ANALYSES OF ACCURACY OF GEAR GRINDING

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Abstract: *The paper deals with the ways of improving gear grinding operations. The technologic factors affecting the radial runout of tooth surfaces at grinding basic holes of gears are investigated. The analytical relations for estimation of a degree of influence of these factors are received. The dominant factors determining non-uniformity of the tooth surface grinding allowance are found out and the ways for its decreasing are discussed.*

Keywords: *gear, tooth surface, hole, accuracy, radial runout, reliability, fixture, locating roller, manufacturing error, grinding allowance, grinding, shaving, honing, checking.*

1. Introduction

Finish machining of precise, and especially hardened, gears includes necessarily their base hole and face grinding followed by tooth surface abrasive machining. The complexity of tools for tooth grinding, their rather low stability, and need in often dressing-profiling have a negative effect on efficiency of the machining and limit opportunities to increase its accuracy. Non-uniform and relatively large machining allowance aggravates lacks of methods for gear finish machining.

It is known that the basic cause of increased allowance for finish machining is an error of workpiece positioning, which components are locating, clamping, and fixture errors [1]. These errors are especially appreciable while locating surfaces are changed and when these surfaces are complicated. Such cases take place at machining gears.

In order to reduce the grinding allowance, it is the gear surface that should be used as the locating one on operations of finish grinding the gear holes and faces. Thus various, frequently exact and expensive, fixtures, for example, self-centering gear holding chucks of membrane or jaw type are applied. Gear workpieces are positioned in these fixtures by the tooth surfaces and by means of locating rollers, balls, or cones. However even with the aid of such devices, it is difficult to ensure required high accuracy and reliability of machining.

In this connection, it is important, for perfection of the technology, to reveal the factors dominantly affecting the result of machining. Using our experience in studying of machining accuracy [1-3], we undertake the present investigation in order to analyse an accuracy of gear wheel grinding on hole grinders. The quality of such operations defines sizes and uniformity of grinding allowances for subsequent tooth surface machining. The paper pursues the didactic purpose as well – to show future (and not only) specialists in gear wheel manufacturing the opportunity of a technique of the differential analysis of accuracy for machining operations.

2. Analyses of accuracy of the hole grinding operation

The radial runout of a tooth surface is one of major parameters of gear kinematics accuracy. At machining, it is formed under influence of a large number of technologic factors including controlled ones [4]. By influencing on the latter, it is possible to control accuracy and reliability of the process.

Let us consider the diagram of positioning of a gear workpiece on a hole grinding machine tool for machining the basic hole (fig. 1). The workpiece to be machined (1) is located from its face and tooth surfaces by using *n* locating rollers (2). This latter contact with both the tooth surface and cylindrical surfaces of *n* fixture jaws (3) made integral with the membrane. The jaws are able to move in a radial direction a little under force *Q* acting on the membrane. It is necessary to create a clearance required for setting up and taking away the workpiece and its centering. The workpiece is clamped up by force P which presses it with its face against flat support pads (4). The fixture is mounted on spindle (5) of the grinder by using its face and exact centering taper hole. Such fixtures provide high accuracy of machining and, therefore, are widely applied in gear manufacturing.

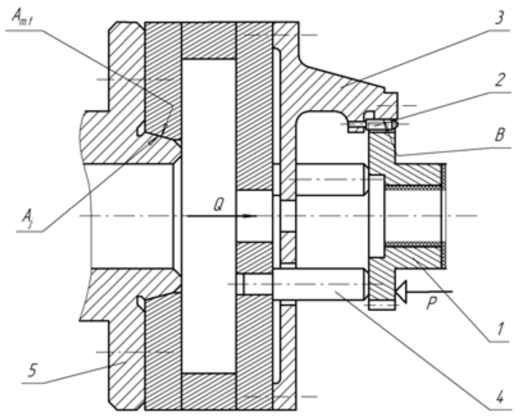

Figure 1. A machining fixture for hole and face gear workpiece grinding.

The analysis of the process of gear hole grinding allows us to reveal the following technologic factors affecting the formation of radial runout of gear tooth surfaces:

1) radial runout $\varepsilon_{m.t}$ of centering spigot $A_{m.t}$, (see fig. 1) of the grinder spindle. For various grades of grinder accuracy $\varepsilon_{m.t}$ makes *3* through *10 μm*;

2) radial runout $\varepsilon_{o.c.j}$ of fixture locating surface *B* related to fixture surface A_j. At accuracy that is usual for tool room manufacturing, $\varepsilon_{o.c.j}$ makes *10 … 20 μm*. Note that a diameter accuracy of surface *B* does not influence the radial runout of the tooth surface concerning the grinded hole of the gear;

3) additional radial runout ε_p of fixture surface *B*, caused by an error of installation of the fixture on the machine tool spindle. ε_p depends first of all on the greatest clearance S_{max} between surfaces $A_{m.t}$ (the machine tool centering spigot) and A_j (the fixture centering hole): $\varepsilon_p=S_{max}$. For accuracy usually applied in practice S_{max} makes *3 … 5 μm*;

4) the tolerance T_D of locating roller diameter *D*. The rollers are usually produced with the accuracy of *IT6* through *IT8*, i.e. $T_D=9 \ldots 12\ \mu m$. The factors stated above in items 2, 3, and 4, form so-called fixture error;

5) form deviation $\varepsilon_{g.f}$ of the tooth surface from its perfect form. $\varepsilon_{g.f}$ causes displacement of an axis of symmetry of the tooth surface in relation to an axis of the hole and, thus, is the reason of occurrence of a locating error at gear wheel positioning in the fixture. The standards specifying the gear accuracy define radial runout F_r. It should be noted that this parameter is complex, i.e. covers both accuracy of the surface form and accuracy of the surface orientation in relation to a datum surface (for example, the basic hole of the gear). It is shown in papers

[1, 3] that an error of the form at least does not exceed *60 %* of the total radial runout. Tooth surfaces after heat treatment (hardening) have usually accuracy of *7* through *9* degrees. For gears (with module *m* up to *6.3 mm* and wheel diameter up to *400 mm*) F_r makes *56 … 100* μm. Thus $\varepsilon_{g.f}$ does not exceed *34 … 60* μm. It should be noted that some local gear surface irregularities (rather significant in values but seldom met with), caused e.g. by welding fine chips up to the tooth surface when heat treating, are not taking into account here;

6) a wear of the fixture working surfaces can essentially lower the accuracy of gears to be machined. It is obvious that the greatest wear should be expected for locating surfaces *B* (see fig. 1) and, especially, cylindrical surfaces of the locating rollers. Though contacts "a roller – locating surface *B*" and "a roller – a tooth surface" are linear, their wear-life, as evidenced by the experience in using of fixtures with such locating elements [1], is rather large. It is owing to the fact that they are not loaded by clamping and cutting forces. Therefore at accuracy calculating, it is enough to limit a roller wear within a roller manufacturing tolerance. Thus, however, it is necessary to accept that the influence of the tolerance on the total tooth surface machining error is regular. At the same time, influence of the other factors has random character.

Other technologic factors practically do not affect the radial runout. So, the force clamping the workpiece in the fixture is directed along its axis. Therefore, it is possible to assume that its action is not accompanied by any gear displacement in a radial direction. At grinding, cutting forces are rather small and their influence on formation of the radial runout is improbable.

Taking into account the above mentioned considerations, the total radial runout of the tooth surface in relation to the hole received by its grinding could be calculate by using the formula

$$\varepsilon = \varepsilon_4 + \sqrt{\varepsilon_1{}^2 + \varepsilon_2{}^2 + \varepsilon_3{}^2 + \varepsilon_5{}^2}\,, \tag{1}$$

where the indexes at the constituent errors of total error ε indicate the number in the list of the above considered manufacturing factors.

It is obvious that $\varepsilon_1=\varepsilon_{m.t}$, $\varepsilon_2=\varepsilon_{o.c.j}$, $\varepsilon_3=\varepsilon_{p.}$ The relations of ε_4 and ε_5 from T_D and $\varepsilon_{g.f}$ will be established below.

A locating roller in a contact to a tooth surface of the workpiece is shown in fig. 2. The roller diameter should be such that its upper generatrix should extended above the addendum circle of the gear, i.e. $R_0>r_a$. It is recommended [5] that rollers should contact with gear surfaces at points placed approximately $0.3{\cdot}m$ from the gear addendum circle (*m* is module of gearing).

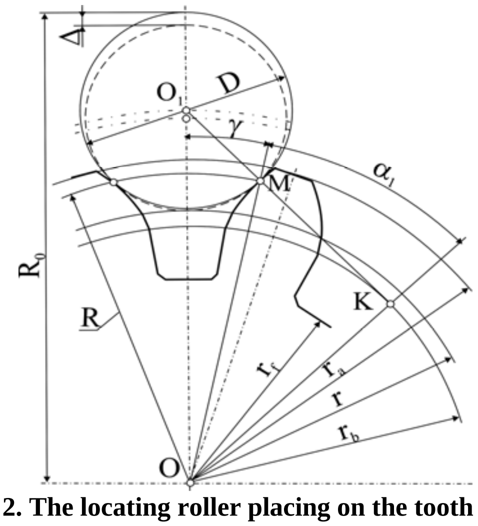

Figure 2. The locating roller placing on the tooth surface.

From fig. 2, the diameter of the roller can be found with [5]

$$D = 2(O_1K - MK) = 2[r_b \cdot \tan(\gamma + \alpha_1) - R \cdot \sin\alpha_1], \qquad (2)$$

where r_b is a radius of the gear base circle;

R is a radius of the circle on which there are points the roller contacts with the tooth surface;

$$\gamma = \frac{\pi}{2Z} - (\tan\alpha_w - \alpha_w) + (\tan\alpha_1 - \alpha_1); \qquad \alpha_1 = \arccos\frac{r_b}{R};$$

α_w is a pressure angle of the gearing;

Z is a number of teeth of the gear.

Thus, radius R_0 of fixture locating surface B may be defined with the formula

$$R_0 = \frac{r_b}{\cos(\gamma + \alpha_1)} + \frac{D}{2}. \qquad (3)$$

With accuracy sufficient to the problem in question, we may believe that angle $(\alpha_1 + \gamma)$ is unaffected by size D within the limits of its tolerance T_D. Therefore, the position of the roller in a tooth space can be considered as its position in a V-location with the angle equals $2(\alpha_1 + \gamma)$. Then changing D within the limits T_D causes changing radius R_0 by Δ. The latter may be found with the expression

$$\Delta = \frac{1}{2}T_D\left(1 + \frac{1}{\sin(\alpha_1 + \gamma)}\right). \qquad (4)$$

Now we establish the effect of Δ on displacement x of the tooth surface axis in respect to its true position. This displacement is required error ε_4, i.e. a constituent of the total radial runout (1). We consider fig. 3 where a simplified diagram of the grinding fixture with three locating rollers is shown. It is quite obvious that the greatest displacement x of the tooth surface axis may be determined by comparing two extreme cases. The first one – all rollers have identical (for example, the greatest possible size D_{max}); the second one – two rollers have the least possible diameter D_{min}, but the third – D_{max}. In the first case, the distance from the tooth surface axis to the locating surface is equal to R_0, in the second – to (R_0-Δ) for the two rollers and to R_0 – for the third roller. In the second case, a radius of the locating surface is decreased by x, i.e. becomes equal to R_0-x. Note that the locating surface can move in a radial direction a little. Neglecting rather small angle changes, we may obtain from fig. 3 the followings:

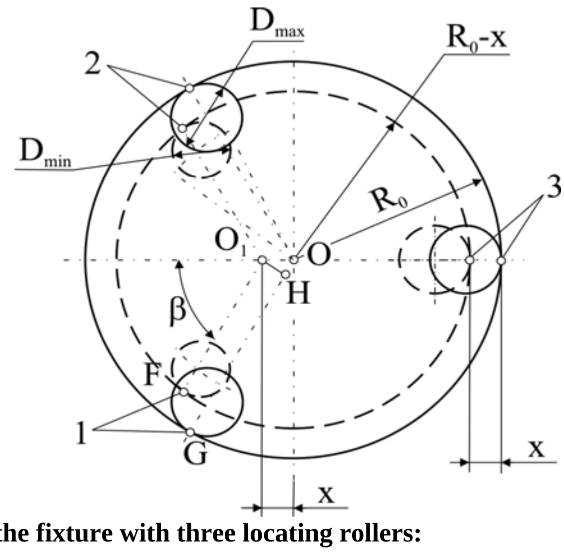

Figure 3. The diagram of the fixture with three locating rollers:
***x* – maximum value of displacement of the tooth surface axis caused by the roller diameter changing; 1, 2, 3 - points of roller contacts with the fixture locating surface.**

$$OO_1 = x; \qquad OH = x \cdot \cos\beta = x\cos\frac{\pi}{3} = 0.5x; \qquad O_1F = R_0 - \Delta = R_0 - x - \frac{x}{2}$$

whence $x = \frac{2}{3}\Delta$, and taking into account (4)

$$\varepsilon_4 = x = \frac{1}{3}T_D\left(1 + \frac{1}{\sin(\alpha_1 + \gamma)}\right). \qquad (5)$$

If a number of the rollers is more than three (for example, six), it is obvious that the greatest displacement of the tooth surface axis also may be determined by using fig. 3 and formula (5).

We will now establish a relation of ε_5 from form accuracy $\varepsilon_{g.f}$ of the tooth surface. Let us examine the diagram of distribution of the form deviations of the tooth surface of a gear to be machined in places of its contacts 1, 2, and 3 with three locating rollers (fig. 4). Assuming random character of distribution, its obeying the normal distribution law

$$y_{(f)} = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{f}{2\sigma^2}}, \qquad (6)$$

where σ is the standard deviation of the distribution;

f is a random displacement of the real tooth surface from its true position, and also that the practical range of displacement scatter f is equal to 6σ, we determine the distribution law of deviation δ of the tooth surface axis from its true position. As may be seen from fig. 4

$$\delta = \frac{1}{2}\left|f_2 - f_1\right|. \qquad (7)$$

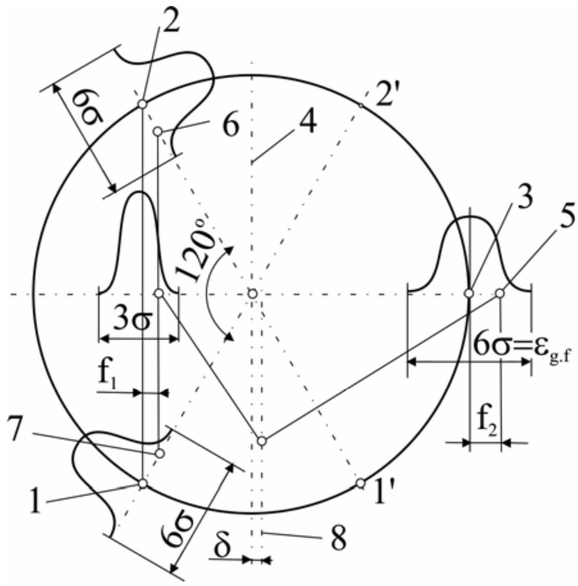

Figure 4. Diagram for definition of the radial runout of the tooth surface:
1, 2, 3 - points of contact of three rollers with the tooth surface (nominal position); 4 - axis of symmetry of the nominal tooth surface; 5, 6, and 7 - a random displacement of the surface (displacement f1 and f2; f1 - total displacement at rollers 1 and 2); 8 - an axis of symmetry about points 5 and 6-7).

The probability density $y(\delta)_3$ of the axis deviation δ equals the product of probabilities of occurrence of displacements f and ($f \pm \delta$) of the form along the whole range of f scatter (index *3* at $y(\delta)$ specifies the number of rollers in the grinding fixture)

$$y(\delta)_3 = \frac{1}{2}\int_{-\infty}^{\infty} y(f)(y(f+\delta)+y(f-\delta))df\,, \tag{8}$$

and after using (6) and (7)

$$y(\delta)_3 = \frac{1}{2\pi\sigma^2}\int_{-\infty}^{+\infty} e^{-\frac{f^2}{2\sigma^2}}\left(e^{-\frac{2(f+\delta)^2}{\sigma^2}} + e^{-\frac{2(f-\delta)^2}{\sigma^2}}\right)df\,. \tag{9}$$

After integration (9) over *f*, we receive the distribution function (or density) of *δ*

$$y(\delta)_3 = \frac{1}{\sigma\sqrt{2{,}5\pi}}e^{-\frac{\delta^2}{2{,}5\sigma^2}}\,. \tag{10}$$

Distribution function (10) has the following characteristics:

- the average value of the deviation $M(\delta)_3 = \int_{-\infty}^{\infty}\delta\cdot y(\delta)d\delta = 0$; (11)
- the dispersion $M(\delta)_3 = \int_{-\infty}^{\infty}(\delta - M(\delta))^2 y(\delta)d\delta = \frac{2.5}{4}\delta^2 = 0.62565^2$; (12)
- the standard deviation $\sigma(\delta)_3 = \sqrt{D(\delta)} = 0.791\sigma$; (13)
- the practical deviation scatter covering *99.73* percent of all possible deviations

$$\varepsilon(\delta)_3 = 4.746\sigma\,. \tag{14}$$

Show that when a number of the rollers is twice increased (that is equal to 6), the accuracy of centering of the tooth surface increases essentially as well. The three additional rollers change diagram 4 so that the right hand part of the diagram with point 3 becomes symmetric about axis 4 with points 1 and 2. I.e. the two additional rollers contact with the locating surface of the fixture in points 1' and 2'. As a result, equations (10) – (14) are transformed into the followings:

$$y(\delta)_6 = \frac{1}{\sigma\sqrt{\pi}}e^{-\frac{\delta^2}{\sigma^2}};\; M(\delta)_6 = 0;\; D(\delta)_6 = \frac{1}{4}\sigma^2;\; \sigma(\delta)_6 = \frac{1}{2}\sigma;\; \varepsilon(\delta)_6 = 3\sigma\,. \tag{15}$$

It is obvious that the increase of a number of the rollers up to a number of teeth on the gear to be machined should ensure its centering with a zero error. However because of excessive complexity, such fixtures are not in use.

Transform formula (1) with regard to the received relations

$$\varepsilon = \frac{1}{3}T_D(1+\frac{1}{\sin(\alpha_1+\gamma)}) + \sqrt{\varepsilon_{m.t.}{}^2 + \varepsilon_{o.c.j}^2 + \varepsilon_p^2 + (K\cdot Fr)^2}\,, \tag{16}$$

where *K=0.47* for the three roller fixture and *K=0.3* for the six roller fixture.

Expression (16) allows estimating radial runout of a gear after grinding its hole. It takes into account accuracy of the machine tool, of some elements of the grinding fixture, and of the tooth surface.

3. Application

We use relation (16) for revealing the dominant factors influencing accuracy of gear machining on grinding operations. At average values of the total error components ε ($\varepsilon_{m.t.}=7$ μm, $\varepsilon_{o.c.j}=15$ μm, $\varepsilon_p=4$ μm, $T_D=15$ μm, $F_r=71$ μm) for a gear wheel with $m=4$ *mm*, $\alpha_w=20°$, $Z=24$, $D=9$ *mm*, $\gamma+\alpha_1=30°$ the calculated radial runout of the tooth surface with respect to the ground hole (at $K=0.3$) makes 42 μm. The share of each partial error is shown in fig. 5.

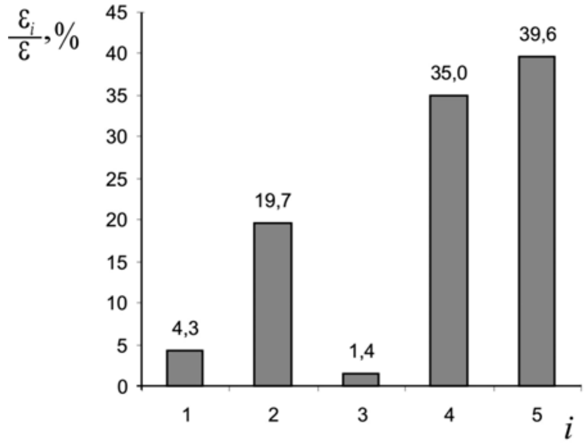

Figure 5. The influence of the technologic factors on radial runout of the tooth surface after hole grinding.

It is visible from diagram 5 that the strongest influence on radial runout of the tooth surface, and therefore, on magnitude and degree of non-uniformity of allowance for subsequent tooth grinding is caused by the tooth accuracy before grinding (share of error ε_5 is equal to *39.6 per cent*) and diameter accuracy of the locating rollers (share of ε_4 equals *35 per cent*). Therefore in order to decrease labour input and to increase reliability of tooth grinding operations, it is necessary, first of all, to raise the tooth surface accuracy before gear grinding and the grinding fixture accuracy. It is necessary to recall that in the used technique of an accuracy estimation of the grinding operations tolerance T_D includes a stock for a roller wearing as well. Hence, by changing roller accuracy, we may control both a gear grinding accuracy and a grinding fixture wear-life.

The best suited method for increasing tooth surface accuracy is, to our mind, a gear shaving. It is rather efficient and, that is more important, shaved surfaces have the least heat treatment distortion. Besides, it may be useful to apply a tooth honing before the hole grinding. This method smoothes off tooth surface macroroughnesses, especially local ones. It enables to reduce non-uniformity of the grinding allowance as well.

A preliminary (before grinding) gear checking may prove to be the simplest but enough efficient. While this checking, one should mark suitable tooth spaces which in subsequent are expedient for using for contact with the locating rollers. The same tooth spaces should be used at gear workpiece setting up for its tooth grinding.

4 Conclusions

The technologic factors affecting the radial runout of the tooth surface at grinding the basic hole of a gear are investigated. The analytical relations for estimation of a degree of influence of these factors are received. The dominant factors determining non-uniformity of the grinding allowance are found out and the ways for its decreasing are discussed.

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