MATHEMATICAL MODELING OF WASTE WATER CLEANING PROCESS IN SEDIMENTATION TANKS

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Abstract *A model of the concentrations of suspended solids (SS) in the aeration tanks and in the effluent from these during sedimentation operation is established. The model is based on simple SS mass balances, a model of the sludge settling and a simple model of how the SS concentration in the effluent from the aeration tanks depends on the actual concentrations in the tanks and the sludge blanket depth.*

The model is formulated in continuous time by means of stochastic differential equations with discrete-time observations. The parameters of the model are estimated using a maximum likelihood method from data from an alternating wastewater treatment plant.

The model is an important tool for analyzing ATS operation and for selecting the appropriate control actions during ATS, as the model can be used to predict the SS amounts in the aeration tanks as well as in the effluent from the aeration tanks.

Key words: *Aeration Tank Settling, mass balance, grey-box models, statistical identification, on-line measurements.*

1. INTRODUCTION

In order to introduce advanced control systems at wastewater treatment plants the demand for mathematical models of the main processes in wastewater treatment plants is increased. The Aeration Tank Settling (ATS) principle introduces settling periods in aeration tanks of alternating plants and enables increased amounts of suspended solids (SS) to be stored in the aeration tanks.

In Figure 1 the flow through the aeration tanks and clarifiers is illustrated. The black and grey lines illustrate alternative flow paths through the aeration tanks. The influent flow and the recirculation flow are denoted Qi and *Qr,* respectively. X_{ssi}, X_{ssr}. And $X_{ssoutat}$ denote SS concentrations in the influent, the return sludge and the effluent from the aeration tanks to the secondary clarifiers.

The dynamics of the water amounts in the aerations tanks are not considered, i.e. it is assumed that the flows to and from each of the aeration tanks are the same (Q_i + Q_r).

Furthermore, the SS concentration in the flow between the two aeration tanks is assumed to be the average SS concentration in the feeding tank. When the feeding tank is fully mixed, this assumption is fulfilled, but when settling occurs it is an approximation.

2. MASS BALANCE EQUATIONS

The mass balance equations for each of the aeration tanks depend on the actual flow path designated f_p. When $f_p = 1$ the influent flow is directed to aeration tank 1, and the effluent flow is taken from tank 2. f_p is 0 when the opposite flow path is applied. With V_{at}, X_{ssm}^1 and X_{ssm}^2 denoting the volume of each of the equally sized aeration tanks and the average SS concentrations in tank 1 and 2, respectively, the mass balance equations can be established.

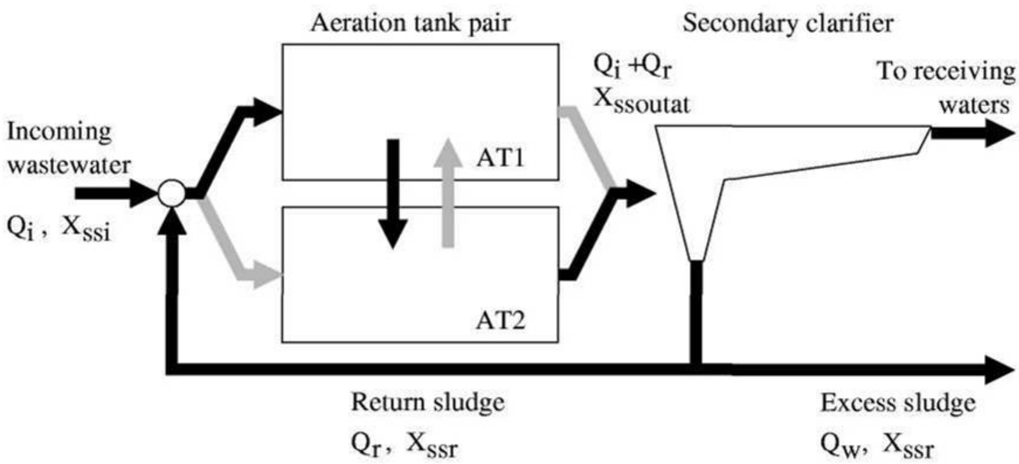

Figure 1. Flow path through aeration tank pair in an alternating WWTP

For $f_p = 1$ the mass balance equations for the aeration tanks are:

$$\begin{cases} \dfrac{dX_{ssm1}}{dt} = \dfrac{Q_i \cdot X_{ssi} + Q_r \cdot X_{ssr} - (Q_i + Q_r) \cdot X_{ssm1}}{V_{at}} & (1) \\ \dfrac{dX_{ssm2}}{dt} = \dfrac{(Q_i + Q_r) \cdot X_{ssm1} - (Q_i + Q_r) \cdot X_{ssoutat}}{V_{at}} & (2) \end{cases}$$

For $f_p = 0$ the mass balance equations are:

$$\begin{cases} \dfrac{dX_{ssm1}}{dt} = \dfrac{(Q_i + Q_r) \cdot X_{ssm2} - (Q_i + Q_r) \cdot X_{ssoutat}}{V_{at}} & (3) \end{cases}$$

$$\frac{dX_{ssm2}}{dt} = \frac{(Q_i \cdot X_{ssi} + Q_r \cdot X_{ssr}) - (Q_i + Q_r) \cdot X_{ssm2}}{V_{at}} \tag{4}$$

The flow path variable can be used to combine equation (1) with (3) and equation (2) with (4) into one equation per aeration tank:

$$\begin{cases} \frac{dX_{ssm1}}{dt} = f_p \cdot \frac{Q_i \cdot X_{ssi} + Q_r \cdot X_{ssr} - (Q_i + Q_r) \cdot X_{ssm1}}{V_{at}} + \\ + (1 - f_p) \cdot \frac{(Q_i + Q_r) \cdot X_{ssm2} - (Q_i + Q_r) \cdot X_{ssoutat}}{V_{at}} \quad (5) \\ \frac{dX_{ssm2}}{dt} = f_p \cdot \frac{(Q_i + Q_r) \cdot X_{ssm1} - (Q_i + Q_r) \cdot X_{ssoutat}}{V_{at}} + \\ + (1 - f_p) \cdot \frac{(Q_i \cdot X_{ssi} + Q_r \cdot X_{ssr}) - (Q_i + Q_r) \cdot X_{ssm2}}{V_{at}} \quad (6) \end{cases}$$

When mixing is stopped in an aeration tank, the suspended solids settle. A simple two layer model, where the water in the layer above the sludge blanket is assumed to be clear water, and the layer under the sludge blanket is assumed to contain all the SS fully mixed, is used. The settling velocity for the sludge blanket is modeled according to

$$\frac{dd_{sb}}{dt} = V_0 e^{-n_v X_{sssl}} \tag{7}$$

where d_{sb} and X_{sssl} denote the sludge blanket depth and the SS concentration in the sludge layer, respectively, see Figure 2, and V_0 and n_v are sludge volume index *(SVI)* dependent parameters.

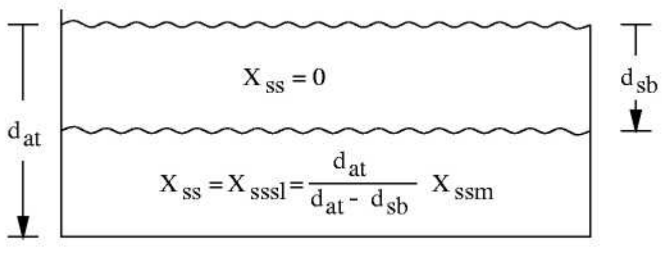

Figure 2. Two layer model of settling in an aeration tank.

For simplicity we use the expressions:

$$\begin{cases} V_0 = (17.4e^{-0.01135SVI} + 3.931) \\ \\ n_v = (-0.9834e^{-0.00581SVI} + 1.043) \end{cases} \tag{8}$$

If sludge blanket depth measurements are available, V_0 and n_v can be estimated.

As the volume of the sludge layer is $\frac{(d_{at} - d_{sb})V_{at}}{d_{at}}$, the average SS concentration in the sludge layer is:

$$X_{sssl} = \frac{d_{at}}{d_{at} - d_{sb}} \cdot X_{ssm} \tag{9}$$

where X_{ssm} is the average SS concentration in the aeration tank.

When the tank is fully mixed, the sludge blanket depth is 0. When mixing is switched on, d_{sb} tends towards zero, which is modeled by:

$$\frac{dd_{sb}}{dt} = -\frac{1}{\tau_{mix}} d_{sb} \tag{10}$$

where τ_{mix} is a mixing capacity dependent time constant.

We introduce the mixing signals *m1* and *m2* for aeration tank 1 and 2, respectively. The mixing signals are 1 when the corresponding aeration tank is mixed and 0 otherwise. The signals can then be used to combine the settling equation (7) with the mixing equation (10) for each of the aeration tanks:

$$\frac{dd_{sb1}}{dt} = l(m_1)(-\frac{1}{\tau_{mix}} d_{sb1}) + (1 - l(m_1))V_0 e^{-n_v X_{sssl_1}} \tag{11}$$

$$\frac{dd_{sb_2}}{dt} = l(m_2)(-\frac{1}{\tau_{mix}} d_{sb2}) + (1 - l(m_2))V_0 e^{-n_v X_{sssl_2}} \tag{12}$$

Here, the aeration tank number is introduced on the sludge blanket depth and average SS concentration variables so that dsb_1, dsb_2, X_{sssl1} and X_{sssl2} designates the sludge blanket depths and average SS concentrations in aeration tank 1 and 2, respectively.

The SS concentration in the effluent from an aeration tank is modeled as a function of the suction depth, d_{suct} and the SS concentration in the sludge layer:

$$X_{ssoutat} = \begin{cases} \frac{d_{suct} - d_{sb}}{d_{suct}} X_{sssl} & pentru \quad d_{suct} \geq d_{sb} \\ 0 & \hat{i}n \quad rest \end{cases} \tag{13}$$

The suction depth is expected to depend on the flow and is modelled as:

$$d_{suct} = d_0 \left(\frac{Q_i + Q_r}{Q_0} \right)^{b_{suct}} \tag{14}$$

where d_0 and b_{suct} are positive parameters and Q0 = 1000 m^3/h. Combining. (9) and (13) yields

$$X_{ssoutat} = \frac{d_{suct} - d_{sb}}{d_{suct}} \cdot \frac{d_{at}}{d_{at} - d_{sb}} \cdot X_{ssm} = \frac{1 - \frac{d_{sb}}{d_{suct}}}{1 - \frac{d_{sb}}{d_{at}}} \cdot X_{ssm} \quad for \quad d_{suct} > d_{sb} \tag{15}$$

To enable smooth changes in $X_{ssoutat}$ when the point $d_{suct} = d_s b$ is passed a smooth threshold function is introduced. Here, the logistic function:

$$l(x) = l(x, a, b) = \frac{1}{1 + e^{\frac{a-x}{b}}} \tag{16}$$

is used. For *x* = *a* the logistic function is 0.5, i.e. the value of *a* determines the midpoint of the switch between 0 and 1. By appropriate selection of a and *b* the change between 0 and 1 of *l(x, a, b)* can be controlled. In Figure 3 the logistic function is shown for *a* = 0 and 3 different values of *b*. In the following *b* > 0 is assumed.

The logistic function (16) is used to calculate $X_{ssoutat}$ while the flow path variable is used to select the discharge tank:

$$X_{ssoutat} = f_p (l(d_{suct} - d_{sb2}) \cdot \frac{1 - d_{sb2} / d_{suct}}{1 - d_{sb2} / d_{at}} \cdot X_{ssm2}) +$$

$$+ (1 - f_p)(l(d_{suct} - d_{sb1}) \cdot \frac{1 - d_{sb1} / d_{suct}}{1 - d_{sb1} / d_{at}} \cdot X_{ssm1}) \tag{17}$$

As d_{suct} is only dependent on the flow, there is no need to consider different suction depths for each of the aeration tanks.

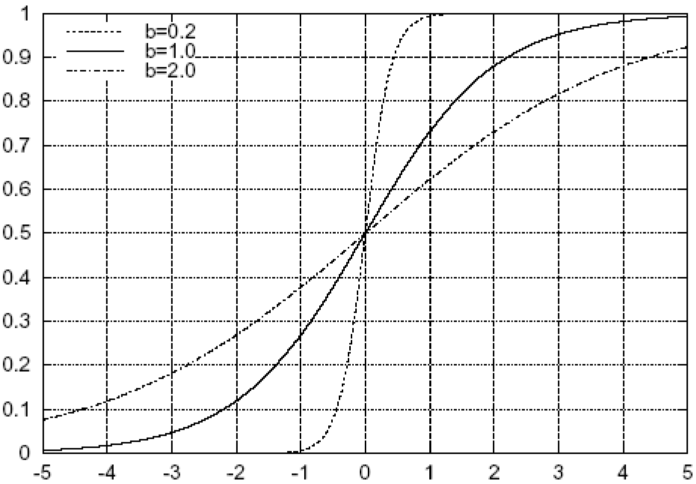

Fig. 3. the logistic function is shown for $a = 0$ and 3 different values of b

3. MATRIX FORM

In order to use a matrix notation, introduce the state vector**X**, the input vector **U** and the observation vector **Y** :

$$\begin{cases} \mathbf{X} = [X_{ssm1}, X_{ssm2}, d_{sb1}, d_{sb2}]' \\ \mathbf{U} = [f_p, m_1, m_2, X_{ssr}, Q_i, Q_r]' \\ \mathbf{Y} = [X_{ssm2}, X_{soutat}]' \end{cases} \tag{18}$$

By use of the vector function f(**X;U; t)** the mass balances and sludge blanket depth equations can be expressed in a vector differential equation:

$$\frac{d\mathbf{X}(t)}{dt} = \mathbf{f}(\mathbf{X}, \mathbf{U}, t) \tag{19}$$

where $\mathbf{f}(\mathbf{X},\mathbf{U},t)$ is easily constructed from equations (7) – (19) and (18).

The measurements are described by the observation equation:

$$\mathbf{Y}(t) = \mathbf{h}(\mathbf{X}, \mathbf{U}, t) \tag{20}$$

where h(**X;U**; t) is constructed from equations (17) and (18).

The parameters in the model formulation can be estimated as random variables using the maximum likelihood parameter estimation method.