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INNER EXCITATION CHARACTERISTICS AND MESHING STIFFNESS FUNCTION OF GEARS

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Abstract: *For studying dynamic behavior of gear trains, description of the inner excitation properties is needed. Kinematic excitation, due to meshing irregularities and profile modifications will be studied, and described by an adequate mathematical model. Dynamic excitation component, introduced mainly by the variable number of tooth pairs in contact, will be combined with the kinematic excitation function, into a general excitation function, called as reduced stiffness function.*
Key words: *gear dynamics, kinematic excitation, gear meshing, inner excitation, tip relief.*

1. INTRODUCTION

Dynamical behavior and noise characteristics of gear transmissions are influenced not only by the gears, as elastic elements with mass, but the gears, as active vibration exciting elements (inner excitation) contribute considerably to these features. Inner excitation comes partly from *kinematical*, partly from *dynamical* effects. The kinematic excitation component is due to the mesh irregularities introduced by deviations of real profiles from the ideal one (manufacturing errors, intended profile modifications, as tip relief etc.), and by tooth deformations under load. Dynamic excitation is due to the variation of the resulting mesh stiffness of teeth in contact, during rolling down. This variation comes from the variation of tooth contact point in tooth height direction, the non-linear stiffness characteristics of teeth [5], and the variation of the number of teeth during contact. These two kinds of excitation effects influence the vibration characteristics of gears in close interaction with each other.
The tip relief [4], for example, has primary effect on kinematic excitation, but, as secondary effect, influences the "contact ratio", which will be load dependent, consequently, the average mesh stiffness varies with the load. Most significant influence can be archived with "long tip relief", where the theoretical contact ratio at zero load is less then one.
Further on, mesh irregularities will be treated, kinematic excitation component formulation, its combination with dynamic component will be presented. Some conclusions will be discussed in case of long tip relief.

2. NOMENCLATURE

A,E - end points of contact of profile pairs, on the contact line, B,D- single tooth contact points, C- pitch point, N_i- points of tangency of the pressure line, with the base circle of index i, F_N - normal force on teeth in mesh, $h(\varphi_1)$ – clearance, $i(...)$ - ratio function, i_n - nominal ratio, J_1, J_2 - moments of inertia, K_j - damping coefficient on j-th tooth pair, n_1 - number of revolution of driver, r_{a1}, r_{a2} - addendum circle radii of gear 1, 2, r_{b1}, r_{b2} - base circle radii of gear 1, 2, r_{bl1}, r_{bl2} - base circle radii of relief profile of gear 1, 2, r_{l1}, r_{l2} - inner radii of relief profile on gear 1, 2, r_{w1}, r_{w2} rolling circle radii of gear 1, 2, T_1, T_2 - torque on gear 1, 2, z_1,z_2 - number of teeth of gear 1,2, $\bar{s}(\varphi_1)$ - spring stiffness of one tooth pair, w_j - deflection of one tooth pair in pressure line direction, α_w - pressure angle on the rolling circle, φ_1,φ_2, - angle of rotation of driver and gear, φ_{2n} - nominal angle of rotation of gear, γ_{1b} - central angle of one pitch on pinion, ω_1 - tooth angular frequency, Ω - rotation angle on the pinion of the period of kinematic excitation function.

3. ANALYTIC FORMULATION OF INNER EXCITATION

Mathematical formulation of kinematic excitation component can be carried out by the application of $\delta_j(\varphi_1)$ *contact functions*, defined for each profile pair combinations as

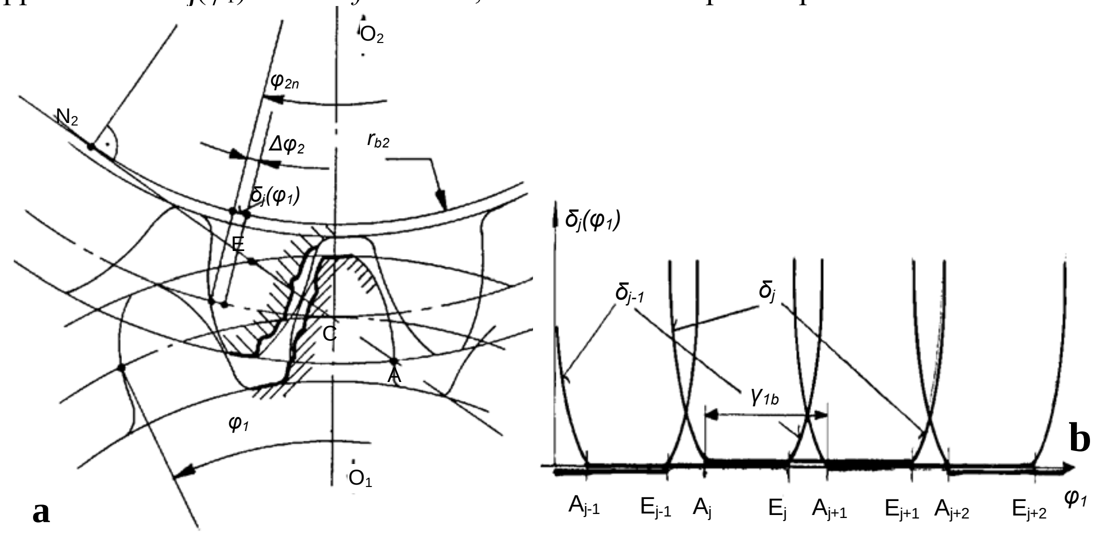

Fig. 1. *Definition of contact function (a), series of contact functions in case of ideal gears (b)*

$\delta_j(\varphi_1)=r_{b2}\cdot(\varphi_{2n}-\varphi_2)$. So, $\delta_j(\varphi_1)$ gives the deviation of real angular position of the driven gear related to its nominal position, measured in *length* on the pressure line, in the function of pinion angular position, in case of contact of the *j*-th profile pair, under zero load, Fig. 1. a. (It is supposed, that the neighboring profiles are taken out, consequently, the gear position is defined by the *j*-th profile pair.) In case of ideal contact of ideal gears, $\delta_j(\varphi_1)=0$ (Fig. 1.b.) on $\overline{A_kE_k}$ pressure line segments, for k=1,2,..j..In case of real gears with real profiles, $\delta_j(\varphi_1)\neq 0$ generally. Similarly, for profile with tip relief, $\delta_j(\varphi_1)\neq 0$ on the contact of relief zone, even in case of ideal conditions [2,3].

In case of *meshing under load*, due to the tooth deflection, the gear lags back related to its nominal angular position. So, the following profile pair enters into contact already before the point A, and the forgoing one, remains in contact even after the point E. In these contact zones, non-conjugate profile pairs are meshing (contact on the top land edge), hence they will be called as *irregular contact zones*, Fig. 2.

3.1. Meshing on the irregular contact zone

Let us consider the mesh position represented on Fig. 2.b., with directions of rotation of Fig. 2.a., for case of $i_n=z_2/z_1>1$. The profile pair 1.,2., are in position of just before entering into contact. Angular position of gears 1., 2. are defined by the rotational angles of points on the

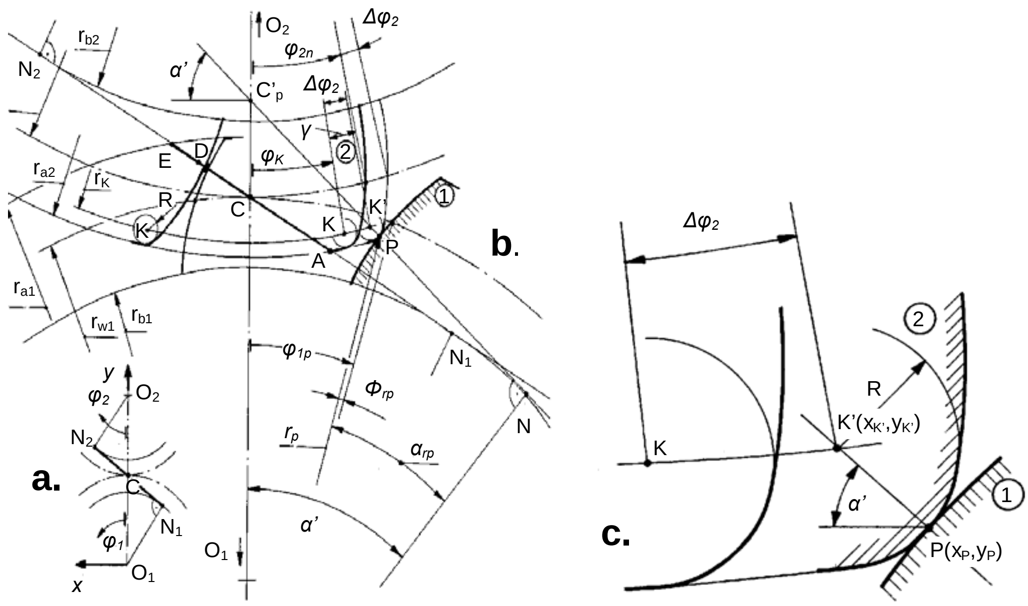

Fig. 2. *Meshing on the irregular contact zone*

rolling circle of profiles 1., 2., so $\varphi_{2n}=\varphi_{1P}/i_n$,. It is supposed, that the top land edge of gear is rounded off with circle of radius *R*, having its center point *K* on a circle of radius r_k Supposing the pinion is fixed, contact of profiles 1., 2. can be archived by rotating the gear in counter-clock wise, with angle $\Delta\varphi_2$, $\varphi_2=\varphi_{2n}+\Delta\varphi_2$, and $\delta_j(\varphi_{1P})= r_{b2}\,\Delta\varphi_2$, is the value of contact function to be determined. Meshing of profiles 1, 2 can take place, if the gear is lagging behind, due ex. by deformations under load on regularly meshing teeth, or due to manufacturing errors.

Let us suppose, that in the given position of pinion, contact will take place in point P of profile 1., with point K being in position of K'. One can write than a set of geometric equations, see [1], for the calculation of unknown parameters of contact of profiles 1, 2, and for calculation of actual ratio, $i(\varphi_{1P})$.

Fig. 3. a. represents the variation of ratio, in function of φ_1. On irregular zone before point A ($\varphi_1<\varphi_{1A}$), the instantaneous ratio is less than the nominal one, hence it is relatively multiplier. In a similar way, one can conclude, that on irregular zone after point E, instantaneous ratio is bigger than the nominal one: the train is relatively decelerator. Values of contact function can

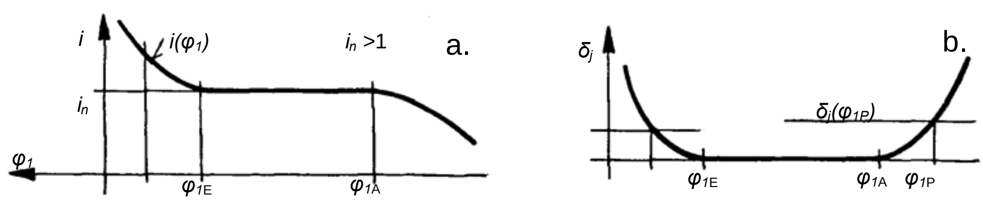

Fig. 3. *The ratio (a) and the contact function, for ideal gears, with irregular contact zones*

be calculated either by calculating the length on the pressure line corresponding to angle with which the gear lags back, or by integrating the ratio function [1].

3.2. Contact function for profiles with tip relief

For gears with long tip relief [4], the tooth profile is composed of two different involute curves [4], consequently the ratio is different on the contact zones of the two profile curve parts. On Fig. 4. ratio function (a) and contact function (b) are represented for case of long tip relief. The irregular zones are not shown.

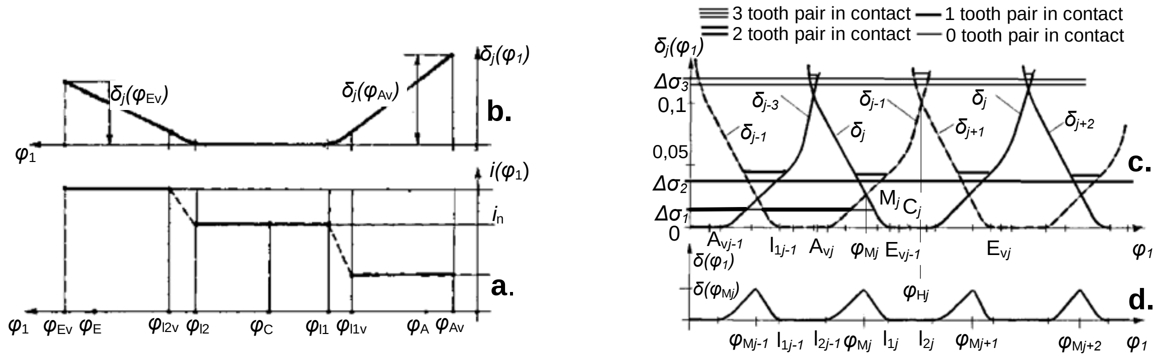

Fig. 4. *Ratio (a) and contact function (b), , series of contact functions (c), contact function of gear train (d), for gear with tip relief [1]*

On Fig. 4.c., series of contact functions with irregular zones are represented, for gears with long tip relief. The rolling down under load, resulting in a constant lagging back of gear with value on the pressure line of $\Delta\sigma = r_{b1}.\varphi_1 - r_{b2}.\varphi_2$=const.>0, can be represented. with a straight line, Fig. 4. c. If $\Delta\sigma(\varphi_1)>\delta_j(\varphi_1)$, then the *j*-th tooth pair is in contact. The deflection of a tooth pair, being in contact at a given angle φ_1 , will be $w_j = \Delta\sigma - \delta_j(\varphi_1)$. One can follow well the number of tooth pairs in contact, at a given $\Delta\sigma$, in function of φ_1., Fig. 4. c. In case of rolling down with zero load, angular position of the gear will be determined by tooth pair in contact position, giving the smallest lagging back of the driver: $\delta(\varphi_1)=\min\limits_{j=1..k}\lfloor\delta_j(\varphi_1)\rfloor$, where $\delta(\varphi_1)$ is the contact function of gear train, Fig. 4.d. It is equal with the standardized kinematic error curve.

3.3. The generalized load dependent contact ratio function $\varepsilon_t(\Delta\sigma)$

Let us consider the contact function $\delta_j(\varphi_1)$, defined on interval $\varphi_{1\,j\,\mathrm{I}}<\varphi_1<\varphi_{1\,j\,\mathrm{S}}$, and let us introduce the indicator function [1]:

$$\mathbf{I}_j(\varphi_1;\Delta\sigma)=\begin{cases}1 & if\ \Delta\sigma\rangle\,\delta_j(\varphi_1)\\ & \varphi_{1jI}<\varphi_1<\varphi_{1jS}\\ 0 & if\ \Delta\sigma\le\delta_j(\varphi_1)\end{cases} \qquad (1)$$

For any angular interval Φ:

$$\varepsilon_t(\Delta\sigma)=\left[\sum_{j=1}^{n}\int_{\Phi} \mathrm{I}_j(\varphi_1)d\varphi_1\right].(\Phi^{-1})\ge 0, \qquad (2)$$

where n is the number of profile pairs in contact. $\varepsilon_t\,(\Delta\sigma)$. gives the average number of profile (tooth) pairs in contact, during rolling down on Φ. Hence the number of tooth pairs in contact is load dependent, see Fig. 4.c., $\varepsilon_t(\Delta\sigma)$ is load dependent as well. So, ε_t is called as *real contact ratio*. For case of the ideal rolling down with zero load of ideal normal gears, with $\Phi=\gamma_{1b}$, applying Eq. (2) (Fig. 5):

$$\varepsilon_t = \frac{\lfloor 2(\varphi_{Ej-1} - \varphi_{Aj}) + (\varphi_{Aj+1} - \varphi_{Ej-1}) \rfloor}{\gamma_{1b}} \quad . \tag{3}$$

Multiplying the nominator and denominator in Eq. (3) with r_{b1} , one can obtain the basic geometric definition of contact ratio ε_α .

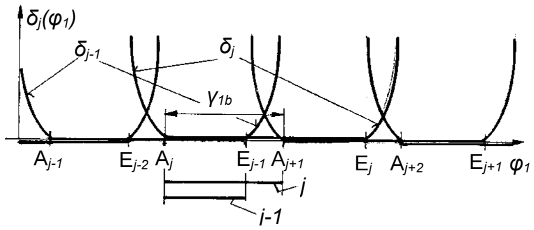

Fig. 5. The ε_t *(=ε_α) real contact ratio for ideal gears with zero load*

4. THE GENERAL EXCITATION FUNCTION

In case of meshing under load, besides the kinematic component, inner excitation properties of gears are influenced by the variation of tooth stiffness and the variation of real contact ratio, i. e. by the actual number of teeth in contact. The two kind of effects appear in close interaction. Their integrated treatment can be carried out by introducing a two variables reduced stiffness function $\hat{s}(\varphi_1;\Delta\sigma)$ as follows:

$$\hat{s}(\varphi_1;\Delta\sigma) = \begin{cases} 0, & if \ \Delta\sigma \le \delta(\varphi_1) \\ \dfrac{\sum_j F_{Nj}}{\Delta\sigma} = \dfrac{\sum_j \bar{s}_j(\varphi_1).w_j}{\Delta\sigma} = \sum_j \bar{s}_j(\varphi_1).\left(1 - \dfrac{\delta_j(\varphi_1)}{\Delta\sigma}\right) & if \ \Delta\sigma \rangle\ \delta(\varphi_1) \end{cases} \tag{4}$$

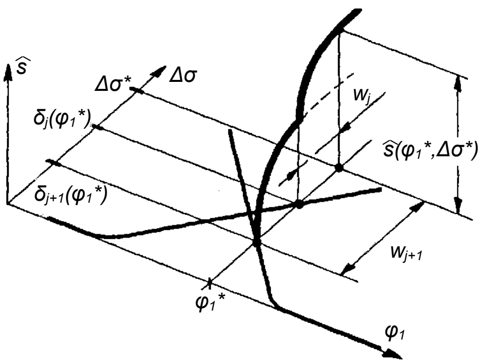

Fig. 6. *Geometric interpretation of reduced stiffness function*

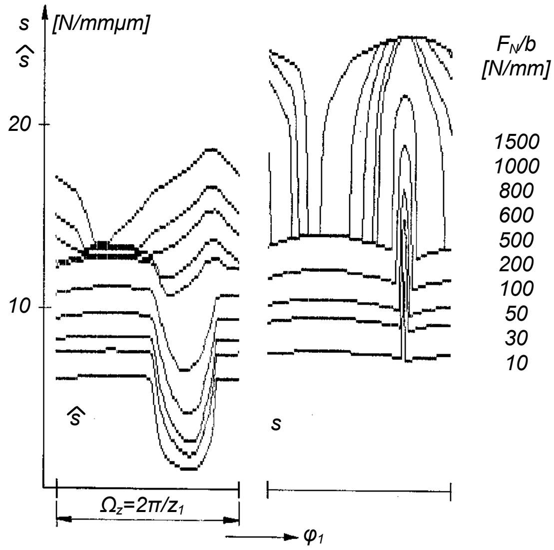

Fig. 7. *Reduced stiffness and mesh stiffness function for gears with tip relief*

In case of non-linear single tooth stiffness curve [5] Eq. (4) is formally more complex, because of the force on one tooth pair, can't be expressed simply as $F_{Nj} = \bar{s}_j(\varphi_1).w_j$.

Geometric interpretation of reduced stiffness function is represented on Fig. 6. for profiles with tip relief, meshing on the relief profile segment.

On left hand side of Fig. 7. one period of reduced stiffness function is represented for ideal gears with long tip relief of number of teeth 53/65, in case of rolling down with constant line pressure values, assuming non-linear single tooth pair stiffness characteristics. Curves reflect well the considerable variation of inner excitation effects. At line pressures of 500-600N/mm, the variation of excitation is rather small, indicating the optimum load range in case of long tip relief.

On right hand side of Fig. 7. the $s(\varphi_1;\Delta\sigma) \neq \hat{s}(\varphi_1;\Delta\sigma)$ mesh stiffness function is represented. $s(\varphi_1;\Delta\sigma)$ [2,3] is the sum of single tooth pair stiffness, and varies as the real contact ratio varies. It is considerably load dependent as well. One can say, that the mesh stiffness c_γ applied in normal gear calculations [4] is the integral mean of $s(\varphi_1;\Delta\sigma)$, for case of ideal gears with ideal rolling down. Hence, $s(\varphi_1;\Delta\sigma)$ is the generalization of mesh stiffness. Further on, one can follow well the variation of real contact ratio, as the load varies, indicating an increasing stiffness of gear contact with increasing load.

5. CONCLUSIONS

Experience shows, that vibration and noise characteristics of gear trains are strongly influenced by intended profile modifications in height and length direction, manufacturing precision and interaction of gear induced excitations and vibration characteristics of other elements of transmissions. For more realistic simulation of dynamic behavior of trains, detailed study and adequate mathematical formulation and description of excitation effects is needed. It was shown, that traditional definitions of some important parameters as contact ratio, mesh stiffness should be generalized for enabling the treatment of load dependency, which is considerably especially for gears with long tip relief. Further on, mathematical formulation of kinematic excitation and integrated interpretation of inner excitation effects allows the taking into consideration of important parameters, as manufacturing errors, profile modifications etc.

6. REFERENCES

1. Márialigeti, J.:Non-linear modelling of vibrations of gears with manufacturing errors. C. Sc. Theses, Budapest. 1990 (in Hungarian)
2. Márialigeti, J. : Simulation Study of the Influence of Involute Tip Relief on Gear Dynamic Behaviour. Periodica Polytechnica Ser. Transp. Eng. Vol. 25. No. 1-2, pp 59-77. 1997
3. Márialigeti J., Lovas L.: Analysis of load dependent dynamic transmission error response of gears with random pitch error, ASME DETC'01 Design Engineering Technical Conferences, Pittsburgh, PA, USA, 9-12 sept. 2001. Abstracts ASME New-York 2001. p.: 635. Full paper edited on the CD of the Conference, in 18th Biennial Conference on Mechanical Vibration and Noise, ASME 2001.
4. Niemann, G., Winter, H.: Machinenelemente, Bd. II. Springer Vlg. Berlin, Heidelberg, New-York. 1985 (in German)
5. Winter, H., Podlesnik, B.: Tooth Stiffness Characteristics of Gears. Part 2. Antriebstechnik. Vol. 22. No. 5., pp 51-57. 1983 (in German)