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THE RIGIDITY MATRIX OF THE DOUBLE EFFECT BALL BEARINGS

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Abstract: *An analytical formulation to simulate the internal mechanical interactions in the double effect ball bearings is presented. The individual constructive particularities of the double effect ball bearings were considered. Many scientists consider that there are no differences between the cases when the inner ring or outer ring is considered rigid, but, these differences exist and are evidenced if the rigidity matrix is correctly constructed in 5 DOF.*

Key words: *Double Effect Ball Bearings, Rigidity Matrix, Outer Ring Rigid (ORR), Inner Ring Rigid (IRR)*

1. INTRODUCTION

The load distribution in double effect ball bearings depends on bearing geometry and the boundary conditions. We consider two variants of boundary conditions. These cases correspond to outer ring rigid case, named "ORR" or to the inner ring rigid case named "IRR". To describe these differences the "Rin" and "Rou" parameters respectively were introduced.

2. ANALITICAL APPROACH

Figures 1, 2, 3 and 4 show some particularities of the double effect ball bearings. For the ORR and IRR cases, the external load vector and the ring displacements, according to Figures 1 to 4, are: $\{F\} = \{F_x, F_y, F_z, M_y, M_z\}$, and $\delta=\{\delta x, \delta y, \delta z, \gamma y, \gamma z\}$. In that analysis the double effect ball bearings presented in Figs 1-4, were abbreviated as DBB1, DBB2, DBB3 and DBB4. An "*r*" index was introduced to describe the bearing rows, so "*r = 1, 2*". The curvature centres are named *Ow, Oi*, *Oe*. To each configuration an inertial system *OXYZ* is attached. The system origin is the geometrical centre of the inner ring. Each rolling element has two degrees of freedom.

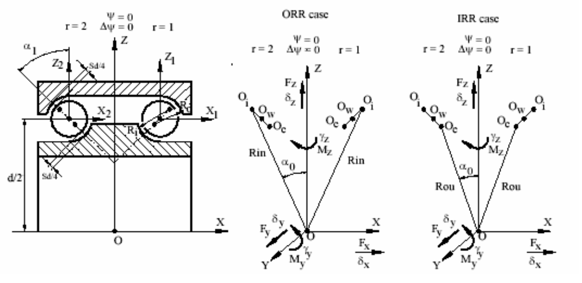

Fig.1. *Characteristics of the DBB1 bearing type*

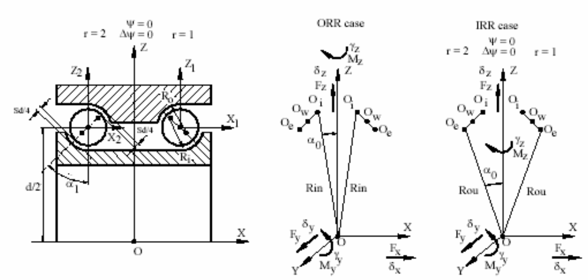

Fig.2. *Characteristics of the DBB2 bearing type*

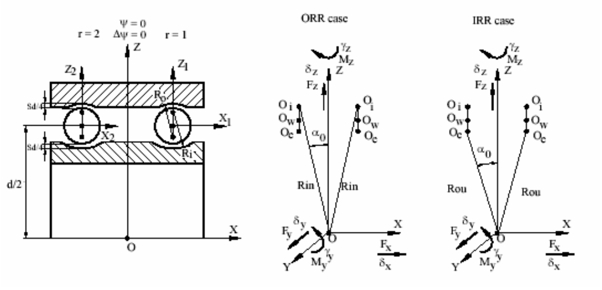

Fig.3. *Characteristics of the DBB3 bearing type*

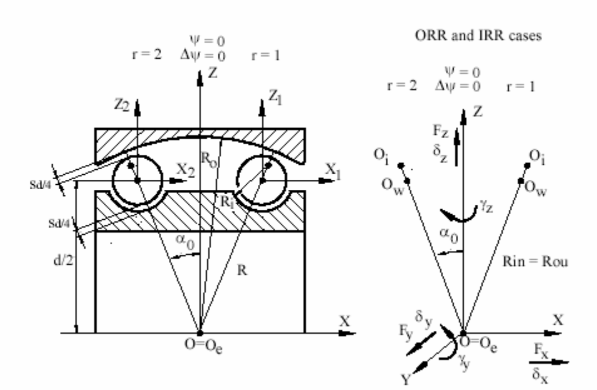

Fig.4. *Characteristics of the DBB4 bearing type*

The differences between the "ORR" and "IRR" concerning the curvature centre displacement are shown in Figs. 5 to 8.
The effect of the ring displacement is evidenced with the < ' > index as following: for the "ORR" case, the *Oi* point becomes <*Oi'* > and for the "IRR" case *Oe* becomes <*Oe*'>.

The load distribution in the DBB 1-4 in the "ORR" and "IRR" cases is function of the *Ow*, *Oi*, and *Oe* point displacements. To create the rigidity matrix, the following functions were constructed: $sgn(r) = \{-1,1\}$, for $r=\{1,2\}$; $\psi = \psi(r,j)$ to describe the rolling element position.

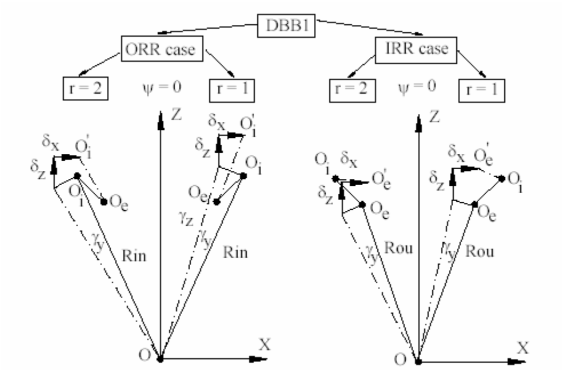

Fig.5. *DBB1 : ORR and IRR dispacements*

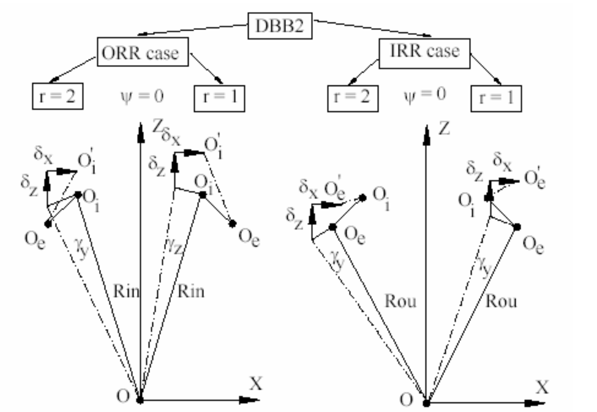

Fig.6. *DBB2 : ORR and IRR dispacements*

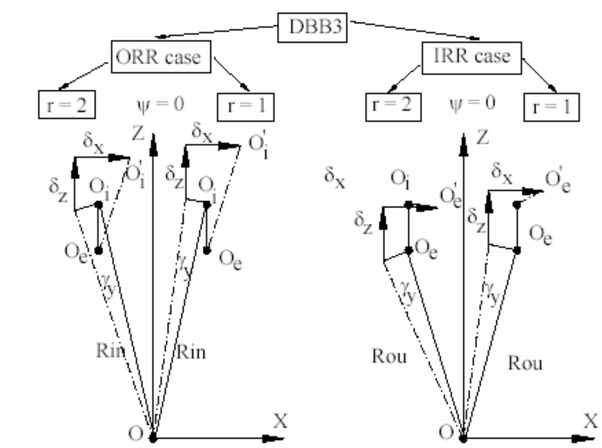

Fig.7. *DBB3 : ORR and IRR displacements*

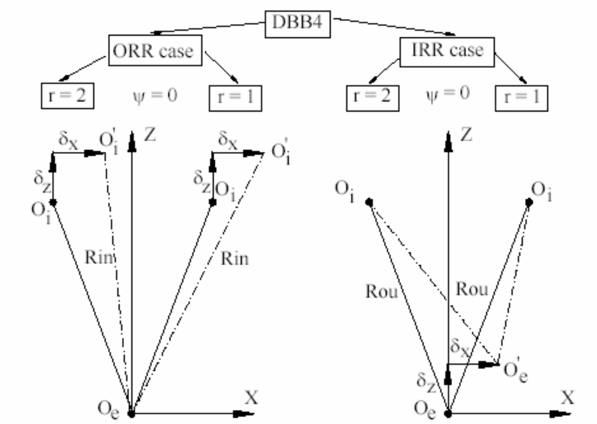

Fig.8. *DBB4 : ORR and IRR displacements*

As function of the studied case “ORR” or “IRR” and the bearing type, the *"α_0"* angle was introduced (see figs. 1-4). The misalignment effect is taken into account with:

$$\alpha(r,j)=\alpha_0+sgn(r).(\gamma_y.cos(\psi)+\gamma_z.sin(\psi) \qquad (1)$$

The static contact deformation for the (r,j) ball from the DBB1-4 structure corresponding to the "ORR case" and "IRR case", is given as:

$$\delta(r,j)=\sqrt{x(r,j)^2+x(r,j)^2}-l_{oi}-l_{oe} \qquad (2)$$

where:

- in the "ORR” case:
  $$z(r,j)=A.(l_{oi}+l_{oe}).\cos(\alpha 1)+\delta_z.\cos(\psi)+\delta_y.\sin(\psi)+Rin.[\cos(\alpha_0)-\cos(\alpha(r,j))] \qquad (3)$$
  $$x(r,j)=B.(l_{oi}+l_{oe}).\sin(\alpha 1)+\delta_x+Rin.[\sin(\alpha_0)-\sin(\alpha(r,j))] \qquad (4)$$

- in the "IRR” case:
  $$z(r,j)=A.(l_{oi}+l_{oe}).\cos(\alpha 1)+\delta_z.\cos(\psi)+\delta_y.\sin(\psi)+Rou.[\cos(\alpha_0)-\cos(\alpha(r,j))] \qquad (5)$$
  $$x(r,j)=B.(l_{oi}+l_{oe}).\sin(\alpha 1)+\delta_x+Rou.[\sin(\alpha_0)-\sin(\alpha(r,j))] \qquad (6)$$

with:

"A" and "B" parameters, form Table 1

Table 1. The A, B, C and D parameters, functions of the "ORR" and "IRR" cases

Bearing type	***r=1***			***r=2***			***r=1,2***	***r=1,2***
	α_1	***A***	***B***	***α_1***	***A***	***B***	***C***	***D***
DBB1	*α_1*	*1*	*1*	*α_1*	*1*	*-1*	*-1*	*1*
DBB2	*α_1*	*1*	*-1*	*α_1*	*1*	*1*	*-1*	*1*
DBB3	*0*	*1*	*0*	*0*	*1*	*0*	*1*	*1*
DBB4	*α_0*	*1*	*1*	*α_0*	*1*	*-1*	*-1*	*0*

and

- $l_{oe}=Ro-D_w/2-Sd/4$, represents the distance between the Oe and Ow points
- $l_{oi}=Ri-D_w/2-Sd/4$, represents the distance between the *Oi* and *Ow* points
- $R_{o,i}$ outer and inner raceway radius
- *Sd* represents the total diametric clearance of the bearing

The contact angle for *(r,j)* rolling element is given as:

$$\alpha_i(r,j)=\alpha_e(r,j)=\arctan\left(\frac{x(r,j)}{z(r,j)}\right) \qquad (7)$$

The contact load for the (r,j) ball is :

$$Q(r,j)=K_{ech}.\delta(r,j)^n \qquad (8)$$

where:

K_{ech} , represents the equivalent rigidity for the point contact type.

The bearing equilibrium equations corresponding to the "ORR" and “IRR” cases are:

$$F_z = \sum_r \sum_j Q(r, j)\cos(\alpha_i(r, j))\cos(\psi(r, j)) = \sum_r \sum_j F_z(r, j) \tag{9}$$

$$F_y = \sum_r \sum_j Q(r, j)\cos(\alpha_i(r, j))\sin(\psi(r, j)) = \sum_r \sum_j F_y(r, j) \tag{10}$$

$$F_x = \sum_j Q(1, j)\sin(\alpha_i(1, j)) + C.\sum_j Q(2, j)\sin(\alpha_i(2, j)) = \sum_j F_x(1, j) + C.\sum_j F_x(2, j) \tag{11}$$

$$M_y = D\sum_{r=1}\left\{\sum_j F_x(1, j).b_z(1, j) + \sum_j F_z(1, j)b_x(1, j)\right\} + D\sum_{r=2}\left\{C\sum_j F_x(2, j).b_z(2, j) + \sum_j F_z(2, j)b_x(2, j)\right\} \tag{12}$$

$$M_z = D\sum_{r=1}\left\{\sum_j F_x(1, j).b_y(1, j) + \sum_j F_y(1, j)b_x(1, j)\right\} + D\sum_{r=2}\left\{C.\sum_j F_x(2, j).b_y(2, j) + \sum_j F_y(2, j)b_x(2, j)\right\} \tag{13}$$

where:

- *Q(j)* represents the load acting on the *(r,j)* roller element;
- b_x, b_y, b_z represents the distance from the point of contact inner raceway - ball to the centre of the inertial system in "ORR" case. For "IRR" case b_x, b_y, b_z represents the distance from the point of contact outer raceway - ball to the centre of inertial system.

For "ORR" case:

$$b_x(r, j) = B1 + \left(\delta_i(r, j) + l_{oi} - \frac{D_w}{2}\right).\sin(\alpha_s(r, j)) \tag{14}$$

$$b_y(r, j) = \left[C1 + \left(\delta_i(r, j) + l_{oi} - \frac{D_w}{2}\right).\cos(\alpha_s(r, j))\right].\sin(\psi(r, j)) \tag{15}$$

$$b_z(r, j) = \left[C1 + \left(\delta_i(r, j) + l_{oi} - \frac{D_w}{2}\right).\cos(\alpha_s(r, j))\right].\cos(\psi(r, j)) \tag{16}$$

with:

- *B1* - represents the distance between the centre of curvature of the inner raceway and the origin of the inertial system along the *OX* axis.
- *C1* - represents the distance between the centre of curvature of the inner raceway and the origin of the inertial system along the *OZ* axis.

For "IRR" case result:

$$b_x(r, j) = B1 + \left(\delta_o(r, j) + l_{oe} + \frac{D_w}{2}\right).\sin(\alpha_s(r, j)) \tag{17}$$

$$b_y(r,j) = \left[C1 + \left(\delta_o(r,j) + l_{oe} + \frac{D_w}{2}\right).\cos(\alpha_s(r,j))\right].\sin(\psi(r,j)) \quad (18)$$

$$b_z(r,j) = \left[C1 + \left(\delta_o(r,j) + l_{oe} + \frac{D_w}{2}\right).\cos(\alpha_s(r,j))\right].\cos(\psi(r,j)) \quad (19)$$

- *B1* - represents the distance between the centre of curvature of the outer raceway and the origin of the inertial system along the *OX* axis.
- *C1* - represents the distance between the centre of curvature of the outer raceway and the origin of the inertial system along the *OZ* axis.

$$\delta_i(r,j)=\delta(r,j).(K_{ech}/K_i)^{1/n} \quad (20)$$

3. THE RIGIDITY MATRIX FOR DBB 1-4 IN THE "ORR" AND "IRR" CASES

The common rigidity matrix for DBB1-4 depends of the (r,j) ball rigidity. That matrix "M", respects the "ORR" and "IRR" case.

$$M = \begin{bmatrix} \frac{\partial Fa}{\partial \delta x} & \frac{\partial Fa}{\partial \delta y} & \frac{\partial Fa}{\partial \delta z} & \frac{\partial Fa}{\partial \gamma y} & \frac{\partial Fa}{\partial \gamma z} \\ \frac{\partial Fry}{\partial \delta x} & \frac{\partial Fry}{\partial \delta y} & \frac{\partial Fry}{\partial \delta z} & \frac{\partial Fry}{\partial \gamma y} & \frac{\partial Fry}{\partial \gamma z} \\ \frac{\partial Frz}{\partial \delta x} & \frac{\partial Frz}{\partial \delta y} & \frac{\partial Frz}{\partial \delta z} & \frac{\partial Frz}{\partial \gamma y} & \frac{\partial Frz}{\partial \gamma z} \\ \frac{\partial My}{\partial \delta x} & \frac{\partial My}{\partial \delta y} & \frac{\partial My}{\partial \delta z} & \frac{\partial My}{\partial \gamma y} & \frac{\partial My}{\partial \gamma z} \\ \frac{\partial Mz}{\partial \delta x} & \frac{\partial Mz}{\partial \delta y} & \frac{\partial Mz}{\partial \delta z} & \frac{\partial Mz}{\partial \gamma y} & \frac{\partial Mz}{\partial \gamma z} \end{bmatrix} \quad (21)$$

To assure a simplified writing for the *M* matrix components, the *X* list is introduced. The *X* list is given as:

$$X = (r,j,ux,uz) \quad (22)$$

With that notation the rigidity matrix components are:

$$\frac{\partial Fa}{\partial\{\delta\}} = \sum_r A.\sum_j \frac{\partial[K_i.\delta_i(X)^n.\sin(\alpha_i(X))]}{\partial\{\delta\}}, \quad (23)$$

$$\frac{\partial Fry}{\partial\{\delta\}} = \sum_r \sum_j \frac{\partial[K_i.\delta_i(X)^n.\cos(\alpha_i(X)).\sin(\psi(r,j))]}{\partial\{\delta\}} \quad (24)$$

$$\frac{\partial Frz}{\partial\{\delta\}} = \sum_r \sum_j \frac{\partial[K_i.\delta_i(X)^n.\cos(\alpha_i(X)).\cos(\psi(r,j))]}{\partial\{\delta\}} \quad (25)$$

$$\frac{\partial M_y}{\{\delta\}} = \frac{\partial \sum_r A.\sum_j F_x(X).b_y(r,j) + \partial \sum_r \sum_j F_z(X).b_x(r,j)}{\partial\{\delta\}}. \qquad (26)$$

$$\frac{\partial M_z}{\{\delta\}} = \frac{\partial \sum_r A.\sum_j F_x(X).b_z(j) + \partial \sum_r A.\sum_j F_y(X).b_x(r,j)}{\partial\{\delta\}}. \qquad (27)$$

and:

- *ux, uz*: are the (*r,j*) ball centre of mass displacement
- $\delta_i(X)$: represent the local contact deformation at the inner ring level for the (*j*) index
- $\alpha_i(X)$: represent the inner contact angle of the ball inner ring contact
- b_x, b_y, b_z refers to the "ORR" or "IRR" case respectively.

$$F_z(X) = \sum_r K_i.\delta_i(X)^{1.5}.\cos(\alpha_i(X))\cos(\psi(r,j))) \qquad (28)$$

$$F_y(X) = \sum_r K_i.\delta_i(X)^{1.5}.\cos(\alpha_i(X))\sin(\psi(r,j))) \qquad (29)$$

$$F_x(X) = \sum_r K_i.\delta_i(X)^{1.5}.\sin(\alpha_i(X)) \qquad (30)$$

4. CONCLUSIONS.

The proposed mathematical model shows the differences between the ORR and IRR cases. The boundary conditions modify bearing rigidity and load distribution.

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