5TH INTERNATIONAL MEETING OF THE CARPATHIAN REGION SPECIALISTS IN THE FIELD OF GEARS

RESULTS OF RESEARCH WORK ON FLEXIBLE GEAR DRIVES II.

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Abstract. *The team-mates of the Department of Machine Elements, University of Miskolc have been working on epicyclic gear drives for decades therefore one of the variants of the epicyclic gear drive got into the group of analysed and tested gear drives. The important parts of the research work are the following: determination of geometric data of the parts of the drive, revealing of phenomenon disturbing the proper operation, kinematic analyses, survey of the influence of load, production of test drive, laboratory tests and development of drives for industrial production. This paper summarises the achievements.*

Key words*: flexible, gear drive, kinematic analyses, laboratory test, radius of curvatur*

1. INTRODUCTION

Although the relation between load and the elastic deformation of bodies due to load has been widely known since the paper published by *Robert Hooke* in 1675, there are relatively few mechanisms the operation of which is based on the elastic deformation of bodies. These mechanisms include flexible drives together with their version of flexible gear drives. In drives, the elastic deformation of gears presents an exciting problem as the rigidity of gears represents a condition for the safe operation of classic gear drives, and in flexible gear drives the flexibility of one or both of the gears represents the basis of operation. The shape of a flexible gear changes under the load on the drive;

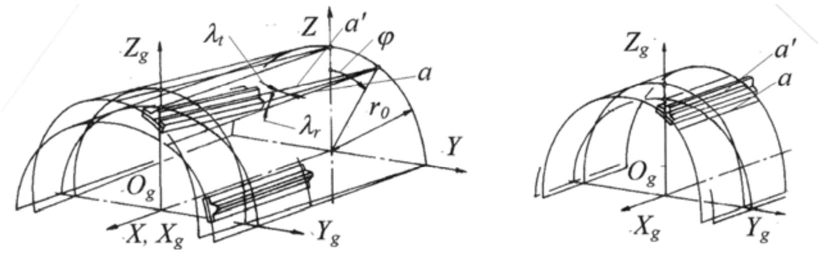
a) the change and its consequences can be best followed through the radius of curvature of the middle surface of the flexible gear,

b) when the relation between the load and radius of curvature is known, it becomes possible to develop the tooth profile expediently.

2. A SPECIAL CASE OF GEAR ENGAGEMENT

2.1. Parts of the flexible gear drive

The basic elements of flexible gear drives are the flexible gear, the rigid gear and the wave generator. The flexible gear is a thin-walled cylinder, its middle surface being mostly a right cylinder with radius r_0. The teeth are located in the direction of the a generatrix of the cylinder.



a) *b)*

Fig. 1. *Middle surface and teeth of a flexible gear a) Cup type flexible gear, b) Ring type flexible gear*

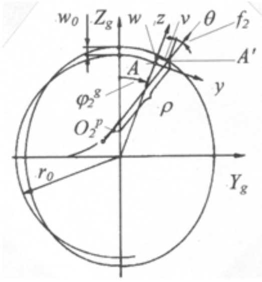
The teeth of the flexible gear remain in the direction of the generatrix also of the gear deformed into an oval shape by the generator; in a cup type flexible gear they are slant as related to the gear axis (*Fig. 1.a)*), and the teeth of the ring type flexible gear are parallel to the gear axis (*Fig. 1.b)*).

2.2. Flexible gear (*Fig. 2.*)

Plane $Y_g Z_g$ of the generator cuts the mean circle with radius r_0 out of the middle cylinder. A is a random point of the mean circle, and its angular coordinate is φ_2^g. After slipping the generator into the flexible gear, point A moves point A', the corresponding radial displacement is $w = w(\varphi_2^g)$, the tangential displacement is $v = v(\varphi_2^g)$, and $\vartheta = \vartheta(\varphi_2^g)$ is the angular displacement of the normal f_2. The radius of curvature belonging to point A' is $\rho = \rho(\varphi_2^g)$, the location of the centre of the osculatory circle on the evolute e of the deformed centre line is O_2^P. The change in the tooth profile of the flexible gear due to load is within the profile error in the range of the working depth of the tooth, that is the teeth can be regarded as rigid. The symmetry axis of a random tooth is the normal f_2 of the centre line. When the flexible gear rotates against the generator, the momentary point of rotation of the tooth is point O_2^P. As the bending rigidity of a flexible gear changes along the circumference because of the teeth and the tooth spaces, the flexible gear is similar to an endless chain, the teeth can be regarded as rigid, and the flexible spaces between the teeth correspond to the chain pivots.

2.3. Meshing of tooth pairs (*Fig. 3.*)

In a general case *the meshing of tooth pairs can be reduced to the meshing of an internal and an external gear.*



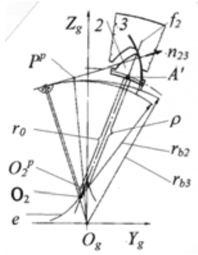


Fig. 2. *Centre line of a flexible gear* ***Fig. 3.*** *Meshing of a tooth pair*

The axes of the gears are out-of-line, the angle of axis deflection and the axle base change according to the rotation of the gear tooth pair against the generator. If the teeth of a flexible gear are parallel to the gear axis (or the slantness of the teeth is ignored), *the meshing of the tooth pair can be reduced to the engagement of a "pair of wheels" with changing centre distance and changing transmission ratio.* The momentary centre distance is the distance between the point of rotation O_3 and the momentary point of rotation O_2^P . The momentary pole of meshing is P^P . The momentary transmission ratio is

$$i_{23}^{g} = P^{p}O_{3} \,/\, P^{p}O_{2}^{p} \tag{1}$$

The location of the momentary point of rotation O_2^P as related to point A' of the deformed centre line is determined by the *radius of curvature* $\rho = \rho(\varphi_2^g)$ on the symmetry axis f_2 of the tooth.

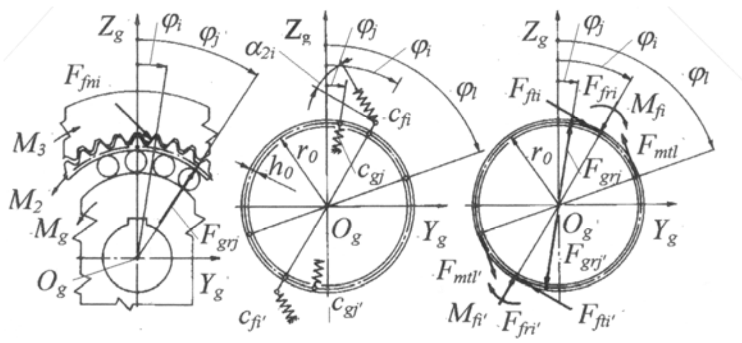


Fig. 4. *Forces and moments acting on a flexible gear*

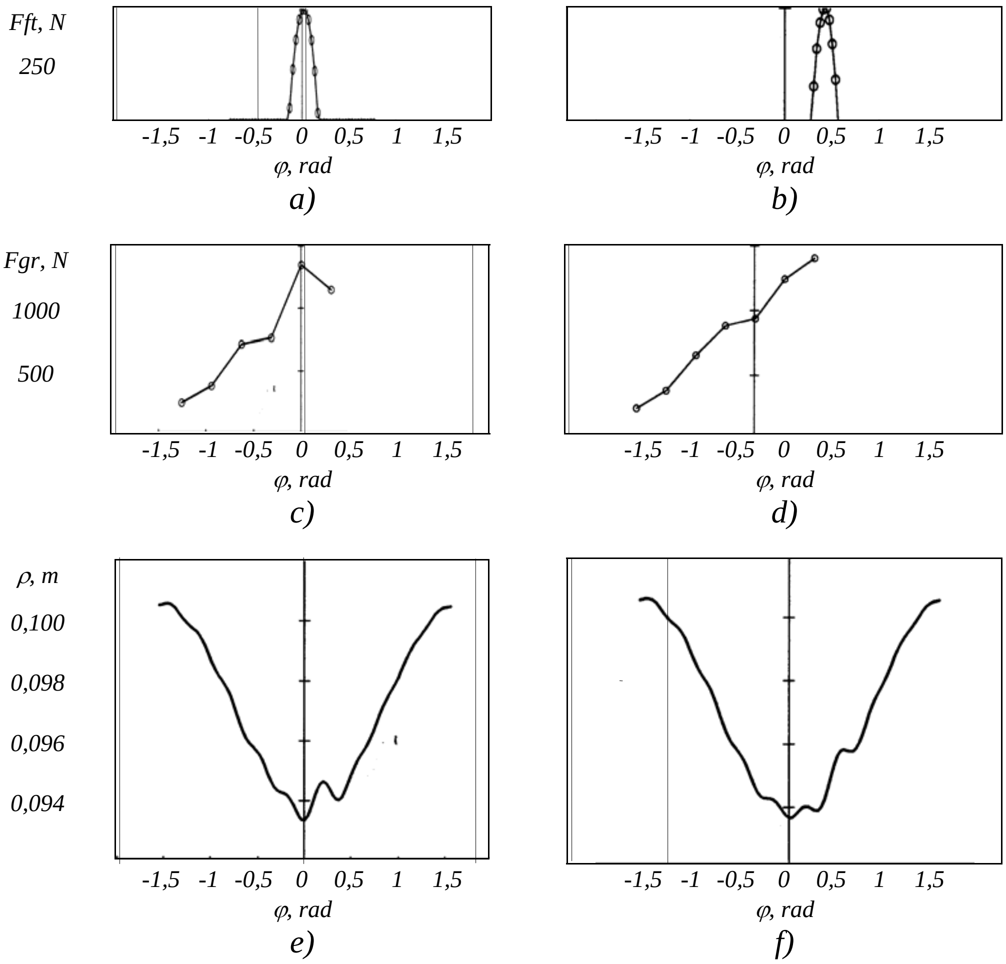


***Fig. 5**. Calculated values. a) and b) tangentional tooth load, c) and d) radial generator load, e) and f) radius of curvature. a), c) and d) wave gear drive. b), d) and f) wave gear coupling.* $w_0/m = 1{,}1$, $m = 1$ *mm*, $r_0 = 97{,}1$ *mm*, $h_e = 3{,}7$ *mm*, $b_e = 80$ *mm*, $M_2 = 600$ *Nm*

3. CALCULATION OF THE RADIUS OF CURVATURE

3.1. Forces and moments acting on a flexible gear *(Fig. 4.)*

In one half of the flexible gear there are *o* number of tooth pairs of mesh, with coordinate φ_i is acted on by force F_{fni}. The flexible gear is supported by rollers with number *p*, or a gear deformed into a polygonal shape is supported by the disc under teeth with number *p* , and the supporting force F_{grj} has a coordinate φ_i. Moment $M_2/2$ is replaced by forces with number *q* along the circumference of the gear, force F_{mtl} has a coordinate φ_l. In the other half of the gear there are similar forces and moments acting, they are, however, not identical with the former, since it is expedient to choose an odd number of teeth in the gear and of the rollers in the generator. If the generator is a disc type one, the deformation waves are automatically asymmetric.

3.2. Determination of radius of curvature by calculation (*Fig. 4*)

The forces and moments along the circumference of the ring can be converted by means of Fourier-series into an equivalent system of force (p_φ, p_Z, m_X) distributed along the circumference. The radial displacement $w = w(\varphi)$ of the points in the centre line can be determined by the differential equation

$$\frac{d^5w}{d\varphi^5} + 2\frac{d^3w}{d\varphi^3} + \frac{dw}{d\varphi} = \frac{r_0^4}{EI_x}\left(\frac{dp_z}{d\varphi} + p_\varphi\right) + r_0^3\left(\frac{d^2m_x}{d\varphi^2} + m_x\right) \tag{2}$$

The relation between the radial displacement of the points of the centre line and the change in the curvature can be given by the expression

$$\kappa_\varphi = -\frac{1}{r_0^2}\left(\frac{d^2w}{d\varphi^2} + w\right) \tag{3}$$

The radius of curvature $\rho = \rho(\varphi)$, if the change in the radius of curvature $\kappa_\varphi = \kappa_\varphi(\varphi)$ is known, can be determined as

$$\rho(\varphi) = \frac{1}{1/r_0 + \kappa_\varphi(\varphi)} \tag{4}$$

or by the strain gauge $(\varepsilon_\varphi = \varepsilon_\varphi(\varphi))$ stuck to the surface of the gear with thickness h_O

$$\rho(\varphi) = \frac{1}{1/r_0 + 2\varepsilon_\varphi(\varphi)/h_0} \tag{5}$$

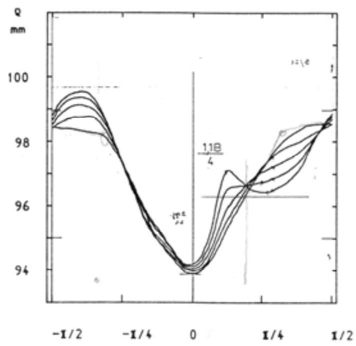
4. MEASUREMENT OF THE RADIUS OF CURVATURE

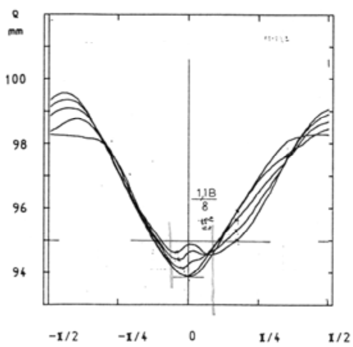
4.1. Development of the measurement site

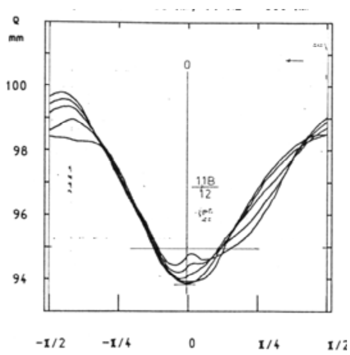
The slow shaft of the drive is acted on by a moment of known magnitude (in the example M_2=*0, 200, 400, 600, 800 Nm*), and of constant direction. Depending on the direction of moment M_2 and on the direction of rotation of the generator, the drive accelerates or decelerates.

4.2. The measured values

a) Using the strain gauges stuck parallel with the face of the flexible gear gives $\rho = \rho(\varphi_g^3)$ as the radius of curvature. b) Sticking the strain gauges on the measurement tooth of the ring gear gives tooth load $F_{23} = F_{23}(\varphi_g^3)$ for determining the location of the meshing zone. c) By the face of the flexible gear, the radial displacement $w = w(\varphi_g^3)$ against the rigid gear is used for detemining the radial rigidity of the chain of generator - flexible gear - ring type gear.







a) *b)* *c)*

Fig. 6. *Measured values of the radius of curvature. a) Flexible gear coupling. b) and c) Flexible gear drive, w_0/m = 1.1, m = 1 mm, r_0 = 97.1 mm*

4.3. Measured value of the radius of curvature (*Fig. 6*)

The flexible gear *H22,* the ring gears *G1, G2, G3* and the wave generators *T1, T2, T3, B* can be set up in several versions: with flexible gear *H22:* a) rigid gears *G21*and *G22*, *G23* can be used to study the difference between the wave coupling and the flexible gear drive. b) Rigid gears *G22*, *G23* can be used to change the size of the backlash. c) Generators *T1, T2* and *T3* can be used to change the wave size (*w_0=1.0, 1.1 és 1,2 mm*). d) Generators *B* and *T1, T2, T3* can be used to study the difference between cam and disc generators.

5. CONCLUSIONS

The radius of curvature will change for identical loads M_2depending on: a) whether the gear is a flexible gear drive or a wave coupling, b) whether the drive accelerates or decelerates, c) the size of the backlash, d) whether the generator is disc type or cam type, e) the size of the wave of deformation.

REFERENCES

[1] Péter J.: Fogazott hullámhajtóművek kapcsolódásának vizsgálata. (Investigation of the Engagement of Flexible Gear Drives.) A Nehézipari Műszaki Egyetem Gépészmérnöki Karára benyújtott egyetemi doktori értekezés. Miskolc. 1983. p.1/216.

[2] Péter 1.: Fogaskerék-hullámhajtómű és tengelykapcsoló kapcsolódásának vizsgálata. (Investigation of the Engagement of Flexible Gear Drives and Flexible Gear Coupling) A Tud. Minősítő Bizottsághoz benyújtott és elfogadott kandidátusi értekezés. Miskolc. 1993. p.111.

[3] Péter J.: Differences between Flexible and Epicyclic Gear Drives According to the Gear

Tooth Action. Proc. of the 6th World C. on the TMM. Vol. II. New Delhi. 1983. p. 846/849.