A NEW METHOD OF CALCULATING THE EQUATION OF THE CONNECTING ROD CURVE

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Abstract: *The paper presents a new method of calculating the equation of the connecting rod curve, based on the Mathematica package. It differs from the traditional methods such as plotting the connecting rod curves of the four-bar mechanisms starting from their implicit equation form, and the harmonic analysis by determining the Fourier values, which allow for rapid stock and retrieval in the synthesis process. The examples concern general and numerical relations for different planar mechanisms.*

Key words: *connecting rod curve, linear system, articulated four-bar, approximate solutions.*

1. INTRODUCTION

The values on the planar mechanisms connecting rods describe trajectories which become more complex if the mechanism comprises more variables. The equation of this curve is used to check observance of the mechanism functioning conditions and to synthesize them. There were several scholars [Roberts, Cebâşev, Sylvester etc.] who studied simple mechanism connecting rod curves since complex mechanism equations are very large and difficult to be used. Broadly speaking, authors [Artobolevskii, Handra-Luca, Pelecudi, Antonescu etc.] analyse the articulated four-bar for which they write geometrical conditions based on the form axis projection, and thus compact but undeveloped relations are obtained. It is common knowledge that the connecting rod equation of the articulated four-bar is a tricyclic sixth which depends on the 9 parameters that define the mechanism.

Popescu [1982, 1987] provides a detailed analysis of the connecting rod equation form to be used in the mechanism synthesis. In what follows, we shall discuss the mechanism in further detail.

2. THE CONNECTING ROD CURVE OF THE ARTICULATED FOUR-BAR MECHANISM

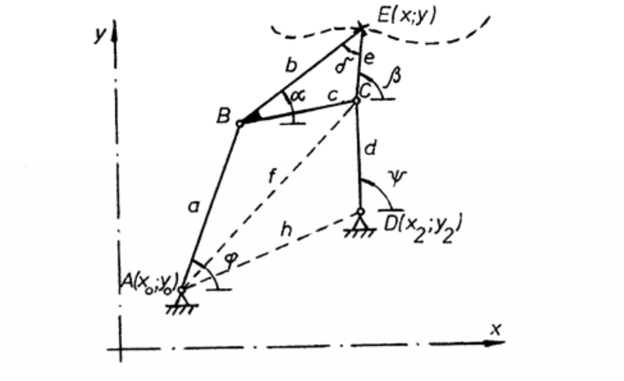

Fig. 1 *The articulated four-bar mechanism*

The E value in Figure 1 projects a trajectory whose equation is to be calculated. Starting from figure 1, we can determine the following relations:

$$\begin{aligned}
&x_0 + a\cos\varphi + b\cos\alpha = x \\
&y_0 + a\sin\varphi + b\sin\alpha = y \\
&x_2 + d\cos\psi + e\cos\delta = x \quad , \quad (1) \\
&y_2 + d\sin\psi + e\sin\delta = y \\
&\beta = \alpha + \delta
\end{aligned}$$

If we omit φ and ψ, then:

$$\begin{aligned}
&(x - x_0 - b\cos\alpha)^2 + (y - y_0 - b\sin\alpha)^2 = a^2 \\
&\left[x - x_2 - e\cos(\alpha+\delta)\right]^2 = \left[y - y_2 - e\sin(\alpha+\delta)\right]^2 = d^2
\end{aligned} \quad , \quad (2)$$

System (2), where α depends on position, is rewritten as:

$$\begin{aligned}
&A_1\cos\alpha + B_1\sin\alpha = C_1 \\
&A_2\cos\alpha + B_2\sin\alpha = C_2
\end{aligned} \quad , \quad (3) \quad \text{where :}$$

$$\begin{aligned}
&Q = A_1 = 2b(x - x_0); K = B_1 = 2b(y - y_0); F = C_1 = (x - x_0)^2 + (y - y_0)^2 - a^2 + b^2; \\
&P = A_2 = 2e\left[(x - x_2)\cos\delta + (y - y_2)\sin\delta\right]; G = B_2 = 2e\left[-(x - x_2)\sin\delta + (y - y_2)\cos\delta\right]; \quad (4) \\
&H = C_2 = (x - x_2)^2 + (y - y_2)^2 - d^2 + e^2
\end{aligned}$$

System (3) provides:

$$\cos\alpha = \frac{\begin{vmatrix} C_1 & B_1 \\ C_2 & B_2 \end{vmatrix}}{\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}}; \sin\alpha = \frac{\begin{vmatrix} A_1 & C_1 \\ A_2 & C_2 \end{vmatrix}}{\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}} \; ; \; (5)$$

and results in:

$$E_i = (C_1 B_2 + C_2 B_1)^2 + (A_1 C_2 - A_2 C_1)^2 - (A_1 B_2 - A_2 B_1)^2 = 0 \ ; \ (6)$$

with $i = 1,...,9.$

Popescu (1982) proposes a 27-variable equation depending on the mechanism parameters. By ordering them according to power, the following form is to be found:

$$\begin{aligned} &Q_6 x^2 + Q_3 y^6 + Q_{22} xy^5 + Q_1 x^4 y^2 + Q_8 x^2 y^4 + Q_{14} x^3 y^3 + Q_{25} x^5 + Q_{21} y^5 + Q_{11} x^3 y^2 + Q_{12} x^2 y^3 \\ &+Q_{13} x^4 y + Q_{17} xy^4 + Q_7 x^4 + Q_7 x^4 + Q_4 y^4 + Q_2 x^2 y^2 + Q_{15} x^3 y + Q_{18} xy^3 + Q_{26} x^3 + Q_{23} y^3 \qquad ; (7) \\ &+Q_{16} x^2 y + Q_{19} xy^2 + Q_9 x^2 + Q_5 y^2 + Q_{20} xy + Q_{27} x + Q_{24} y + Q_{10} = 0 \end{aligned}$$

The relations between the 27 variables are determined by numerical methods and the final form is:

$$\begin{aligned} &Q_1 (x^6 + y^6 + x^4 y^2 + x^2 y^4) + Q_2 (x^2 y^2 + x^4) + Q_4 (y^4 - x^4) + Q_5 y^2 + Q_9 x^2 + Q_{10} + \\ &+Q_{10} + Q_{11} (x^5 / 2 + x^3 y^2 + xy^4 / 2) + Q_{12} (y^5 / 2 + x^2 y^3 + x^4 y / 2) + Q_{15} (x^3 y + xy^3) + \ ; \qquad (8) \\ &+Q_{16} x^2 y + Q_{19} xy^2 + Q_{20} xy + Q_{23} y^3 + Q_{24} y + Q_{26} x^3 + Q_{27} x = 0 \end{aligned}$$

Starting from these considerations, we aim to determine the precise equation form by using more recent data packages. The equation is 11 page-long (2 columns); for economy reasons, only a fragment is provided.

4 b^2 d^4 x^2 - 8 b^2 d^2 e^2 x^2 + 4 b^2 e^4 x^2 - 8 b^2 d^2 x^4 + 8 b^2 e^2 x^4 + 4 b^2 x^6 - 8 b^2 d^4 x xa + 16 b^2 d^2 e^2 x xa - 8 b^2 e^4 x xa + 16 b^2 d^2 x^3 xa - 16 b^2 e^2 x^3 xa - 8 b^2 x^5 xa + 4 b^2 d^4 xa^2 - 8 b^2 d^2 e^2 xa^2 + 4 b^2 e^4 xa^2 - 8 b^2 d^2 x^2 xa^2 + 8 b^2 e^2 x^2 xa^2 + 4 b^2 x^4 xa^2 + 16 b^2 d^2 x^3 xd - 16 b^2 e^2 x^3 xd - 16 b^2 x^5 xd - 32 b^2 d^2 x^2 xa xd + 32 b^2 e^2 x^2 xa xd + 32 b^2 x^4 xa xd + 16 b^2 d^2 x xa^2 xd - 16 b^2 e^2 x xa^2 xd - 16 b^2 x^3 xa^2 xd - 8 b^2 d^2 x^2 xd^2 + 8 b^2 e^2 x^2 xd^2 + 24 b^2 x^4 xd^2 + 16 b^2 d^2 x xa xd^2 - 16 b^2 e^2 x xa xd^2 - 48 b^2 x^3 xa xd^2 - 8 b^2 d^2 xa^2 xd^2 + 8 b^2 e^2 xa^2 xd^2 + 24 b^2 x^2 xa^2 xd^2 - 16 b^2 x^3 xd^3 + 32 b^2 x^2 xa xd^3 - 16 b^2 x xa^2 xd^3 + 4 b^2 x^2 xd^4 - 8 b^2 x xa xd^4 + 4 b^2 xa^2 xd^4 + 4 b d^2 y - 4 b e^2 y - 4 b x^2 y + 8 b x xd y - 4 b xd^2 y - 8 b^2 d^2 x^2 y^2 + 8 b^2 e^2 x^2 y^2 + 8 b^2 x^4 y^2 +16 b^2 d^2 x xa y^2 - 16 b^2 e^2 x xa y^2 - 16 b^2 x^3 xa y^2 - 8 b^2 d^2 xa^2 y^2 + 8 b^2 e^2 xa^2 y^2 + 8 b^2 x^2 xa^2 y^2 - 16 b^2 x^3 xd y^2 + 32 b^2 x^2 xa xd y^2 - 16 b^2 x xa^2 xd y^2 + 8 b^2 x^2 xd^2 y^2 - 16 b^2 x xa xd^2 y^2 + 8 b^2 xa^2 xd^2 y^2 - 4 b y^3 + 4 b^2 x^2 y^4 - 8 b^2 x xa y^4 + 4 b^2 xa^2 y^4 - 4 b d^2 ya + 4 b e^2 ya + 4 b x^2 ya - 8 b x xd ya + 4 b xd^2 ya + 4 b y^2 ya +16 b^2 d^2 x^2 y yd - 16 b^2 e^2 x^2 y yd - 16 b^2 x^4 y yd - 32 b^2 d^2 x xa y yd + 32 b^2 e^2 x xa y yd + 32 b^2 x^3 xa y yd + 16 b^2 d^2 xa^2 y yd - 16 b^2 e^2 xa^2 y yd - 16 b^2 x^2 xa^2 y yd + 32 b^2 x^3 xd y yd - 64 b^2 x^2 xa xd y yd + 32 b^2 x xa^2

xd y yd - 16 b^2 x^2 xd^2 y yd + 32 b^2 x xa xd^2 y yd - 16 b^2 xa^2 xd^2 y yd + 8 b y^2 yd - 16 b^2 x^2 y^3 yd + 32 b^2 x xa y^3 yd - 16 b^2 xa^2 y^3 yd - 8 b y ya yd - 8 b^2 d^2 x^2 yd^2 + 8 b^2 e^2 x^2 yd^2 + 8 b^2 x^4 yd^2 + 16 b^2 d^2 x xa yd^2 - 16 b^2 e^2 x xa yd^2 - 16 b^2 x^3 xa yd^2 - 8 b^2 d^2 xa^2 yd^2 + 8 b^2 e^2 xa^2 yd^2 + 8 b^2 x^2 xa^2 yd^2 - 16 b^2 x^3 xd yd^2 + 32 b^2 x^2 xa xd yd^2 - 16 b^2 x xa^2 xd yd^2 + 8 b^2 x^2 xd^2 yd^2 - 16 b^2 x xa xd^2 yd^2 + 8 b^2 xa^2 xd^2 yd^2 - 4 b y yd^2 + 24 b^2 x^2 y^2 yd^2 - 48 b^2 x xa y^2 yd^2 + 24 b^2 xa^2 y^2 yd^2 + 4 b ya yd^2 - 16 b^2 x^2 y yd^3 + 32 b^2 x xa y yd^3 - 16 b^2 xa^2 y yd^3 + 4 b^2 x^2 yd^4 - 8 b^2 x xa yd^4 + 4 b^2 xa^2 yd^4 - 8 a^2 b d^2 e x^2 Cos[de] + 8 b^3 d^2 e x^2 Cos[de] +……………………..………….= 0 .

There are also equations of the connecting rod curve for some particular cases of the mechanism. Again, for economy reasons, these equations are not discussed in the paper. Nevertheless, it is worth mentioning that these equations are simpler. The equation of a real mechanism (in general) is calculated, i.e. numerical data for the mechanism parameters are provided. The following data were used:

$$a = 30, b = 50, d = 40, e = 40, x_A = 10, y_A = 10, x_D = 100, y_D = 20, \delta = 60$$

The data were divided into 10 groups (outside the angle) so as to not use large numbers (we are dealing with a sixth degree equation) and the resulting equation was:

(3.272069459180975*^6 - 403917.81839644327x + 1.0291177647383088*^6x^2 - 320794.25163305853 x^3 + 43316.058002125144x^4 - 2957.431961543907x^5 + 83.99987743596876x^6 - 3.5109850529900985*^6 y + 668102.9952181454xy - 304175.05080927734x^2 y + 44460.26475634218x^3 y - 2039.0748415112719x^4 y + 0. x^5 y + 1.2677451705825222*^6 y^2 - 358975.97026955767x y^2 + 68747.01091271496x^2 y^2 - 5914.863923087814x^3 y^2 + 251.99963230790624x^4 y^2 - 201799.7687668981y^3 + 44460.26475634218x y^3 - 4078.149683022544x^2 y^3 + 0. x^3 y^3 + 25430.95291058982 y^4 - 2957.4319615439076x y^4 + 251.99963230790624x^2 y^4 - 2039.074841511272y^5 + 0.x y^5 + 83.99987743596876y^6 = 0.

The equation is simpler as compared to the case in which letters are used. The equations for several particular cases are to be found following the same procedure. Thus, if $xa = ya = yd = 0$ and the connecting rod triangle is equilateral, then the equation becomes:

2.5599999999999995*^6 + 1.2320026718958812*^6 x + 289300.0085800688x^2 - 192999.91267324513x^3+34999.95786862013x^4 - 2999.9954038488277x^5 + 99.99984679496093x^6 - 3.0206950657787398*^6y - 311770.3129407689xy - 129903.61154848206x^2 y + 34640.99846076536x^3 y - 1732.0499230382688 x^4 y + 0. x^5 y

+ 1.1092989667846297*^6 y^2 - 192999.91267324513x y^2 + 49999.94637824807x^2 y^2 - 5999.990807697656x^3 y^2 + 299.99954038488283x^4 y^2 - 129903.61154848206y^3 + 34640.99846076537xy^3 - 3464.0998460765377x^2 y^3 + 0.x^3 y^3 + 14999.988509627936 y^4 - 2999.995403848828x y^4 + 299.99954038488283 x^2 y^4 - 1732.0499230382684y^5 + 0.x y^5 + 99.99984679496093y^6 = 0.

Popescu [1995] used the same data to check the results and the equation which was found was identical to the one determined by traditional methods; consequently, the two procedures were validated.

3. RECCURENT PROBLEMS IN THE MECHANISM SYNTHESIS BASED ON THE EQUATION OF THE CONNECTING ROD CURVE

The research aim was to determine an equation to be used in the mechanism synthesis. Consequently, the final equation should include 9 variables depending on the mechanism parameters. For a set number of values, we can determine the numerical values of the variables in the equation of the connecting rod curve and solve the non-linear equations that result from the equalizing of the variables and the values.

The problem was solved partially [Popescu, 1995], some relations between the 27 variables being established while 16 variables were still independent. Therefore, new relations were to be determined between the 16 variables in order to reduce them to 9.

The methods based on Mathematica result in too many variables. What we need are numerical correlations between the variables to be used in real cases of four-bar mechanisms. After numerous trials and errors, new correlations were found, but later on these correlations were assumed to have resulted from truncation errors (calculation inaccuracies). For more precise calculations, the results were different. These were accounted for by the non-linear variables in the equation of the connecting rod, which depend on the system parameters.

As shown, the equation of the connecting rod curve cannot be used for the mechanism synthesis: if Q(i) are given values, the 9 variables cannot be determined. The reason is that the system is too complex and the equations are difficult to solve as too large. In fact, this system is more complex than the one used for determining the equation of the connecting rod curve and it is difficult to use in the synthesis because it is non-linear.

Table 1 shows the variables of the simplified equation (8) for a real mechanism. It is obvious that there are no correlations that lead to the equation simplification.

*Table 1.*Solutions

i =1	C(1) = -5.360991E-04	i = 9	C(9) = 28.67414
i = 2	C(2) = 3.85965	i = 10	C(10) = 42.85653
i = 3	C(3) = -2.350439	i = 11	C(11) = -706.2414
i = 4	C(4) = -18.15872	i = 12	C(12) = 19.93094
i = 5	C(5) = 852.3191	i = 13	C(13) = 650.3323
i = 6	C(6) = -0.2270539	i = 14	C(14) = -116.1669
i = 7	C(7) = 0.2633739	i = 15	C(15) = -743.5965
i = 8	C(8) = -3.51949		

There were several attempts to find more correlations between the variables by using different data, different methods and different types of correlations. Yet, the results were not better as approximate numerical data were used, which allowed for calculation errors. Thus, the equations cannot be accurately validated.

4.CONCLUSIONS

The articulated four-bar synthesis cannot be made on the basis of the equations of the connecting rod curve because:

- the analytical form of the equation is too complex to be used for the mechanism synthesis and the resulting non-linear system would be more complex than the initial one;
- the equation of the connecting rod curve is of a high degree (6th) and comprises many variables, thus allowing for serious calculation errors (a 27 equation system);
- the solutions are but approximate, there is no accurate validation of the equation, i.e. for some other values on the curve different variables are to be found;
- there is no improvement if the equation is simplified – calculation errors are still possible;
- if the equations of the articulated four-bar are so complex, it is obvious that the equations of more complex systems are much larger (tens of pages) and cannot be used for synthesis.

4. REFERENCES

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