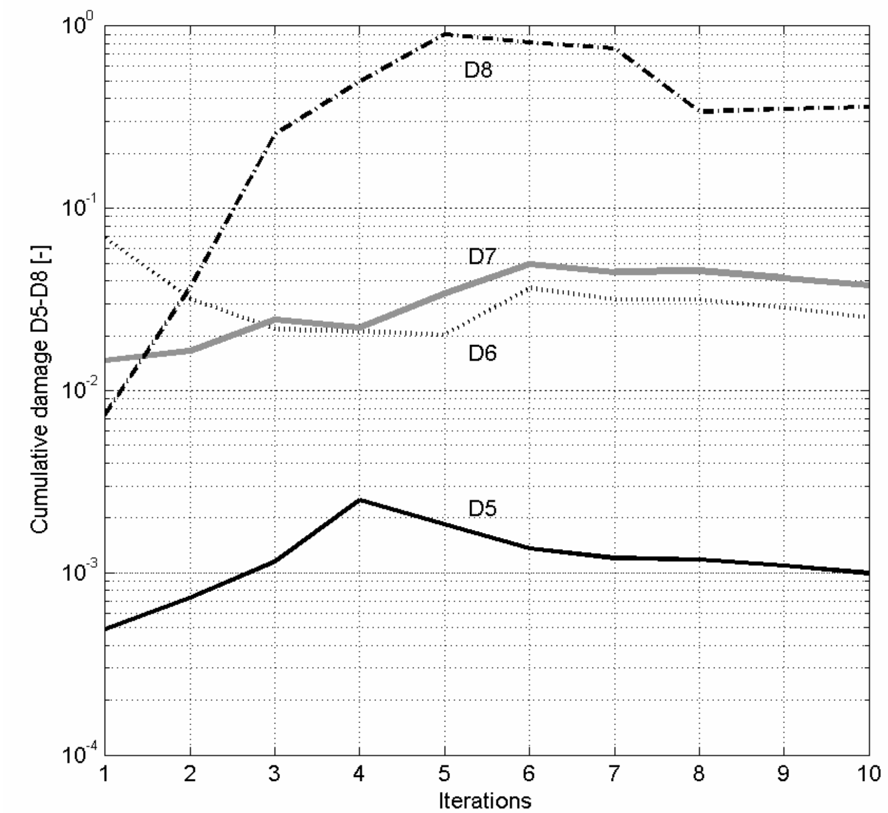

Fig. 5 *History of the opt. process for cumulative damage in semi-logarithmic coordinates*

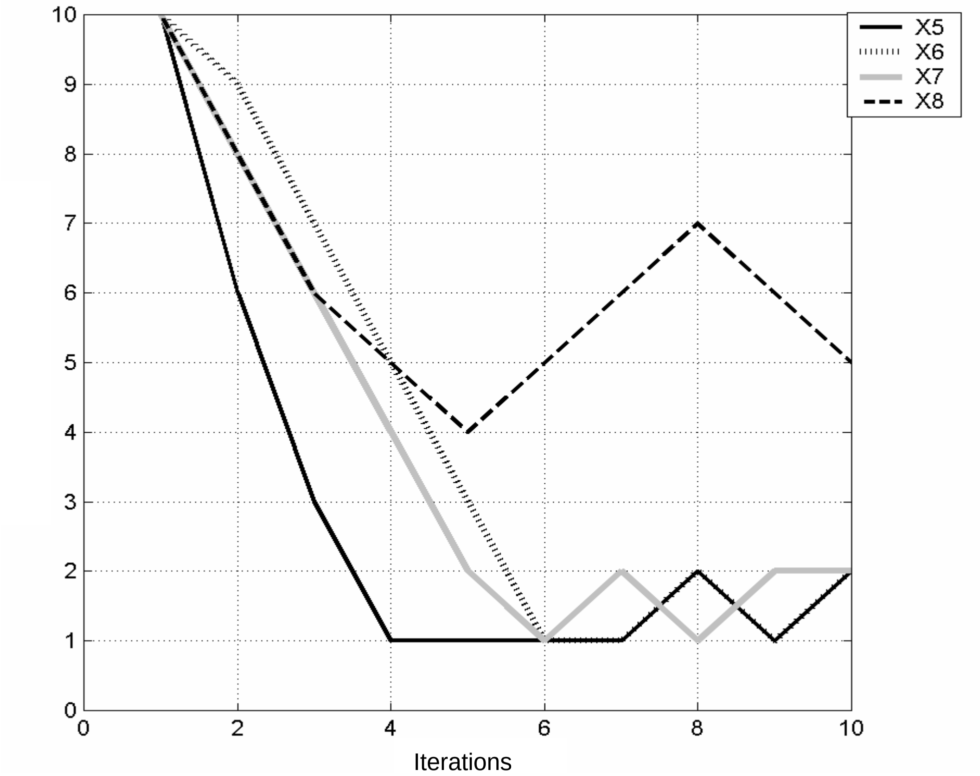

Fig. 6 *History of the optimizing process for design variables X5 - X8*

6. CONCLUSION

The paper discusses the optimal mass design subjected to the prescribed fatigue life. The suggested method of discrete optimisation is simple but effective for complicated dynamic tasks of engineering mechanics. The problem presented in our paper, is interesting for designers of the frame structures of the vehicles chassis, special trucks, cranes, or conveyers. The process of the dynamic strength dimension is often more complicated as a classic static optimization. By this way, we can solve the problems of the fatigue dimension, or service life dimension of the structures.

7. REFERENCES

1. **Bathe K. J., Wilson E. L.**: *Numerical Methods in Finite Element Analysis, New Jersey, Prentice-Hill, 1976*
2. **Balda M.**: *Prediction of Damage Cumulation in Vibrating Rotors, Proc. of IFToMM Fifth International Conference on Rotor Dynamics, Darmstadt, 1998*
3. **Carpinteri A., Spagnoli A., Vantadori S.**: *Critical plane approach for multiaxial fatigue of metals, Zeszyty Naukowe Politechniki Opolskiej, seria Mechanika z. 67, 269/2001*
4. **Dressler K., Köttgen V. B., Kötzle H.**: *Tools for fatigue evaluation of non-proportional loading, In proceedings of Fatigue Design'95, Helsinky, Finland 1995*
5. **Hans, M.**: *A nearly strict fuzzy arithmetic for solving problems with uncertainties, 19th Int. Conf. of the North American Fuzzy Information Processing Society – NAFIPS'2000, pp. 439-443, Atlanta, USA, 2000*
6. **Kalinčák, D.**: *The power spectral densities of the measured track irregularities, Intern. Scient. Conf. „The transport of the 21st century", Warszawa, Poland, 2002, pp. 111-118*
7. **Pitoiset X., Preumont A., Kernilis A.**: *Tools for a Multiaxial Fatigue Analysis of Structures Submitted to Random Vibrations, Proceedings European Conference on Spacecraft Structures Materials and Mechanical Testing, Braunschweig, Germany, 1998*
8. **Preumont A., Piéfort V.**: *Predicting random high cycle fatigue life with finite elements, Lournal of Vibration and Acoustics, 116, 1994, pp. 245-248*
9. **Sága M.**: *Mass Minimising of Truss Structures Subjected to Prescribed Fatigue Life, Machine Dynamics Problems, Vol. 28, No. 4, 2004, pp. 101 -106*