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APPLICATION OF MULTIAXIAL FATIGUE ANALYSIS TO DAMAGE PREDICTION OF TRANSPORT MACHINERY

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A***bstract:*** *The paper proposes the application of the optimising procedure to multiaxial rainflow analyse of non-proportional loading of randomly excited structures. Cumulative damage is calculated by Corten-Dolan hypothesis. Applied methods are implemented to finite element solver FEM_MRFA created in MATLAB. The numerical application and short comparison is presented by finite element analyse of the randomly oscillated frame structure of the track maintenance machine DELTA.*

Key words: *multiaxial fatigue analysis, rainflow analysis, cumulative damage, random vibration.*

1. INTRODUCTION

It's generally problematic to calculate the fatigue life for machines parts where a dynamic load results are considerable changed in the principal stresses. This fact is any topical problems in vehicles dynamics. By FE analyse we can obtain generally six components of the stress-time function (multiaxial stress) but it is very complicated to define the computational hypothesis for equivalent uniaxial load spectrum. In this case, the rainflow analysis for random stresses, known in classic uniaxial form as von Mises or Tresca hypotheses, is impossible.

The goal is to propose the optimising approach application to estimate the high-cycle fatigue damage for multiaxial stresses caused by random vibration, from finite element analyse.

2. MULTIAXIAL RAINFLOW ANALYSIS

This problem can be found in many areas of technology, however, such as in a bodywork structure, axle components, crankshafts, rotary blades for wind power station etc. Assumed the high number of criteria existing in the literature, we'll consider only following techniques:

- general technique – version 1,
- general technique – version 2 and,
- critical plane approach.

General technique – version 1

Let define the multiaxial rainflow counting [4], [5]: Let $\boldsymbol{\sigma}(t)=[\sigma_x,\sigma_y,...]^T$ be a random *n*-dimensional vector, the *n* signals may be thought of either as load components of external forces acting on the structure or as components of the local stress or strain tensor at a given point. The fundamental idea is to count raniflow cycles on all linear combinations $\sigma_{MRF}(t)$ of the random vector components of the form:

$$\sigma_{MRF}(t) = \sum_{i=1}^{n} c_i \cdot \sigma_i(t) = \begin{bmatrix} c_1 & \dots & c_6 \end{bmatrix} \cdot \begin{bmatrix} \sigma_x \\ \vdots \\ \tau_{zx} \end{bmatrix} = \mathbf{c} \cdot \boldsymbol{\sigma} , \qquad (1)$$

where c_i belong to a hypersphere $\sum_{i=1}^{n} c_i^2 = 1$.

Practically, when the stress state is biaxial, the stress components can by written under the form of three dimension vector $\boldsymbol{\sigma} = [\sigma_x, \sigma_y, \tau_{xy}]^T$. A set of linear combinations

$$\sigma_{MRF}(t) = c_1 \cdot \sigma_x(t) + c_2 \cdot \sigma_y(t) + c_3 \cdot \tau_{xy}(t) \qquad (2)$$

can be chosen for values of c_i such as $c_1^2 + c_2^2 + c_3^2 = 1$ thus defining a sphere [2]. The goal is to find extreme value of the estimated damage or life for vector **c**.

General technique – version 2

The finding process of the vector **c** components is effective, if the number of components c_i is minimal. Hence, we can modify the general (six components) stress problem (1) to principal stresses problem (only three components) by relation

$$\sigma_{MRF}(t) = c_1 \cdot \sigma_1(t) + c_2 \cdot \sigma_2(t) + c_3 \cdot \sigma_3(t) , \qquad (3)$$

where $\sigma_1 \geq \sigma_2 \geq \sigma_3$ are principal stresses.

Critical plane approach

For such multiaxial stress fields, the fatigue phenomenon is generally regarded as being governed by a combination of the shear and normal stress acting on critical plane. Hence, let define

$$\sigma_{MRF}(t) = \mathbf{c}^T \cdot \mathbf{T} \cdot \mathbf{c} , \qquad (4)$$

where **T** is the classic stress tensor, and **c** is the vector of the direction cosines of the critical plane,

$$\mathbf{c} = \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix}^T, \quad c_1 = \cos(\alpha_{crit.}), \; c_2 = \cos(\beta_{crit.}), \; c_3 = \cos(\gamma_{crit.}) .$$

In this case the "damage equivalent stress function" is defined by normal stress in critical plane. The critical plane is possible to locate by numerical (optimising) way or to define at the beginning of the computing.

Considering other way, we can write the Dang Van criterion [1] as follows

$$\sigma_{DV} = \frac{\sigma_{1,a} - \sigma_{3,a}}{2} + b \cdot \sigma_{h,\max} , \qquad (5)$$

where $\sigma_{h,max}$ is hydrostatic stress, $\sigma_{1,a}$ and $\sigma_{3,a}$ are the largest and smallest amplitudes of principal values of the stress tensor, b is a material constant. Dang Van assumes critical plane in plane of maximum shear stress.

3. IMPLEMENTATION OF OPTIMISING TECHNIQUES TO RAINFLOW ANALYSIS AND FATIGUE LIFE PREDICTION

Finite element analysis is the traditional numerical technique for the stress estimation of the oscillating mechanical structures. We shall assume

- random excitation,
- geometrically and physically linear FE model,
- Miner's linear law for cumulative damage.

Subject to the normality condition of $\mathbf{c}$, the optimising variables vector is normalised by relation $\mathbf{c}=\frac{\mathbf{c}'}{\sqrt{\mathbf{c}'^T\cdot\mathbf{c}'}}$. Our aim is to find the optimum vector $\mathbf{c}_{opt}$ for the objective function

$$F=\max(D(\mathbf{c}))=\max(\sum_{i=1}^{nc}\frac{n_i}{N_i}) \quad \text{or} \quad F=\min(T(\mathbf{c}))=\min(\frac{t}{3600\cdot D(\mathbf{c})}) \quad [\text{hour}], \qquad (6a,b)$$

where t is time interval of realisation, D is cumulative damage, T is fatigue life in hours, N_i is number of cycles to failure, n_i is number of cycles at one particular stress level. By rainflow decomposition of σ_{MRF} $(t,\mathbf{c})$, we obtain N_i and n_i. Considering Corten-Dolan modification of Wohler curve (Fig. 1) [6], we can write

$$N_i(\mathbf{c})=10^{\left[\frac{\log(\sigma_{A\max})\cdot\log(N_0(\mathbf{c}))-\log(\sigma_{Ai})\cdot[\log(N_0(\mathbf{c}))-\log(N_A)]}{\log(\sigma_{A\max})}\right]}, \qquad N_0(\mathbf{c})=10^{\left[\frac{\log(\sigma_{Ci}(\mathbf{c}))\cdot\log(N_A)-\log(\sigma_{A\max})\cdot\log(N_{Ci}(\mathbf{c}))}{\log(\sigma_{Ci}(\mathbf{c}))-\log(\sigma_{A\max})}\right]}$$

$$N_{Ci}=N_A\cdot\left(\frac{\sigma_{A\max}}{\sigma_{Ci}(\mathbf{c})}\right)^{k\cdot m} \quad \text{and} \quad \sigma_{Ci}(\mathbf{c})=\sigma_C\cdot\frac{R_m-\sigma_{mi}(\mathbf{c})}{R_m} \quad \text{(Goodmann)},$$

where σ_{mi}, σ_{Ai} are mean stress and stress amplitude after rainflow decomposition of $\sigma_{MRF}(t,\mathrm{c})$, R_m is tensile strength. Parameters N_A, σ_{Amax}, N_C, σ_C, N_0, N_i, σ_{Ai} are presented on Fig. 1.

The searching process is realised by computational program FEM_MRFA created in MATLAB. Program contains stress analyse by FEM and optimising procedure by MATLAB function FMINS for chosen element or node.

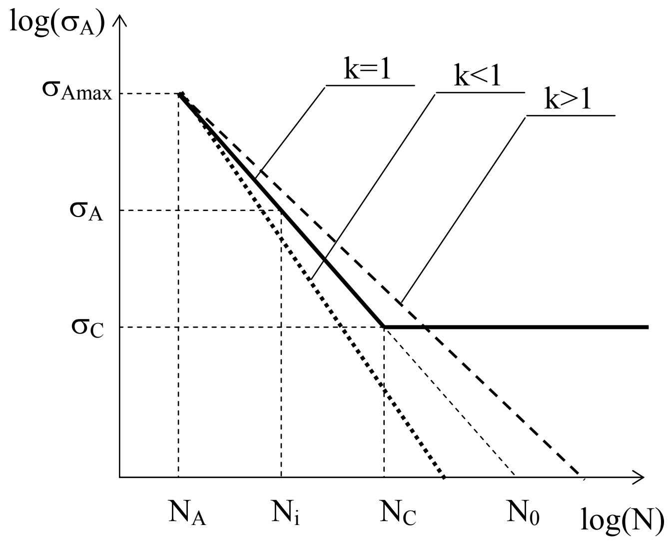

Fig. 1 Wohler σ-N curve

4. FE APPLICATION

Assuming finite element analyse, we can usually simulate the vehicles frames by 3D beam or shell finite elements.

For 3D beams is the uniaxial stress analyse realised in marginal points of the cross section (Fig. 2) by relationship

$$\sigma_x=\frac{N_x}{A}-\frac{M_z}{I_z}\cdot y+\frac{M_y}{I_y}\cdot z \qquad (7)$$

where y, z are the centroidal principal axes, A is the cross section area, I_y, I_z are centroidal principal moments of inertia for the elements section, N_x ,M_y , M_z are internal axial force and bending moments. Considering the introduced

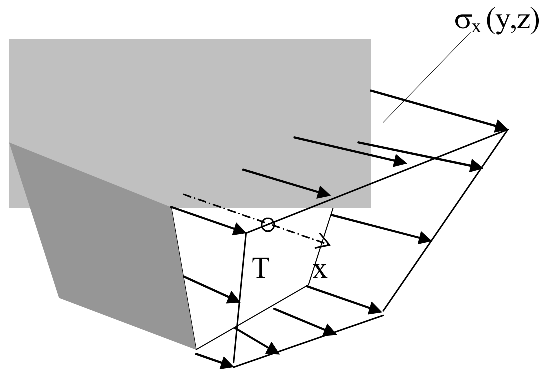

Fig.2 Distribution of the stress σ_x in a beam

facts, we can to observe the inapplicability of the multiaxial technique for this finite element.

On the other hand the application of the shell finite elements to construction of the computational model is defined as a plane stress problem. The stress computing is realised by superposition of the membrane and bending stress components

$$\sigma_{mxx} = \frac{F_{xx}}{t} \quad , \quad \sigma_{myy} = \frac{F_{yy}}{t} \quad , \quad \sigma_{mxy} = \frac{F_{xy}}{t} \tag{8a}$$

and

$$\sigma_{bxx} = \pm\frac{6 \cdot M_{xx}}{t^2} \quad , \quad \sigma_{byy} = \pm\frac{6 \cdot M_{yy}}{t^2} \quad , \quad \sigma_{bxy} = \pm\frac{6 \cdot M_{xy}}{t^2} \, , \tag{8b}$$

where t is the thickness of the element, F_{xx}, F_{yy}, F_{xy}, M_{xx}, M_{yy}, M_{xy} are componets of internal forces and moments per unit width. The stress components on top and bottom surface (Fig. 3) are following

$$\sigma_{xx} = \sigma_{mxx} + \sigma_{bxx}, \quad \sigma_{yy} = \sigma_{myy} + \sigma_{byy}, \quad \sigma_{xy} = \sigma_{mxy} + \sigma_{bxy}. \tag{8c}$$

It's mean that the "damage equivalent stress" is

$$\sigma_{MRF}(t) = c_1 \cdot \sigma_{xx}(t) + c_2 \cdot \sigma_{yy}(t) + c_3 \cdot \sigma_{xy}(t) \tag{9}$$

or by using of the principal stresses

$$\sigma_{MRF}(t) = c_1 \cdot \sigma_1(t) + c_2 \cdot \sigma_2(t) \, . \tag{10}$$

The last relationship is very effective subject to minimum number of the parameters c_i.

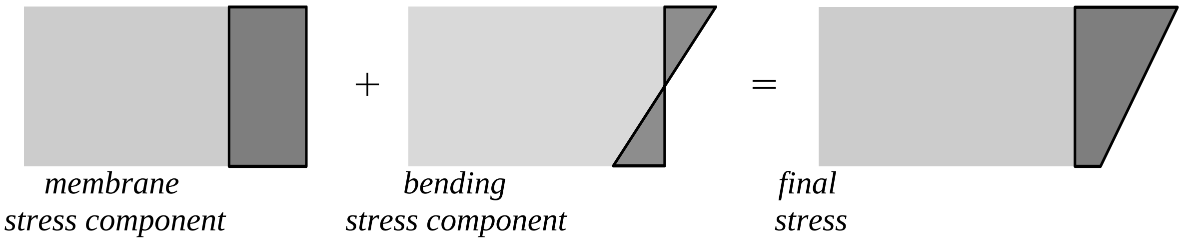

Fig. 3

Fatigue life estimation of the track maintenance machine DELTA

Concrete application of the introduced theory is realised on the finite element model of the track maintenance machine DELTA (Fig. 4). Let's consider the computational parameters:

- Young's modulus of elasticity $E=2.10^{11} Pa$,
- Poisson ratio $\mu=0,3$,
- Density $\rho=7800 kg/m^3$,
- Elastic limit $R_k=\sigma_y=247\ MPa$,
- Tensile strength $R_m= 370\ MPa$,
- Point of Wohler curve $N_A=10^4$ *cycles*, $\sigma_{Amax}=217\ MPa$,
- Fatigue limit $\sigma_C=68,7\ MPa$,
- Constant $k=0,9$,
- Exponent of Woher's curve $m=5,2$,
- Time interval $t\in<0,500>$ [sec],
- Time increment $\Delta t=0,05$ [sec]
- Vehicle speed $v=100$ [km.h^{-1}].

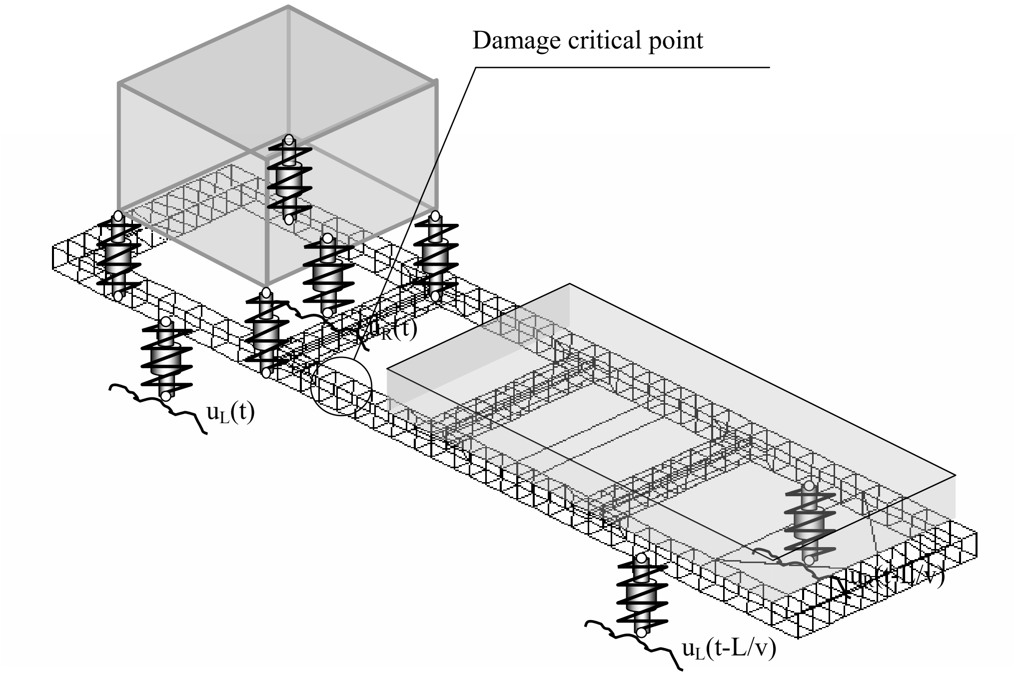

Fig.4 Computational model of the machine DELTA with identification of the critical point subject to cumulative damage

The functions $u_L(t)$ and $u_R(t)$ are the zero-mean random functions of kinematical excitation in vertical direction, which we know from measurement on real railway line. The power spectral density of $u_L(t)$ and $u_R(t)$ is presented on figure 5.

Since the rainflow analyse is possible to realise in only time domain, we'll use the Monte Carlo simulation of the functions $u_L(t)$ and $u_R(t)$ (Fig. 6).

With the advent of recent computational facilities, this method becomes ever more attractive. The results are determined from the series of numerical analyses (approximately 100-1000 realisations of random excitation). It is recommended to generate about 5000-10000 random values of excitation function (defined by power spectral density $S_{ff}(\omega)$) for each realisation.

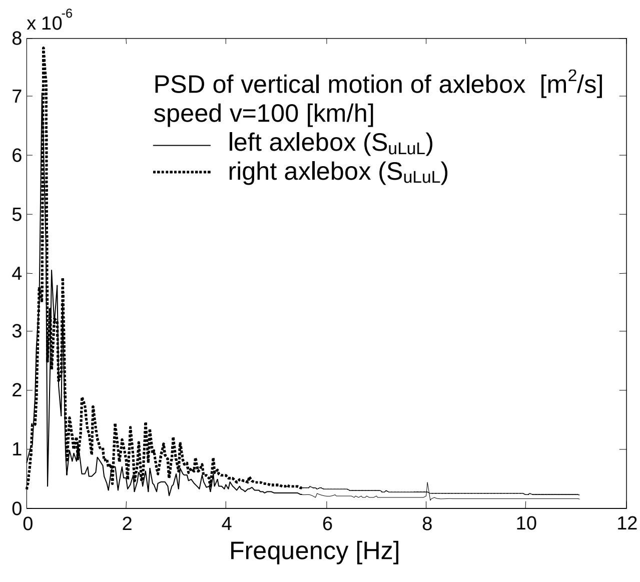

Fig. 5

Simulation of input stationary Gaussian process $u(t)$ with zero mean can be formulated by

$$u(t) = \sqrt{2} \cdot \sum_{k=1}^{N} \sqrt{S_{uu}(\omega_k) \cdot \Delta\omega} \cdot \cos(\omega_k \cdot t - \varphi_k)\,, \tag{11}$$

where φ_k is uniformly distributed random number ($0 \leq \varphi_k \leq 2.\pi$), $u(t)$ is zero mean stationary process with power spectral density $S_{uu}(\omega)$. The generated time behaviours of the $u_L(t)$ and $u_R(t)$ are shown in figure 6.

The finite element model contains 632 shell, 32 beam and 8 spring-damper elements. The displacements were computed in 651 nodes. The damage and fatigue life prediction was calculated and compared for selected – critical nodes (The best critical node was node 75). The chosen results for critical point are presented in Table 1.

Table 1 Optimising variables for multiaxial fatigue analysis in critical point (node 75)

Optimising variables				
	Initial value	Final value	Damage D	Fatigue life T [hour]
c_1	1	0,69032	$1{,}085.10^{-6}$	63991
c_2	0	0,72351		

The time behaviour of the principal stresses in critical point is shown in figure 7 and the principal stresses correlation is presented on figure 8.

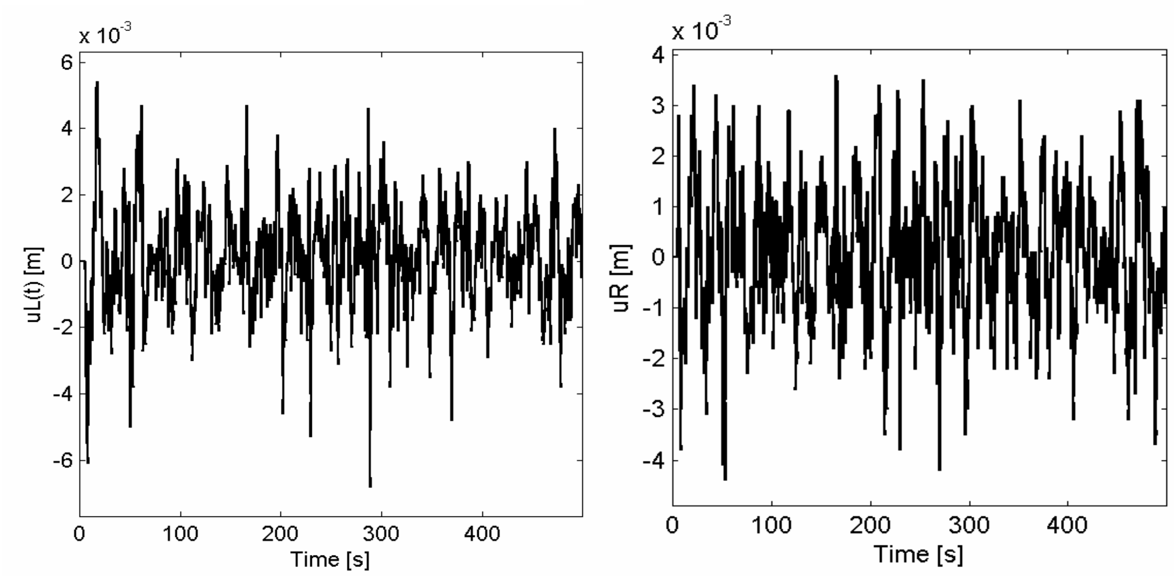

Fig. 6 Generated time behaviors of the vertical random axlebox displacements $u_L\ (t)$ and $u_R\ (t)$

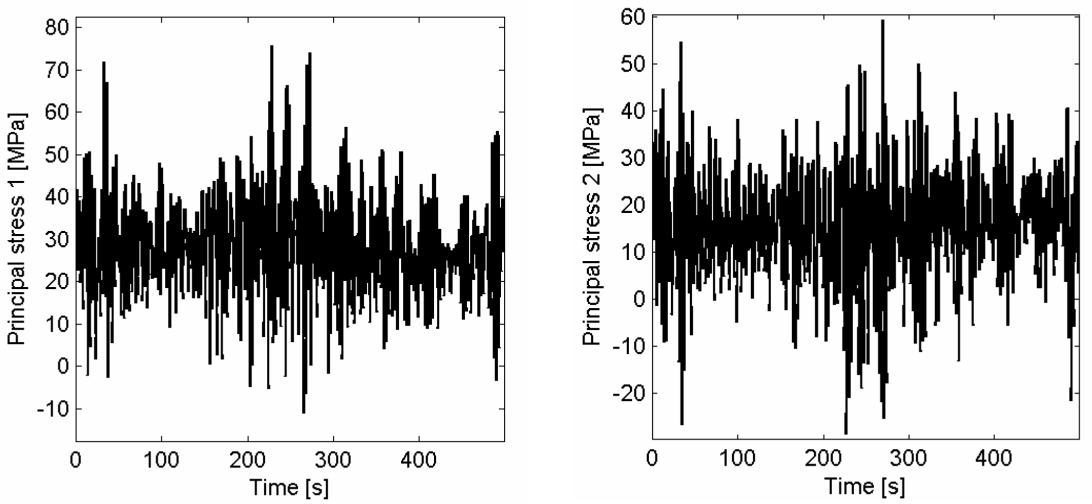

Fig. 7 Time behaviors of principal stresses σ_1 (t) and σ_2 (t) in damage critical

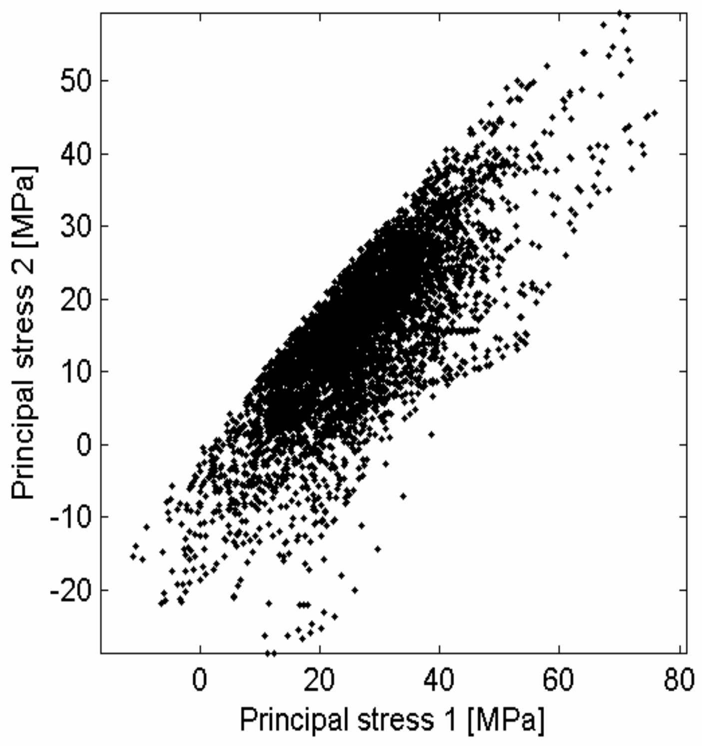

Fig. 8 Non-proportionality of principal stresses in critical point

5. CONCLUSION

This paper discusses multiaxial approach for the estimation of fatigue damage or fatigue life of the shell structural models. It is introduced the chosen ways of the multiaxial rainflow analyse of random stress tensor. The suggested and applied approach is based on equation (1) and leads to an optimising problem of a few (minimal two) variables c_i *(i=2,…6)*. The application of multiaxial rainflow analyse is presented by the cumulative damage calculation of the randomly excited frame of the track maintenance machine DELTA.

6. REFERENCES

[1] Balda M.: Prediction of Damage Cumulation in Vibrating Rotors, Proc. of IFToMM Fifth International Conference on Rotor Dynamics, Darmstadt, 1998.

[2] Dressler K. - Köttgen V. B. - Kötzle H.: Tools for fatigue evaluation of non-proportional loading, In proceedings of Fatigue Design'95, Helsinky, Finland 1995.

[3] Papuga J. - Růžička M.: Uniaxiální a multiaxiální metódy řešení životnosti y hlediska algoritmizace, 17. konferencia Výpočtová mechanika 2001, Nečtiny 2001, (269-278).

[4] Pitoiset X.- Preumont A. - Kernilis A.: Tools for a Multiaxial Fatigue Analysis of Structures Submitted to Random Vibrations, Proceedings European Conference on

Spacecraft Structures Materials and Mechanical Testing, Braunschweig, Germany, 1998.

[5] Preumont A. - Piéfort V.: Predicting random high cycle fatigue life with finite elements, Lournal of Vibration and Acoustics, 116, 1994, (245-248).

[6] Trebuňa F. - Bigoš P.: Intenzifikácia technickej spôsobilosti ťažkých nosných konštrukcií, VIENALA, Košice 1998.