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SELF-REGULATING TWO D-O-F CONNECTED PLANETARY GEAR DRIVES HAVING CONSTANT TRANSMISSION RATIO

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Abstract: *The study below deals with the two degree-of-freedom connected drives having constant transmission ratio. These connected systems consist of two differential gear drives and two one degree-of-freedom gear drives. We define the necessary kinematic- and inner ratios, than we determine the possible powerflows of the connected system. The study shows expressively the evolution of the inner motion state which meets the moment- and power equlibrium conditions. In this state the connected planetary gear drive works with optimal efficiency.*
Key words: *differential gear drive, connected drive, self regulation, two degree-of-freedom system*

The study examines the working conditions of two degree-of-freedom connected drives having constant transmission ratio. These connected systems consist of two differential gear drives and two one degree-of-freedom gear drives. Using the kinematic equations we define the necessary kinematic- and inner ratios, then – based on the kinematic- and dynamic relations of the differential drive – we determine the possible powerflows of the connected system. The study shows expressively the forming of the inner motion state which meets the moment- and power equlibrium conditions. In this state the connected planetary gear drive having constant kinematic transmission ratio works with optimal efficiency.

1. KINEMATIC RELATIONS

Let's examine the connected system, seen on *Fig.1.*, in which the base elements of the D_1 and D_2 are denoted by p_1,q_1,r_1 and p_2,q_2,r_2 (p,q toothed base elements, r arm). The elements r_1,p_2 are joined by H_1 drive, and the elements q_1,q_2 are joined by $H2$ drive. Let's assume, that the input element of the system is $0\equiv p_1$, while the output element is $\infty\equiv r_2$. Since D_1 and D_2 differential drives have $s_1=s_2=2$ degree-of-freedom before coupling, and after coupling the system has two connected (p_2r_1 and q_1q_2) base elements ($k=2$), the connected system has $s_k=s_1+s_2-k=2+2-2=2$ degree-of-freedom as well.

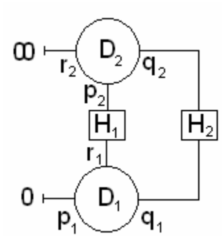

Fig.1.

Let's choose the inner ratios of the D_1 and D_2 differential drives $i_{p1q1}=i_{p2q2}=-1$, and the kinematic transmission ratio of the connected drive $k_{0\infty}=k_{p1r2}=-2$. On the basis of inner ratios and denoting the angular velocities of base elements by ω (e.g. ω_{p1} is the angular velocity of p_1) we can write:

$$\omega_{p1} + \omega_{q1} - 2\omega_{r1} = 0; \quad (1)$$

$$\omega_{p2} + \omega_{q2} - 2\omega_{r2} = 0. \quad (2)$$

From these, by $k_{H1}=\omega_{p2}/\omega_{r1}=-2$ ratio of H_1 drive and $k_{H2}=\omega_{q2}/\omega_{q1}=1$ ratio of H_2 drive, it can be derived, that the connected system has – independently of the state of motion – $k_{0\infty}=-2$ constant transmission ratio.

Since the powerflows are determined by k_{p1r1}; i_{p1q1} and k_{p2r2}; i_{p2q2} inner ratios [1], the ranges of k_{p1r1};k_{p2r2} and their relation are necessary for the examination of the powerflows and working possibilities of the connected system. The relation $k_{p1r1}=f(k_{p2r2})$ can be derived easily:

$$k_{0\infty}\, k_{H1}=(-2)(-2)=\omega_{p1}/\omega_{r2}*\omega_{p2}/\omega_{r1}= k_{p1r1}*k_{p2r2},$$

consequently:

$$k_{p1r1}=4/k_{p2r2} \quad (3)$$

2. POSSIBLE POWERFLOWS

According to [1] by $i_{p1q1}=-1$ in the field of $0<k_{p1r1}<1$ theoretically the $r_1 \rightarrow p_1q_1$ and $p_1q_1 \rightarrow r_1$ powerflows are possible. But here only the $p_1q_1 \rightarrow r_1$ powerflow can be realized since p_1 is input element. So in this range the connected system works as shown on *Fig.2.*. It can be seen, that in this case the D_2 differential's powerflow can be – assuming that $r_2 \equiv \infty$ output element – only $p_2 \rightarrow q_2 r_2$. According to (3) it belongs to $4 < k_{p2r2} < \infty$. We note that the powerflow of D_2 is $p_2 \rightarrow q_2 r_2$ according to [1] as well.

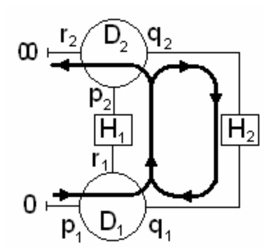

Fig.2.

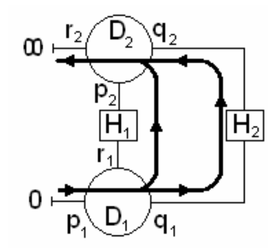

Fig.3.

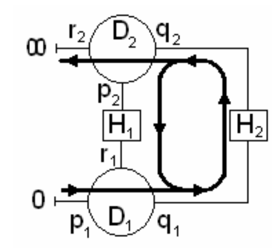

Fig.4.

In the field of $1 < k_{p1r1} < 2$ the powerflow is also $p_1 q_1 \rightarrow r_1$ [1], so *Fig.2.* shows the connected system's powerflow also for this case. The transmission ratio range of D_2 differential drive is $2 < k_{p2r2} < 4$ according to (3). In case of $k_{p1r1} = k_{p2r2} = 2$ kinematic ratio the $q_1 \equiv q_2$ central elements are stationary, so the $q_1 \rightarrow q_2$ powerflow ceases, and closed powerflow doesn't circulate in the system.

In the field of $2 < k_{p1r1} < 4$ the powerflow can be $p_1 \rightarrow q_1 r_1$ and $p_2 q_2 \rightarrow r_2$, and $1 < k_{p2r2} < 2$ (*Fig.3.*).

In the field of $4 < k_{p1r1} < +\infty$ the powerflow is also $p_1 \rightarrow q_1 r_1$ and $p_2 q_2 \rightarrow r_2$, and $0 < k_{p2r2} < 1$. In case of $k_{p2r2} = 1$ limit value the D_2 differential drive works as a clutch.

Finally, in case of $-\infty < k_{p1r1} < 0$ the powerflow is $p_1 r_1 \rightarrow q_1$ and the transmission ratio of D_2 is $-\infty < k_{p2r2} < 0$, to which belongs $q_2 \rightarrow p_2 r_2$ powerflow. We note, that when $k_{p1r1} = -\infty$ the D_1 differential drive works as a simple drive and because of $\omega_{p2} = 0$ the $p_2 \rightarrow r_1$ powerflow stops (*Fig.4.*).

On *Fig.2-4.* it can be seen, that – according to the change in the state of motion – from a circulating powerflow (*Fig.2.*) a branched- (*Fig.3.*) then a reverse circulating powerflow can evolve (*Fig.4.*).

3. DYNAMIC RELATIONS

The question arises, that which state of motion will stay up by stable running. This question can be answered after the analysis of the moment-relations. The moments acting on base elements (M) can be determined, if the η_0 efficiency (efficiency when the arm is fixed) of D_1 and

D_2 differential drives, the w_1 and w_2 indexes (determine the direction of powerflow), and η_{H1} and η_{H2} efficiencies of H_1 and H_2 drives are known [1],[2]. For the sake of simplicity let's assume, that $M_0 \equiv M_{p1} = 1Nm$, $\eta_0 = \eta_{H1} = 0.98$ and $\eta_{H2} = 0$ (since q_1 and q_2 are connected elements).

The determination of the moments acting on base elements is shown for the transmission ratio range of $0 < k_{p1r1} < 1$, $4 < k_{p2r2} < \infty$. The valid powerflow can be seen on *Fig.2.*. In this case $w_1 = -1$, $w_2 = 1$ [1], so in D_1 drive:

$$M_{q1} = -i_{p1q1} \cdot \eta_0^{w_1} \cdot M_0 = 1 \cdot 0{,}98^{-1} \cdot 1 = 1{,}020408 Nm\,;$$

$$M_{r1} = (i_{p1q1} \cdot \eta_0^{w_1} - 1) \cdot M_0 = (-1 \cdot 0{,}98^{-1} - 1) \cdot 1 = -2{,}020408 Nm\,;$$

and in D_2 differential drive:

$$M_{p2} = -\frac{\eta_{H1}}{k_{H1}} \cdot M_{r1} = -\frac{0{,}98}{-2} \cdot (-2{,}020408) = -0{,}99 Nm\,;$$

$$M_{q2} = -i_{p2q2} \cdot \eta_0^{w_2} \cdot M_{p2} = 1 \cdot 0{,}98^{1} \cdot (-0{,}99) = -0{,}9702 Nm\,;$$

$$M_{r2} = -(M_{p2} + M_{q2}) = -(-0{,}99 - 0{,}9702) = 1{,}9602 Nm$$

moments are necessary to get equilibrium. But with these relations the M_{q1} and M_{q2} acting on $q_1q_2 \equiv q$ connected element are not equal. The difference $M_{dq} = M_{q1} + M_{q2} = 1.020408 - 0.9702 = 0.050508Nm$ generates a moment which decelerate the ω_q of element q (together with ω_{p2} and ω_{r1}) until clutch state of D_1 differential ($\omega_{p1} = \omega_q = \omega_{r1} = 1$). We mention – without detailing –, that in $0 < \omega_q < 1$ section an $M_{dq} = 0.029204Nm$ decelerating moment works. Due to this moment the element q stops, and than accelerates in the opposite direction until reaching $\omega_{p2} = \omega_{r2} = \omega_q = -0.5$ clutch state. If $\omega_q < -0.5$, an $M_{dq} = -0.1Nm$ moment works, and because of this $\omega_q >> -0.5$. So the system set in $k_{p2r2} = 1$ and $k_{p1r1} = 4$ ratio values by self-regulation, so D_2 works in clutch state while D_1 works as a differential gear drive with $k_{p1r1} = 4$.

4. EFFICIENCY

The efficiency of the connected planetary gear drive can be calculated on the basis of loss-analysis. Let's choose the angular velocity of input element $\omega_0 \equiv \omega_{p1} = 1/s$, and its moment $M_0 \equiv M_{p1} = 1Nm$, consequently the input power $P_0 = 1W$.

The efficiency of D_1 differential [1]:

$$\eta_{p1 \to q1r1} = \frac{1 - (1 - k_{p1r1}) \cdot \eta_0^{w_1}}{k_{p1r1}} = \frac{1 - (1 - 4) \cdot 0{,}98^{1}}{4} = 0{,}985 Nm\,.$$

The loss power of D_1 differential:

$$P_V = (1-\eta_{p1\to q1r1})\cdot P_0 = (1-0{,}985)\cdot 1 = 0{,}015W\ .$$

From $k_{p1r1}=4$ kinematic ratio in case of $\omega_{p1}=1/s$ the angular velocity of r_1 is $\omega_{r1}=0.25/s$, and the moment on r_1 arm is:

$$M_{r1} = (i_{p1q1}\cdot\eta_0^{w_1}-1)\cdot M_{p1} = (-1\cdot 0{,}98^1-1)\cdot 1 = -1{,}98Nm\ .$$

So the loss power in H_1 drive:

$$P_{VH1} = -(1-\eta_{H1})\cdot M_{r1}\omega_{r1} = (1-0{,}98)\cdot 1{,}98\cdot 0{,}25 = 0{,}0099W\ .$$

Since D_2 differential drive works in clutch state, its loss is $P_{V2}=0$. So the total power loss of the connected system:

$$\sum P_V = P_{V1} + P_{VH1} = -0{,}015-0{,}0099 = -0{,}0249W\ .$$

And the efficiency of the system is (considering only the tooth losses):

$$\eta_{0\infty} = \frac{P_0-\sum P_V}{P_0} = \frac{1-0{,}0249}{1} = 0{,}9751\ ,$$

to which belongs

$$M_{r2} = \frac{-\eta_{0\infty}}{\omega_{r2}} = \frac{-0{,}9751}{-0{,}5} = 1{,}9502Nm$$

moment.

5. STRUCTURAL CONSTRUCTION

The kinematic sketch of the connected drive can be seen on *Fig.5.*, when bevel gear differential drives are used, and the arrangement is not coaxial. *Fig.6.* shows the structure in case of coaxial arrangement. The kinematic sketch when the differential drive consists of cylindrical gears and the arrangement is coaxial can be seen on *Fig.7.*.

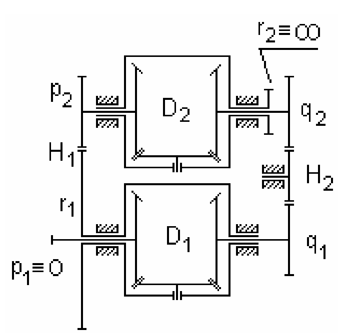

Fig.5.

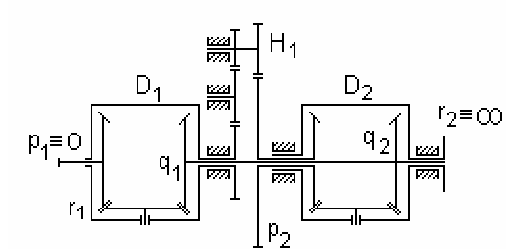

Fig.6.

In *Fig.6-7.* solutions the H_2 drive is missing, because its $k_{q1q2}=1$ kinematic transmission ratio is assured by direct junction of q_1q_2 base elements.

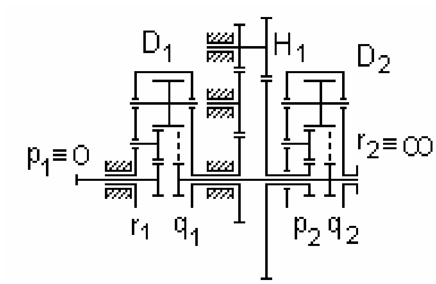

Fig.7.

6. CONCLUSION

The examination of connected differential gear drives reveals that connected systems with arbitrary constant transmission ratio can be designed by proper selection of inner ratios of D_1, D_2 and kinematic transmission ratios of H_1, H_2 one degree-of-freedom drives. Main property of these systems is that their inner motion set by self regulation in such a state – from the infinite number of possible states –, wich has the less power loss, so they work with optimal efficiency. It's necessary to mention, that by proper selection of various parameters the two degree-of-freedom connected planetary gear drives can be used as self-regulating continuously variable drives.

7. REFERENCES

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