OPTIMALISING PLANETARY GEARS BUILT IN DRUMS WITH HIGH GEAR-RATIO

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Abstract: *Planetary gears with internal and external toothed wheels are used to perform low weight drives of high gear-ratio. Their weight and dimensions are influenced by the number of stages and by the gear-ratio of each planetary gear. The aims of the investigation are to find the proper stages' number and gear-ratios leading to optimal solutions where the weight of planetary gears built in drums are minimum (The outer diameters of inner toothed rings should be equal).*
Key words: *List*

1. INTRODUCTION

The planetary gears are widespread in industry, because of their beneficial properties. Their power densities are high and they can add up or divide powers. They can operate with low or high gear-ratios and with high efficiency. Planetary gears containing internal and external toothed wheels (type KB) are used plenty of times because the efficiency of meshing gears is more than 90% at A gear-ratio of 700 [1]

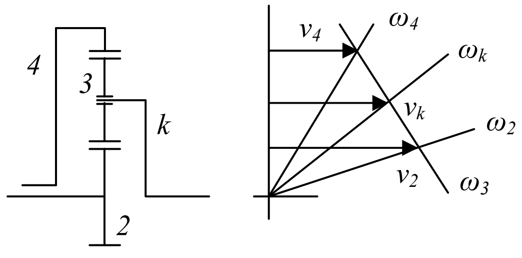


Figure 1.
Velocity diagram of planetary gears with internal and external toothed wheels

$$\omega_4 = \left(1 - \frac{1}{i_b}\right) \cdot \omega_k + \frac{1}{i_b} \cdot \omega_2 \qquad (1)$$

Transferring more power is needed to place more planetary gears between the inner toothed ring and the sun wheel. However, increasing the number of planetary gears limits the ratio of planetary gear drives. Using three planetary gears in a type KB planetary gear drive its maximum gear ratio can reach the value of 11-12 depending s on the high of teeth. To produce planetary gear drives with higher gear-ratios make it necessary to couple more planetary gear drives or choose other types having generally lower efficiency.

The aims of this investigation is to find an optimal solution of planetary gear drives built in drums for driving belt conveyors. The optimizing condition is to reach the lower weight of the planned planetary gear drives. The planetary gear drives built in the drums have to contain three planetary wheels, a sun wheel and an inner wheel. The task is to find the best gear-ratios for each planetary gear coupled together to perform the drives of the drum with the required gear ratio and to reach the lowest weight. The planetary gear drives have to have such constructions that every inner gear drives the drum, so they have the same angular velocity as the drum. For this reason a motor drive the sun wheel of the first planetary stage and its planet carrier drives the sun wheel of the next planetary stage. The planet carrier of the last planetary stage does not rotate.

2. EQUATION DESCRIBING THE GEAR RATIO OF A PLANETARY GEAR DRIVE BUILT IN A DRUM

Using the velocity diagram of planetary gears types KB (see in Figure 1.) the motion equation of motion (1) can be written easily for the two planetary gears. Using these equations the correlation between the angular velocities of the elements of a planetary gear drives type KB+KB (see in Figure 2.) can be determined.

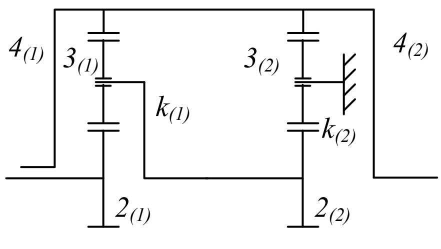


Figure 2.

Planetary gears drive having two planetary stages types KB

To reach a high gear ratio the following power flow has to perform:

- driving element is the sun gear of the first planetary stage;
- the carrier of the first planetary stage drives the sun gear of second planetary stage;

- the inner gears of both planetary stages rotate the drum;
- therefore the carrier of the second planetary stage has to stand.

The general equation of motion (1) can be written for the two planetary gears (2;3), and the modified version of them can be used for the whole drive ($\omega_{4(1)} = \omega_{4(2)}$; $\omega_{k(1)} = \omega_{2(2)}$).(4;5).

$$\omega_{4(1)} = \left(1 - \frac{1}{i_{b(1)}}\right) \cdot \omega_{k(1)} + \frac{1}{i_{b(1)}} \cdot \omega_{2(1)} \quad and \quad \omega_{4(2)} = \left(1 - \frac{1}{i_{b(2)}}\right) \cdot \omega_{k(2)} + \frac{1}{i_{b(2)}} \cdot \omega_{2(2)} \tag{2;3}$$

$$\omega_{4} = \left(1 - \frac{1}{i_{b(1)}}\right) \cdot \omega_{k(1)} + \frac{1}{i_{b(1)}} \cdot \omega_{2(1)} \quad and \quad \omega_{4} = 0 + \frac{1}{i_{b(2)}} \cdot \omega_{k(1)} \tag{4;5}$$

Joining equations (4) and (5) the relation between angular velocity of driving and driven elements of planetary gear drives type KB+KB (6) and the gear- ratio of these drives (7) can be determined (6) if the internal gear ratios $i_{b(1)}$ and $i_{b(2)}$ are known.

$$\omega_{4} = \left(1 - \frac{1}{i_{b(1)}}\right) \cdot i_{b(2)} \cdot \omega_{4} + \frac{1}{i_{b(1)}} \cdot \omega_{2(1)} \tag{6}$$

$$i_{drive} = \frac{\omega_{2(1)}}{\omega_{4}} = i_{b(1)} - i_{b(1)} \cdot i_{b(2)} + i_{b(2)} \tag{7}$$

3. DETERMINING THE OPTIMAL GEAR RATIOS

Volume of a gear box depends on the center distance of gear pairs and also on the gear ratio. The center distance of a gear pair in planetary gear type KB is determined by the load carrying capacity of the spur gears (sun wheel and planetary wheel pair), so it is necessary to know the gear-ratio (u_{32}) these geared wheels to calculate their center distance (8):

$$u_{32} = \frac{i_b + 1}{-2} \tag{8}$$

If applied torque T_2 loads the driving sun gear and gear-ratio u_{32} are known, the center distance of gear pairs can be calculated using the equation (9):

$$a_{w23} = \sqrt[3]{\frac{b}{d} \cdot T_2 \cdot \frac{(u_{32}+1)^4}{u_{32}} \cdot \frac{1}{2 \cdot k} \cdot \frac{1}{\sin 2\alpha_w}}\,, \tag{9}$$

where b/d is the ratio of the width and pitch diameter of smallest gear, *k* the allowable surface pressure, α_w angle of action. However, there is a connection between the center distances of gear pairs of both planetary stages in order to reach the requirement that the radii of both inner gears are the same. Taking this fact into consideration the pitch diameter of inner gear can be derived in the following form, using the center distances $a_{w23(1)}$, $a_{w23(2)}$ and gear ratios $u_{32(1)}$ $a_{32(2)}$ (10):

$$d_{w4} = \frac{2 \cdot a_{w23(1)} \cdot \left(2 \cdot u_{32(1)} + 1\right)}{u_{32(1)} + 1} = \frac{2 \cdot a_{w23(2)} \cdot \left(2 \cdot u_{32(2)} + 1\right)}{u_{32(2)} + 1} \tag{10}$$

This equation can be transformed as follows (11):

$$d_{w4} = \frac{2 \cdot a_{w23(1)} \cdot u_{43(1)}}{u_{43(1)} - 1} = \frac{2 \cdot a_{w23(2)} \cdot u_{43(2)}}{u_{43(2)} - 1} \tag{11}$$

Expressing the pitch diameter d_{w4} with inner gear-ratios $i_{b(1)}$ and $i_{b(2)}$:

$$d_{w4} = \frac{2 \cdot a_{w23(1)} \cdot \dfrac{2 \cdot i_{b(1)}}{i_{b(1)} + 1}}{\dfrac{2 \cdot i_{b(1)}}{i_{b(1)} + 1} - 1} = \frac{2 \cdot a_{w23(2)} \cdot \dfrac{2 \cdot i_{b(2)}}{i_{b(2)} + 1}}{\dfrac{2 \cdot i_{b(2)}}{i_{b(2)} + 1} - 1} \tag{12}$$

Comparing the equation (7) and (12) the center distance of gear pairs of second planetary stage is:

$$a_{w23(2)} = a_{w23(1)} \cdot \frac{i_{b(1)}}{i_{drive} + i_{b(1)}} \cdot \frac{i_{drive} + 1}{i_{b(1)} - 1}. \tag{13}$$

The center distance $a_{23(1)}$can be calculated using equation (9) with data $T_{2(1)}$ and $u_{32(1)}$ and also the center distance $d_{23(2)}$. But during the calculation it have to make certain of the proper load carrying capacity of the second planetary stage, and if it is necessary the center distance $a_{w23(2)}$ have to be increased. For the calculation of center distance $a_{w23(2)}$ the equation (9) can be used, in which instead of $u_{32(1)}$ an $T_{2(1)}$ the gear ratio $u_{32(2)}$ and the increased applied torque $T_{2(2)}=(1-i_{b1})T_{2(1)}$ have to substitute.

Knowing the center distances of gear pairs the equations to determine the pitch diameters of each toothed wheel can be expressed with the gear ratio (u_{32}).

Pitch diameters of sun gears are:

$$d_{w2} = \frac{2 \cdot a_{w23}}{u_{32} + 1} \tag{14}$$

Pitch diameters of planetary gears are:

$$d_{w3} = \frac{2 \cdot a_{w23} \cdot u_{32}}{u_{32} + 1} \tag{15}$$

Pitch diameters of ring gears are:

$$d_{w4} = \frac{2 \cdot a_{w23} \cdot (2 \cdot u_{32} + 1)}{u_{32} + 1} \tag{16}$$

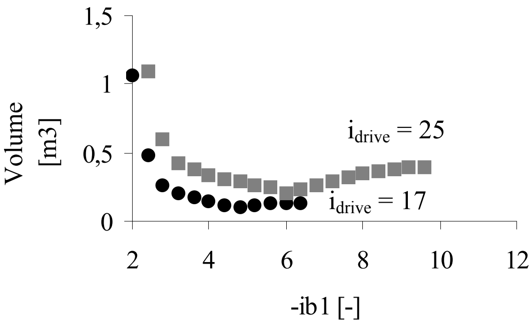
The detailed structure of the planetary gear drive is unknown so the determination of its volume was calculated using the following assumptions:

- mass of house of a planetary gear drive is three times as big as mass of inner gear;
- the driving shaft and the driven shaft have the summa mass of the sun gears;
- the mass of carriers and bearings is equal to the mass of planetary gears;
- the width of gears is determined by the diameter of the smaller meshing gears.

To achieve high load carrying capacity, the ratio of width and pitch diameter of small gear is equal 1 assuming precision manufacture of the gears. Using the determined geometrical dates of gears the volume of planetary gear drive can be calculated.

According to this calculation the variation of the volume of planetary gear drive in function of inner gear ratio of the first planetary stage be seen in Figure 3 and 4. In the calculation the following data were used, which do not influence the results of the optimization of the sizes of planetary gear:

- torque input: $M = 3000$ [Nm];
- bearing pressure: $k = 7$ [N/mm^2];
- ratio between width and diameter of gear: $b/d_w = 1$ [-];
- angle of action: $\alpha_w = 20^\circ$;
- number of planetary gears: N = 3 [db].



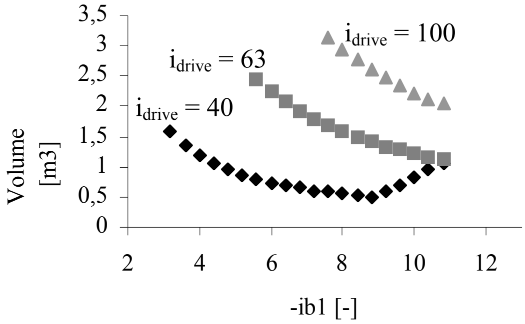


Figure 3.
Volume of drive with the ratio of first stage in low ratio of drive

Figure 4.
Volume of drive with the ratio of first stage in high ratio of drive

In the Figure 3 it can be seen that choosing the inner ratio of first stage -4.8 the volume of driving has minimum if the gear-ratio of planetary gear drive is 17. When the gear-ratio of planetary gear drive is 25, the volume has a minimum if the inner ratio of first stage is -6. When the gear-ratio of planetary gear drive is 40, the volume has a minimum if the inner ratio of first stage 8.8. (Figure 4.)

If the two stage planetary gear is planned for higher gear ratio than above mentioned, there is no optimal inner gear ratio of the first planetary stage because it has to be too high values which leads to collide the three planetary wheels. It can be seen in Figure 4. showing the continuous decreasing of the volume without reaching its minimum in the range of the applicable inner gear ratio, when the gear ratio of planetary gear drive is 75 or 100.

Maybe in these cases a planetary gear drive having three stages can be the optimal solution: its volume can be smaller.

4. SUMMARY

According to the calculations, in case of more stage planetary gears it is useful to choose an optimal value for ratio by particular drives to get the minimal volume and mass what can be achieved. Sometimes it is more advantageous to realize the drive with more stages than it is needed to get a planetary drive with lower mass.

5. ACKNOWLEDGMENTS

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