MATHEMATICAL MODELLING OF ARCH FORMATION IN GRANULAR MATERIALS

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Abstract: *The mathematical modelling of formation, stability and damage of self-supporting stagnant arch like structures inside bulk material discussed in this paper. Some experimental methods will also be presented, for the measurement of material properties used in the mathematical model. We present an algorithm for the determination of critical outlet size for a simplified model silo.*

Keywords: *granular material, arching, finite element method, triaxial apparatus*

1. INTRODUCTION

Arcing occurs mostly during the discharge of silos, but pressure acting on underground structures is also influenced by arching, and smaller than the weight of the overburden. Earth pressure applied on retaining wall sometimes also smaller than the theoretical value, and this phenomena are also caused by the arching action. The arch formation inside silos can change the wall pressure distribution in a large scale, and sometimes the whole silo collapses because of this change in pressure distribution. The outflow of granular materials from containers is also considered to a sequence of formation and collapse of arches. The evaluation of stresses in granular assemblies without taking the arching action into consideration, always gives false, inaccurate results.

2. MATHEMATICAL MODEL

The classical method, used for the evaluation of stresses within the arch uses two theoretical solutions [1]. The first one evaluates the stresses that consolidate the granular material and give rise to its strength, the second one analyzes the stresses that act in an arch regarded as a structural member. The evaluation of stresses within these speculative arches is based on assumptions on the shape of these arches. The classical definition of arching follows from these aspects: “arching refers to spontaneous formation of an arch like supported stagnant mass of bulk material [1] “, which is capable of bearing the pressure originating from the mass above it.

For the mathematical model, first we need to determine the stresses inside our model silo. The model silo was a simple rectangle shaped container, with variable outlet size on its bottom. For the determination of stresses acting inside the granular mass, we used a used linear elastic, isotropic, homogenous continuum material model for the granular assembly. Our assumptions were:

1. The granular material fills out continuously the container, and its material properties are independent of space coordinates, time and orientation.
2. We assumed, that the connection between the stress and the deformation tensor is linear in every point of our model silo.
3. The load originated only from material self weight.
4. The wall-material friction can be neglected.
5. The rigidity of the silo side wall supposed to be infinite.
6. We assumed plain strain state.

Our aim was to determine the critical outlet size of this model silo. Critical outlet size means a border-line case, namely if the outlet size is bigger than this critical values, stable arches ca not be taken form.

Fig. 1: Triaxial apparatus

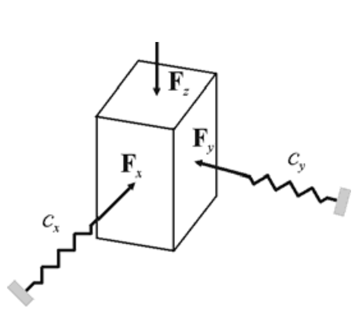

Fig. 2: Triaxial compression

2.1. Material properties and failure criteria

For the determination of stresses, the specimen's Young modulus (E) and the Poisson's ratio (ν) had to be determine. The measurement process of these material properties described in [2]. We needed an other material property for the description of the collapse of the arch. This is the critical stress belonging to biaxial stress state. The arch collapses, when on its free boundary – where the material is in biaxial stress state – the compressive stress exceeds the critical value σ_K .

We used a special triaxial apparatus (*fig. 2*) for the determination of material properties. The description of this apparatus can be found in [3]. To measure the critical stress belonging to biaxial stress state, we applied two different kind of load on the granular material. In the first step we applied a triaxial pre-compression on the specimen. Then we removed one of the lateral springs (*fig. 3*), and increased the vertical load until the collapse of the specimen. With the removal of one of the lateral springs, we realized the biaxial stress state, needed for the measurement of the critical stress.

Knowing the material properties, the stresses arising inside the model silo can be determined using finite element method. The finite element model of silo is in *fig 3*. After the determination of stresses inside the model silo, we have to analyze the process of arch formation. For this, we have to determine the failure criterions "controlling" the arch formation and collapse process. The failure criterion for arch formation is simple for this rectangle shaped model silo: granular assemblies are unable to resist tension. It is also simple to formulate this conditions in mathematical form. After the evaluation of stresses, we have to compute the eigenvalues of the stress tensor in every point of the granular material. These eigenvalues are the so called principal stresses. If the biggest principal stress is positive, than in that point of the material tension occurs, and the

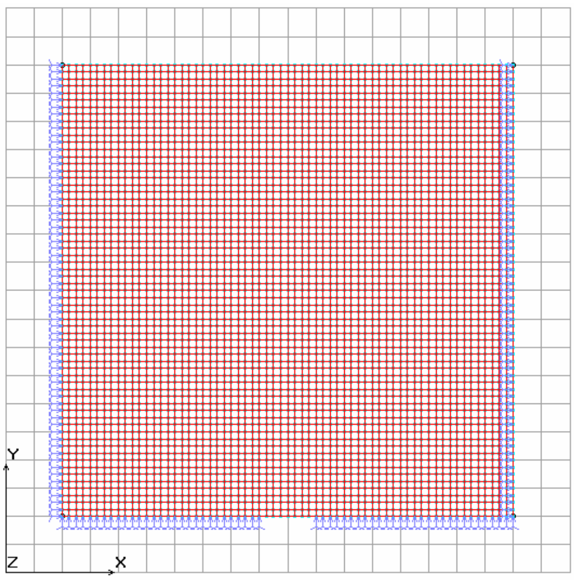

Fig 3: The finite element model

continuity of the material fails. The material is falling out from these point through the open outlet.

The stability and collapse of the arches depends on two different failure conditions. The firs failure condition occurs, when on the materials free boundary – where the material is in biaxial stress state – the compressive stress exceeds the critical value σ_K. The mathematical formulation of this failure criterion is also in connection with the principal stress values. If the third (smallest, and in our case negative) principal stress value is smaller then the critical σ_K, in any points of the granular assembly's boundaries, than the failure of the arch is due to happen. The value of this critical stress depends on the magnitude of pre-compressing stresses. The second failure criterion is about the shear stresses acting inside the granular material. The value of the shear stresses inside granular materials cannot be higher than a critical value. The critical shear stress value is – usually considered to be linear – function of the friction angle and cohesion of the granular material.

Using this failure conditions, it is possible to create an algorithm for the numerical simulation of the arching process (*fig 4*).

2.2. The algorithm

1. First we open the outlet to an initial size.
2. The determination of stresses inside the granular material comes next. Using finite element method, this is possible; numerically.
3. Knowing the stresses, the eigenvalues of stress tensors must be computed.
4. Using the biggest eigenvalues, the failure criterion for arch formation must be applied.
5. After the removal of material elements, where the first failure criterion prevailed, the domain, where the stresses were evaluated changed, so the stresses must be evaluated again.
6. Knowing the new stresses, the eigenvalues must be computed again, and the failure condition for arch formation must be applied. This goes until there are no more points inside the material, where tension occurs.
7. When there is no more tension, the two failure criterion for arch collapse must be taken into account. If none of them comes to be true, then a stable arch formed. This arch belongs to the outlet size adjusted in step 1.
8. The outlet size can be enlarged, and then the whole process starts again from step 2.
9. The algorithm runs until one of the arch failure criterions comes to be true.
10. When the arch collapse criterion occurs, the critical outlet size is determined.

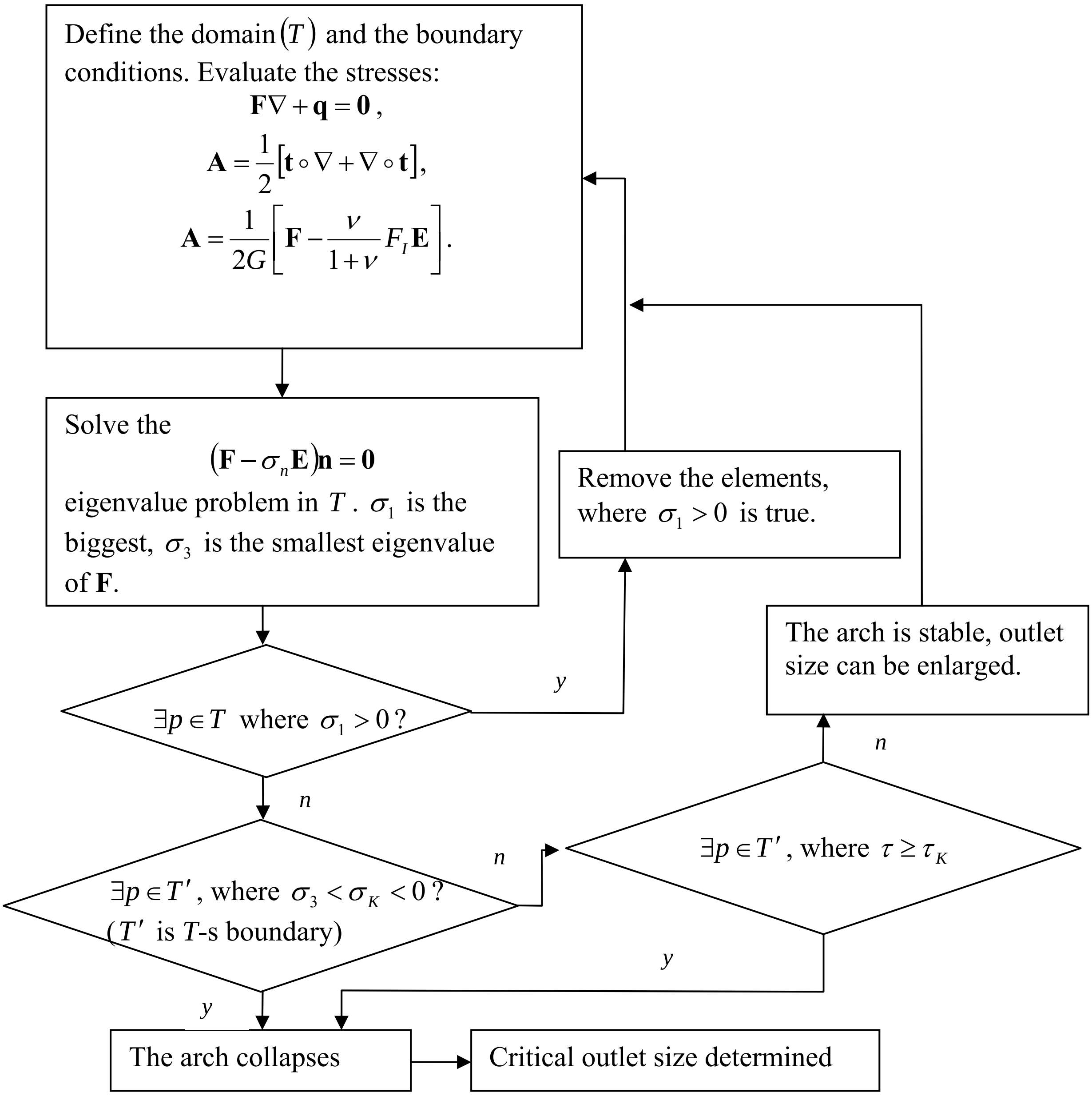

Fig 4.: The algorithm for determination of critical outlet size

3. RESULTS AND FURHTER RESEARCH

An iterative method for modelling the arching action in granular assemblies was developed. This method is different, and more efficient than any methods existing in the literature for determination of critical outlet size in silos. Our further research will include the conic shaped containers, where the possible sliding of the material at the container wall must be also taken into account.

4. REFERENCES

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