SPECIAL ASPECTS IN TRIANGULATION AND MAPPED MESHING

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Abstract: *The purpose of this paper is to focus on special aspects with regard to the triangulation and mapped meshing. A presentation of the concepts and basic theory is offered, which was intended as an adequate answer for the questions like what, how and why.*
Keywords: *mesh generation, triangulation, Octree, Delaunay, point insertion, boundary constraint triangulation*

1. INTRODUCTION

Computational mesh generation represents an important procedure in numerical analysis of complex processes. With the aid of Computational Fluid Dynamics (CFD), one can analyze intricate flow problems. The accuracy of the numerical results is strongly influenced by the mesh mapping and the boundary conditions.

The computational domain can be mapped with either structured or unstructured meshes, or with a combination of theirs; also, a multi-block technique can be successfully applied.

Mesh types are used to mapping a computational domain, there are some important theoretical aspects that should be taken into account. These can be considered as a “philosophy” of an adequate how-to-do and offer both the justification and the rightful answer at the question what. The triangulation and the quad/hexahedral meshing represent problems of the very kind. In this section, we consider triangular meshes in two dimensions.

2. TWO-DIMENSIONAL TRIANGULATIONS

By triangulation one refers to building polygons when knowing a set of nodes and eventually to filling the domain with matching elements, in accordance with certain criteria and procedures. There are four distinct types of input domains, all of which can be viewed as polygonal regions of the plane, because their boundaries consist of straight line segments.

The task is to partition the domain into triangles, that meet edge to edge, and that may be required to satisfy some other optimality properties. For all four types of domains, a single parameter, the number of vertices, is meant to measure the input complexity.

2.1. Simple polygon and polygon with holes

The domain is a polygonal region of the plane, and its boundary forms a simple, polygonal, closed curve. The triangulation must use the edges of the boundary as edges in the triangulation. In a Steiner triangulation problem, extra vertices may be added to the interior or on the polygon; hence, the edges of the boundary may be subdivided to form several collinear edges in the triangulation.

The polygon with holes differs from the previous case in that the boundary may form several disjoint polygonal Jordan curves. These curves surround holes within the polygon.

2.2. Point set

The input is a set of points in the plane. Without Steiner points, the vertices of the triangulation are exactly the input points, and the boundary of the triangulation is the convex hull. With Steiner points, the vertices of the triangulation are a superset of the input points, and the boundary of the triangulation is a convex region that may be larger than the convex hull.

2.3. Planar straight line graph (PSLG)

The input is a set of vertices and non-crossing line segments in the plane, which must be used as edges in the triangulation. This most general input occurs in practice for „multiple domains",that is, domains that include boundaries between different materials.

2.4. Octree

With this method, cubes containing the geometric model are recursively subdivided until the desired resolution is reached. Irregular cells are then created where cubes intersect the surface, often requiring a significant number of surface intersection calculations.

The Octree technique does not match a pre-defined surface mesh, as the advancing front or Delaunay mesh might; rather surface facets are formed wherever the internal Octree structure intersects the boundary. The resulting mesh also will change as the orientation of the cubes in the Octree structure is changed and can also require.

Following a recent survey, only four of the 38 codes generating tetrahedral meshes reported using some form of octree technique. SCOREC [1] at Rensselaer develops a set of mesh generation tools called MEGA that uses the Octree technique that is available through their partners program. A public domain octree mesh generator called QMG is available from Steve Vavasis [2] at Cornell.

2.5. Delaunay

By far the most popular of the triangle and tetrahedral meshing techniques are those utilizing the Delaunay [3] criterion. The Delaunay criterion, sometimes called the "empty sphere" property simply stated, says that any node must not be contained within the surrounding sphere of any tetrahedral within the mesh. A circum-sphere can be defined as the sphere passing through all four vertices of a tetrahedron.

The Delaunay criterion does not represent an algorithm for generating a mesh, but it helps and guide how to connect a set of existing points in the space. Therefore, it is necessary to provide a method for generating node locations within the geometry. A typical approach is to first mesh the boundary of the geometry to provide an initial set of nodes. The boundary nodes are then triangulated according to the Delaunay criterion. Nodes are then inserted incrementally into the existing mesh, redefining the triangles or tetrahedrons locally as each new node is inserted to maintain the Delaunay criterion.

2.6. Point insertion

The simplest point insertion approach is to define nodes from a regular grid of points covering the domain at a specified nodal density. In order to provide for varying element sizes, a user specified sizing function can also be defined and nodes inserted until the underlying sizing function is satisfied. Another approach is for nodes to be recursively inserted at triangle or tetrahedral centroids.

Similar to the circumcircle point insertion method, another technique is the so-called, Voronoi-segment point insertion method. A Voronoi segment can be defined as the line segment between the circumcircle centers of two adjacent triangles or tetrahedra. The new node is introduced at a point along the Voronoi segment in order to satisfy the best local size criteria. This method tends to generate very structured looking meshes with six triangles at every internal node. One straightforward method used by INRIA in their mesh generator GSH3D, is point insertion along edges. A set of candidate vertices is generated by marching along the existing internal edges of the triangulation at a given spacing ratio. Nodes are then inserted incrementally, discarding nodes that would be too close to an existing neighbor. This process is continued recursively until a background sizing function is satisfied.

2.7. Boundary constrained triangulation

In many finite element applications, there is a requirement that an existing surface triangulation be maintained. In most Delaunay approaches, before internal nodes are generated, a three dimensional tessellation of the nodes on the geometry surface is produced. In this process, there is no guarantee that the surface triangulation will be satisfied. In many implementations, the approach is to tessellate the boundary nodes using a standard Delaunay algorithm without regard for the surface facets. A second step is then employed to force or recover the surface triangulation. Obviously, if doing so, the triangulation may no longer be strictly "Delaunay", hence the term "Boundary Constrained Delaunay Triangulation".

In two dimensions the edge recovery is relatively straightforward. The process is considerably more complex in three dimensions, since after recovering all edges in the surface triangulation, there is no guarantee that the surface facets themselves will be recovered. Additional side recovery operations can be required to maintain the surface triangulation. While the two dimensional recovery process is guaranteed to produce a boundary conforming triangulation, there are cases in 3D where a valid triangulation can not be defined without first inserting additional vertices. This fact increases the complexity of any three dimensional boundary recovery procedure. In the first approach defined by George[7] and implemented in INRIA's GSH3D software, edges are recovered by performing a series of tetrahedral transformations by swapping two adjacent tetrahedrons for three.

2.8. Free form deformation (FFD)

The main idea of FFD method is to deform a simple quasi-orthogonal mesh until it takes the shape of the desired domain to become discrete. The origin of this method is in „linear/nonlinear analysis” of structures. The hypotheses used to explain the behavior of the structure are the well-known formulas of the elasticity theory.

The advantages of this method consist in a simplicity and automatic mesh generation. When starting with a regulate mesh, the deformation process preserves the mesh regularity. The control of the mesh generation is possible by the local changing of the elastic characteristics of structure. The main disadvantage remains the relatively great computational effort in case of 3D meshes. It is necessary an optimized solver for large systems of equations.

2.9. The advancing front method

Another very popular family of triangle and tetrahedral mesh generation algorithms is the advancing front, or moving front method. Two of the main contributors to this method are Rainald Lohner [4], [5] at George Mason University and S. H. Lo [6] at the University of Hong Kong. A form of the advancing front method, which is sometimes called "advancing layers", is also used for generating boundary layers for CFD in Navier-Stokes applications.

This method is able to control the element sizes near the boundary. Another model presents a method wherein the elements are stretched in the direction of the boundary, the expected direction of fluid flow. A public domain version of Pirzadeh's code, VGRID is available from NASA, Langley.

3. QUAD/HEXAHEDRAL MESHING

Mapped meshing

Whenever the geometry of the domain is suited for, quad or hex mapped meshing will generally produce the most desirable result. Although mapped meshing is considered a structured method, it is quite common for unstructured codes to provide a mapped meshing option. Such that mapped meshing can be applied, the opposite edges of the area to be meshed must have equal numbers of divisions. In 3D, each opposing face of a topological cube must have the same surface mesh. This can often be impossible for an arbitrary geometric configuration or can involve considerable user interaction to decompose geometry into mapped meshing regions and assign boundary intervals.

4. CONCLUSIONS

Topical aspects of great interest that are issued from the computational mesh generation problem are the triangulation mainly and the quad/ hexahedral meshing. The present work offers a detailed presentation of the triangulation, laying the stress upon polygons, point set, planar straight line, the Octree technique, the Delaunay criterion, point insertion, boundary constraint triangulation, free form deformation and the advancing front method.

This paper represents just a section of the studies regarding the global problem of computational mesh generation, which has been carried on by the authors. Eventually, the authors would like to mention the successful development of their own 2D/3D codes dedicated to grid generation, which are supposed to be applied in interested areas.

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