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THE ENVELOPE OF THE INDUSTRIAL ROBOT'S MOTION WITH THE END MEMBER MOTION ALONG A CONICAL AREA

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Annotation: *The paper is focused on the possibilities of straight line surfaces machining based on the use of a robotic system. The solving method of the presented problem is illustrated by the specific example, i.e. the movement of the end member of a robot with conical surfaces machining.*
Key words: *robotic mechanism, conical surface, orientation of a robot end element, transform matrix, inverse problem*

1. INTRODUCTION

With the performance of various technological operations (welding, machining, etc.) it is necessary that the end member of a robot moves along the defined path and, at the same time, the tool fulfils certain criteria in relation to the machined surface. The contribution deals with the application of the end element of a robot to conical surfaces machining. The method derived in this paper can be applied to the other straight line surfaces, e.g. the cylindrical one.

2. THE DESIGN OF THE ROBOTIC MECHANISM

The designed robotic mechanism is presented in fig.1. The mechanism has six degrees of freedom, a simple open kinematic structure with rotary kinematic couples only.

We have chosen this design of the mechanism because of the aim of its application and optimality with the regard to the analysis and the solution of the so-called inverse problem. On the other hand the position and the orientation of the end member (tool) is given by six parameters and thus the mechanism must have minimally six degrees of the freedom.

By means of a computer simulation, the robot's manipulation area is determined (envelope of its motion).

3. GEOMETRICAL DESCRIPTION OF THE MECHANISM

In this part we take interest in the geometrical description of the considered mechanism. We denote its edges successively by $P_i, i = 0,1,\cdots,6$. Then the location and the orientation of the end member P_5P_6 are uniquely given by six so-called generalized coordinates $\varphi_i, i = 1, 2, \cdots, 6$. Those are the angles of rotations the meaning of which is obvious from fig. 2. In order to make the analytical description of the mechanism, we express the related points and vectors by the coordinates in the orthonormal anticlockwise oriented systems $S_i = (P_i; \vec{e}_i, \vec{f}_i, \vec{g}_i), i = 0, 1, \cdots, 6$ related to the members of the mechanism as it is shown in fig. 2. Besides these coordinate systems we consider the system S_Z connected to the base (by a workpiece) and S_W connected to the tool. In order to simplify the following notations, we denote $S_Z = S_{-1}$ and $S_W = S_7$. In the sequel we denote the coordinates of the point X by $X^{[i]} = \left[\overrightarrow{P_iX} \cdot \vec{e}_i, \overrightarrow{P_iX} \cdot \vec{f}_i, \overrightarrow{P_iX} \cdot \vec{g}_i\right]^T$ and the coordinates of the vector $\vec{p}$ by $\mathrm{p}^{[i]} = (\vec{\mathrm{p}} \cdot \vec{\mathrm{e}}_\mathrm{i}, \vec{\mathrm{p}} \cdot \vec{\mathrm{f}}_\mathrm{i}, \vec{\mathrm{p}} \cdot \vec{\mathrm{g}}_\mathrm{i})^\mathrm{T}$ in the coordinate system $S_i, i = -1, 0, 1, \cdots, 6, 7$. Between the coordinates of the points in the individual coordinate systems the following transform equations will hold.

$$X^{[j]} = T_{j,i} X^{[i]} + P_i^{[j]}, \quad (1)$$

were

$$T_{j,i} = \left[e_i^{[j]}, f_i^{[j]}, g_i^{[j]}\right] = \begin{bmatrix} \vec{e}_i \cdot \vec{e}_j & \vec{f}_i \cdot \vec{e}_j & \vec{g}_i \cdot \vec{e}_j \\ \vec{e}_i \cdot \vec{f}_j & \vec{f}_i \cdot \vec{f}_j & \vec{g}_i \cdot \vec{f}_j \\ \vec{e}_i \cdot \vec{g}_j & \vec{f}_i \cdot \vec{g}_j & \vec{g}_i \cdot \vec{g}_j \end{bmatrix}, i, j = -1, 0, 1, \cdots, 6, 7 . \quad (2)$$

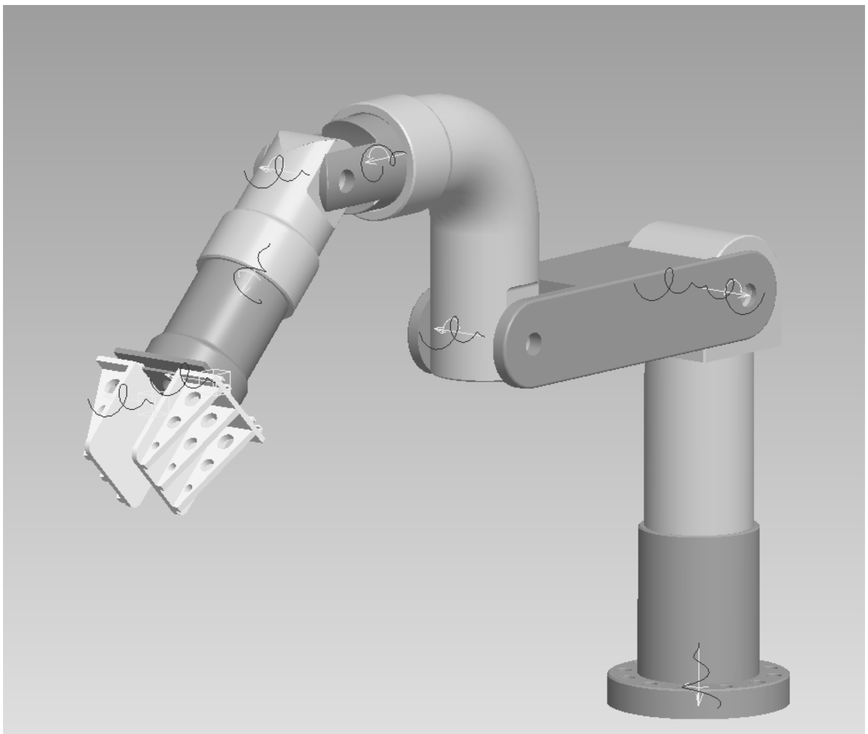

Fig. 1 Mechanism of a robot

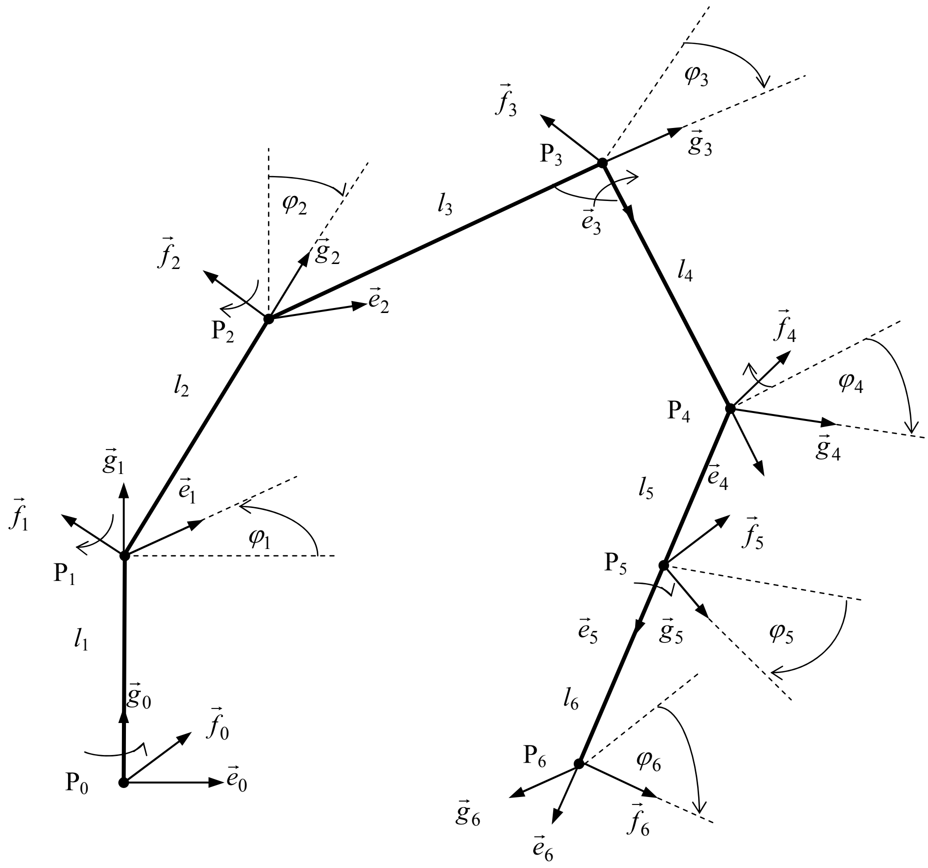

Fig. 2 Coordinate systems and the generalized coordinates

If we apply the notation of the so-called extended coordinates and transform matrices

$$\overline{X}^{[i]} = \left[\begin{array}{c} X^{[i]} \\ \hline 1 \end{array}\right] = \left[\begin{array}{c} \overrightarrow{P_i X} \cdot \vec{e}_i \\ \overrightarrow{P_i X} \cdot \vec{f}_i \\ \overrightarrow{P_i X} \cdot \vec{g}_i \\ \hline 1 \end{array}\right], \overline{T}_{j,i} = \left[\begin{array}{c|c} T_{j,i} & P_i^{[j]} \\ \hline 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|c} \vec{e}_i \cdot \vec{e}_j & \vec{f}_i \cdot \vec{e}_j & \vec{g}_i \cdot \vec{e}_j & \overrightarrow{P_i P_j} \cdot \vec{e}_i \\ \vec{e}_i \cdot \vec{f}_j & \vec{f}_i \cdot \vec{f}_j & \vec{g}_i \cdot \vec{f}_j & \overrightarrow{P_i P_j} \cdot \vec{f}_i \\ \vec{e}_i \cdot \vec{g}_j & \vec{f}_i \cdot \vec{g}_j & \vec{g}_i \cdot \vec{g}_j & \overrightarrow{P_i P_j} \cdot \vec{g}_i \\ \hline 0 & 0 & 0 & 1 \end{array}\right], \quad (3)$$

we can rewrite the transform equations (1) in the compact form

$$\overline{X}^{[j]} = \overline{T}_{j,i} \overline{X}^{[i]}, i, j = -1, 0, 1, \cdots, 6, 7 . \quad (4)$$

Since the repeated transition from one coordinate system to another one is given by the product of the related extended transform matrices, it is easy to verify that the following relations hold

$$\overline{T}_{j,i} = \overline{T}_{j,j+1} \overline{T}_{j+1,j+2} \cdots \overline{T}_{i-2,i-1} \overline{T}_{i-1,i} , \quad \overline{T}_{i,j} = \left(\overline{T}_{j,i}\right)^{-1}, -1 \le j < i \le 7 . \quad (5)$$

4. THE INVERSE PROBLEM AND ITS SOLUTION

Let us suppose that there are given the lengths of the individual members of the mechanism $l_1, l_2, \cdots, l_6$ and the extended transform matrices

- $\overline{Z} = \overline{T}_{-1,0}$ - the given location and orientation of the mechanism with respect to the base,
- $\overline{W} = \overline{T}_{6,7}$ - the given location and orientation of the tool with respect to the end member,
- $\overline{T} = \overline{T}(t)$ - the required location and orientation of the tool with respect to the base at the given time instant t.

The aim is to determine the values of the generalized coordinates $\varphi_i = \varphi_i(t), i = 1, 2, \cdots, 6$ so that the transform matrix $\overline{T}_{-1,7} = \overline{T}_{-1,7}(\varphi_1(t), \varphi_2(t), \cdots, \varphi_6(t))$ holds the equality

$$\overline{T}_{-1,7} = \overline{T} \quad . \tag{6}$$

This equality leads to twelve nonlinear equations with six unknowns.

5. THE MOTION OF THE END MEMBER WITH THE PRODUCTION OF CONICAL SURFACES

Let k be a spatial curve which is given at the basic coordinate $S_Z = S_{-1}$ by the parametric equations

$$\begin{aligned} x &= x_z = x_{-1} = \varphi(t), \\ y &= y_Z = y_{-1} = \psi(t), \\ z &= z_Z = z_{-1} = \xi(t), \; t \in < t_0, t_1 > \end{aligned} \tag{7}$$

with the parameter t chosen so that the motion along this curve has in advance given velocity, i.e. there is given the velocity magnitude

$$v(t) = \sqrt{\varphi'^2(t) + \psi'^2(t) + \xi'^2(t)} \, .$$

Let V be the given point in the space which in the basic coordinate system has the coordinates $V = [v_1, v_2, v_3]^T$. Then the generalized conical surface given by the curve k and the point V has the parametric equations (with respect to the basic coordinate system)

$$\begin{aligned} x &= \varphi(t) + (v_1 - \varphi(t))s, \\ y &= \psi(t) + (v_2 - \psi(t))s, \\ z &= \xi(t) + (v_3 - \xi(t))s, \; t \in < t_0, t_1 >, s \in < 0, 1 > . \end{aligned} \tag{8}$$

We shall require that the motion of the tool is so that its working point is at the time t at the related point of the curve k, i.e.

$$P^{[-1]} = [\varphi(t), \psi(t), \xi(t)]^T$$

and the tool is oriented so that $\vec{e} \uparrow\uparrow \overrightarrow{PV}$, the vector $\vec{f}$ is orthogonal to the considered conical surface, and the vector $\vec{g}$ completes the system as the orthonormal and anticlockwise oriented one, i.e.

$$\begin{aligned}
e^{[-1]} &= \frac{(v_1 - \varphi(t), v_2 - \psi(t), v_3 - \xi(t))^T}{\sqrt{(v_1 - \varphi(t))^2 + (v_2 - \psi(t))^2 + (v_3 - \xi(t))^2}}, \\
u^{[-1]} &= (\varphi'(t), \psi'(t), \xi'(t))^T, \\
v^{[-1]} &= e^{[-1]} \times u^{[-1]}, \\
f^{[-1]} &= \frac{v^{[-1]}}{\left|v^{[-1]}\right|}, \\
g^{[-1]} &= e^{[-1]} \times f^{[-1]}.
\end{aligned} \tag{9}$$

Hence, at any time instant t there is the transform matrix determined

$$\overline{T}(t) = \left[\begin{array}{ccc|c} e^{[-1]} & f^{[-1]} & g^{[-1]} & P^{[-1]} \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

and the values of the generalized coordinates can be determined by the procedure from the previous section.

6. THE MOTION ENVELOPE

In many cases the designers have to know the boundaries of the mechanism motion within all moving members of the mechanism. The motion envelope determines the industrial robot's manipulation area for specific operations thus enabling to create a model of a robotized workplace so that if can suit the ideas of a future operation. The envelope of the industrial robot's motion with the end member motion along a conical area can be seen in fig.3.

7. CONCLUSION

Nowadays, automation and robotization represent a worldwide trend in industrial development. Robotization has become one of the strategic orientations in the development of manufacturing processes. Industrial robots are being applied to all basic technologies of both productive and unproductive branches. Their principal role is to create conditions for replacing human labour especially in harmful environment, with monotonous and exhausting work as well as for productivity rate increase.

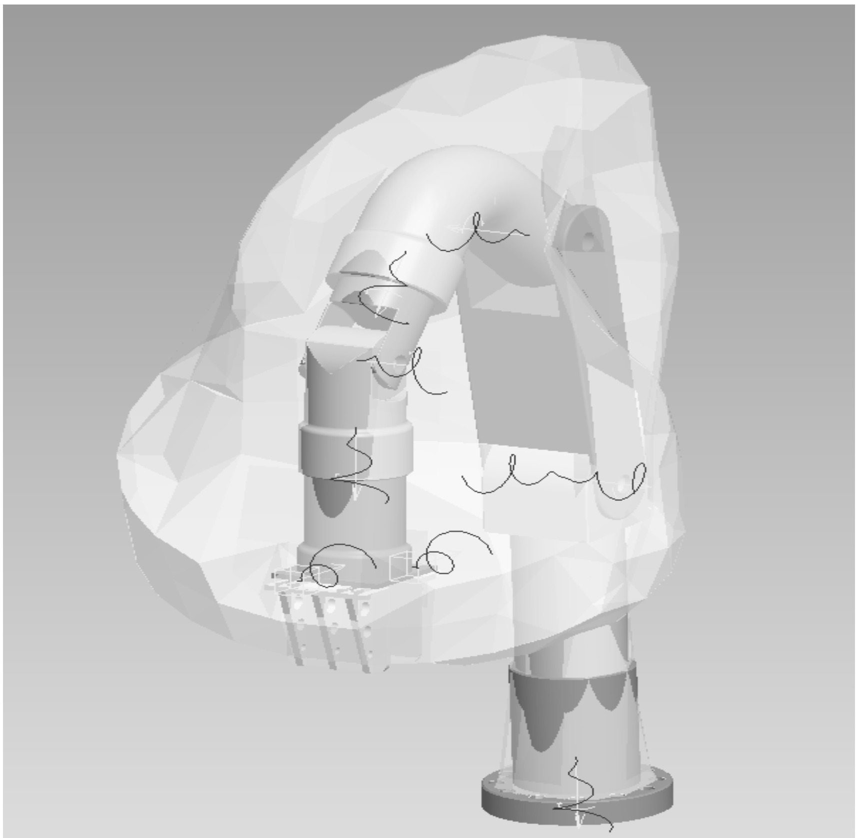

Fig. 3 The envelope of the industrial robot's motion with the end member motion along a conical area

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