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IN THE FIELD OF GEARS

ASPECTS OF TECHNOLOGICAL DECISION FOR EVALUATING TOOTH PROFILE DEVIATION

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Abstract: The paper studies the precision problem of a gear control parameter, the tooth profile deviation. By statistically approaching the worked gear precision problem through grounding by the abrasive worm tool, the tolerance field for each part, the active duration of the tool between two sharpenings a new could increase, without diminishing the overall quality of the gearing from the tooth profile deviation point of view. The paper also presents an approach method to technological decision, basid on the fuzzy logic, founded upon knowledge.

Keywords: technological decision, profile deviation statistical approach, fuzzy

1. Generalities

We worked in a gear manufacturing system. In design it is very easy to prescribe for technology the precision parameters recommended by standards and even achievable in manufacturing. We deem it necessary to ask ourselves: „What are the costs?” Should we analyze the problem from the viewpoint of precision taking into account its evaluation method and hence, implicitly, the value imposed to this precision that will influence costs?

The working by grinding with abrasive worm tool is generally costly. Any manufacturer is interested in increasing the life cycle of the abrasive worm tool between two sharpenings (economy of abrasive tools, diamond tools, machine idle time etc.). The life cycle of the abrasive worm tool between two profilings is given by the spliting duration, where the value of tooth profile deviation is registered in the tolerance prescribed in the documentation. The tooth profile deteriorates along with the number of parts worked with the abrasive worm tool. The tool wear fundamentally influences this parameter. The exit from the tolerance field of tooth profile deviation leads to the technological precision regarding the sharpening or profiling of the abrasive worm tool.

In paper [2] we demonstrated that, with a cylindrical gearing, the deviation of the flank form improves by 35-45% after the first hours of load-working. In the operating system the real kinematic conditions of rolling of involute flanks are created, the wear of running in occurs, an involute profile improvement of the tooth following.

2. Statistical Criterial Approach

A cylindrical gearing, up to the precision stage 8 according to the smooth operation criterion has the tolerance of tooth profile deviation imposed [4]. With ground gears of small and medium dimensions and modules, this tolerance is of micronic level.

We consider: f_{fr1}, f_{fr2} - deviation of gear 1 and 2 profiles;

f_{f1}, f_{f2} - tolerance of gear tooth profile deviation of gears 1 and 2.

Following the manufacturing process, the two gears will be individually characterized by the deviation of tooth profile f_{fr1} and f_{fr2}. This means that the gearing will be characterized by a sum of the two profiles' deviation, which we propose to symbolize by f_{frA}.

$$f_{fr1} \leq f_{f1}; \quad f_{fr2} \leq f_{f2}; \tag{1}$$

Normally, this sum will characterize the gearing from the viewpoint of teeth profile deviation. Actually, this is a result and not a specific constraint to be taken into account in design. For a gearing to be precise, corresponding to a precision stage (stage 8 maximum on the smooth operation criterion), the individual deviations of gears will have to fit into the tolerance of tooth profile deviation.

$$f_{fA} = f_{f1} + f_{f2}; \tag{2}$$

In the extreme case, the deviation is the value of tooth profile deviation tolerance itself. In this case, the gearing will be characterized by a theoretical tolerance equal to the sum of profile deviation tolerances of the two gears that make up the gearing.

The grinding process by abrasive worm tool is a process of normal running, stable, with only systematic errors, caused by tool wear, between its two sharpenings. The form deviation of gear flanks ground by abrasive worm tool will be distributed in the adjustment value (which is the minimal value, and at its extreme it can be 0) and the value of tooth profile deviation tolerance, according to the Gauss-Laplace law of normal distribution. The system of statistical control applied in practice at gear grinding by abrasive worm tool at ANGRED S.A. has proved this distribution at the substantiation of checking on statistical bases, and at the analysis of technological system stability. The paring of ground gears 1 and 2 from a production batch, and implicitly the value of the deviation sum of the two gears' profiles will be distributed according to the same law of normal distribution.

When determining the deviation of the square average of the deviation sum for the two profiles of gearing σ_{frA} , we have to take into account the fact that the assembling of the gears in the series and mass production systems is a compound accidental event, while the tooth profile deviation at each gear is an independent accidental event, determining the compound one.

We mark:

- σ_{ffr1} şi σ_{ffr2} - the average square deviation of gear tooth deviation 1, respectively 2;
- σ_{frA} - the average square deviation of the sum of the gearing's two profiles;
- w_1, w_2 - the spreading interval of gear tooth profile deviation 1, respectively of gear 2, which has value f_{f1} at limit, respectively f_{f2}, under conditions of smooth operation of the technological process of grinding;
- w_{frA} – the spreading interval of the deviation sum of gearing's two profiles.

In this context [1]:

$$w_A = 6\sigma_{frA}; \quad w_1 = 6\sigma_{ffr1}; \quad w_2 = 6\sigma_{ffr2}; \tag{3}$$

According to [1], at the study of working errors by statistical methods, the square average deviation of a sum of accidental independent values is equal to the square root of the sum squares of square average deviations for the given accidental values.

$$\sigma_{frA} = \sqrt{\sigma_{ffr1}^2 + \sigma_{ffr2}^2} \quad ; \tag{4}$$

$$6\sigma_{frA} = \sqrt{(6\sigma_{ffr1})^2 + (6\sigma_{ffr1})^2}; \tag{5}$$

$$6\sigma_{ffr1} = f_{f1}; \qquad 6\sigma_{ffr2} = f_{f2}; \tag{6}$$

$$w_A = \sqrt{(f_{f1})^2 + (f_{f2})^2} \quad ; \quad but: \; w_A = f_{fA\,pr.}; \tag{7}$$

$$f_{fA\,pr.} = \sqrt{(f_{f1})^2 + (f_{f2})^2}; \tag{8}$$

Rezults:

This $f_{fA\ pr}$ is the probable or practical tolerance of the two profiles' deviation sum. Comparing the probable (practical) tolerance of the deviation sum of the two profiles, with the theoretical tolerance of the sum of the two profiles deviation, we observe the following:

The facts demonstrated analytically above can be graphically represented in figure 1. The difference between the size of curve *c* surface compared to *a* and *b* is caused by the fact

$$f_{fA\,pr.} = \sqrt{(f_{f1})^2 + (f_{f2})^2} < f_{fA} = f_{f1} + f_{f2}; \tag{9}$$

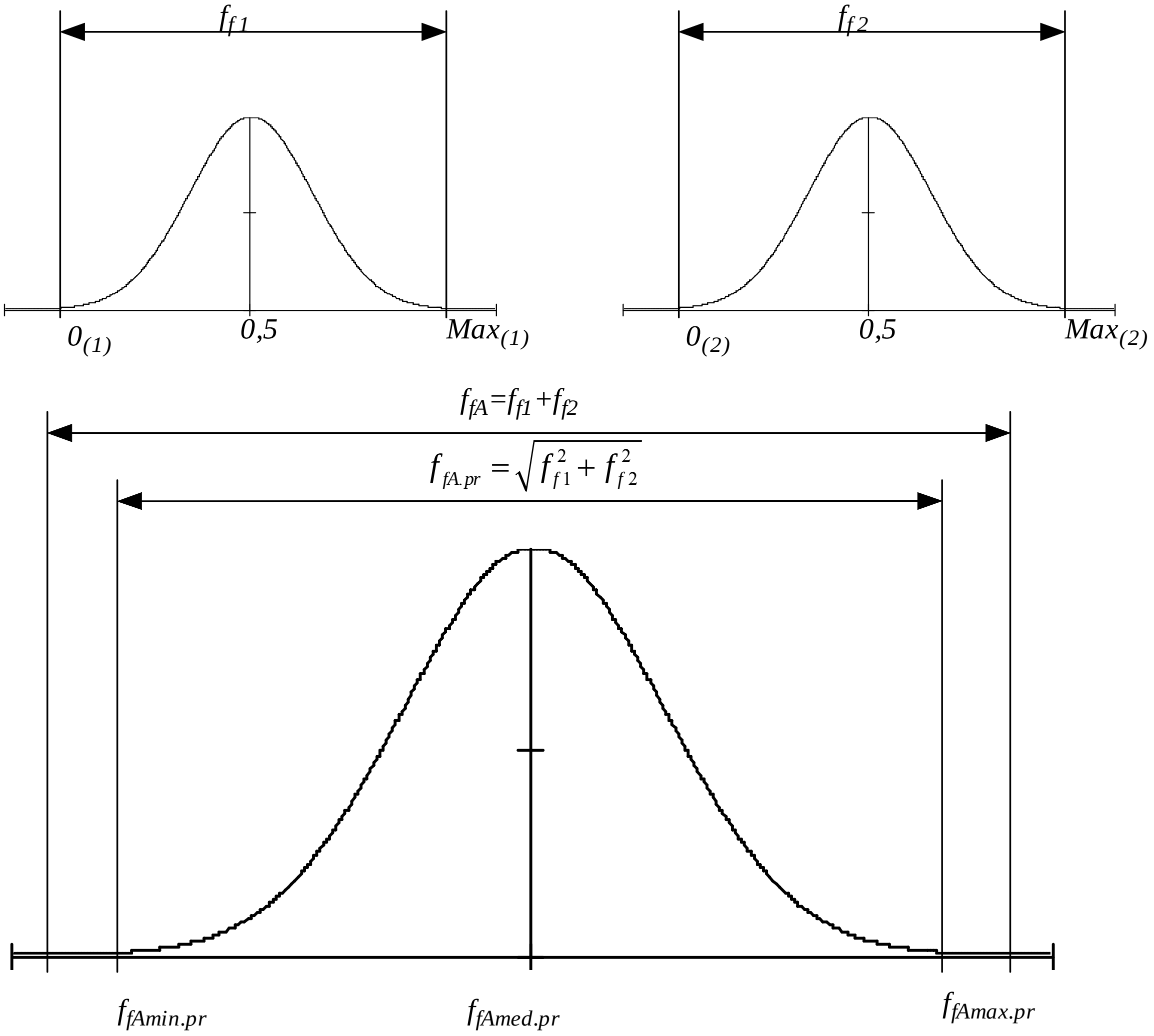


Fig. 1. Distribution of the tooth profile deviation (a.- gear 1; b.- gear 2; c.-gear pair.)

that the size of the first surface is determined by the total possible number of pairings between gears 1 and 2, whereas the other two are determined by the actual number of manufactured gears 1 and 2.

As a normal and immediate consequence, the deviation limits of the profile deviation sum for the two gears will not be *0* and $f_{f1}+f_{f2}$, but:

$$f_{fA\min.} = 0 + \frac{f_{fA} - f_{fA\,pr.}}{2} = \frac{f_{fA} - f_{fA\,pr.}}{2}; \tag{10}$$

$$f_{fA\min.} = f_{fA} - \frac{f_{fA} - f_{fA\,pr.}}{2} = \frac{f_{fA} + f_{fA\,pr.}}{2}; \tag{11}$$

Example: We consider a gearing with gears 1 and 2, with the following main parameters: $m=3;\quad z_1=22;\quad z_2=53;\quad f_{f1}=11\mu m;\quad f_{f2}=13\mu m.$

$$f_{fA} = f_{f1} + f_{f2} = 11 + 13 = 24\mu m.$$

That means that during design, indirectly, a sum of deviations of the gearing's two profiles in the maximum value of 24μm has been envisaged. The practical tolerance of the deviation sum of the two profiles in the stable system of grinding by abrasive worm tool, will be:

Thus the dispersion of the two profiles' deviation sum will be in the domain of 3,5÷20,5 μm . Further on, we try to approach the problem oppositely.

$$f_{fA\,pr.} = \sqrt{f_{f1}^2 + f_{f2}^2} = \sqrt{11^2 + 13^2} \cong 17\mu m;$$
$$f_{fA\,pr.} < f_{fA};$$
$$f_{fA\min.} = \frac{f_{fA} - f_{fA\,pr}}{2} = \frac{24-17}{2} = 3{,}5\mu m;$$
$$f_{fA\max.} = \frac{f_{fA} + f_{fA\,pr}}{2} = \frac{24+17}{2} = 20{,}5\mu m.$$

From the design stage, a tolerance of the two gears' profile deviation sum of 24μm is given (not otherwise than as sum of the two gears' profile flank deviation tolerances). The two gears are grounded by the method of working by the abrasive worm tool (Reishauer), a method of technological stability between two sharpenings, the systematic errors being caused by tool wear, and the deviations of profile form will occur according to a Gauss-Laplace distribution. The problem stands in determining the value of admitted individual tolerances for each gear, so that we can achieve a dispersion of the sum for the two gears' profile deviation within the limits of 24 μm:

$$f_{fA\,pr.} = \sqrt{f_{f1}^2 + f_{f2}^2};$$
$$24 = \sqrt{f_{f1}^2 + f_{f2}^2};$$
$$\textit{we are considering}: f_{f1} = f_{f2} - 2;$$
$$24 = \sqrt{f_{f1}^2 + (f_{f2} - 2)^2}; \quad 24 = \sqrt{2}(f_{f2} - 1)$$
$$f_{f2} = 18\mu m; \quad f_{f1} = 16\mu m.$$

Hence we can conclude that in a techonological system of grinding by abrasive worm tool, where the system has a normal evolution between two resharpening, and where only systematic errors occur, based upon a statistical approach of tooth profile precission issue, we can extend the tolerance field at the flank form deviation for each gear, according to the method mentioned above. This decision will lead to a substantial increase in the number of parts worked between two sharpenings of the abrasive worm tool, and implicitly to the reduction of manufacturing costs. It is however necessary to ensure a compatibility of the type and quality of the abrasive disk to the material and thermal treatment of the gear.

3. Approaching precision issues by knowledge and fuzzy logic criteria

Within the context where we demonstrated that the deviation of the flank form improves by 35-45%, after the first hours of operation, we can approach the decision of meeting quality conditions on the basis of fuzzy logic.

In the theory of classical arrays [3], the characteristic function of an array A in the universal array U is defined by:

$$\chi_A(x) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases} ; \tag{12}$$

This characteristic function has two possible values to model the idea that *„statement x belongs to A”* is either true or false for each element in U (reference to array A is done as in case of an exact array).

A simple extension of this concept is the so-called function of appartenence μ_A, of

$$0 \le \mu_A(x) \le 1 \quad \textit{for any } x \in A, \tag{13}$$

vague array A, which models the idea that statement *x belongs to A*, which is not only true or false by all means (uniquely true or false). On the contrary, it can be a gradual idea between true and false [3].

where μ_A represents the degree of truth, subjective (or rational) attributed by human arbitration (quality evaluator).

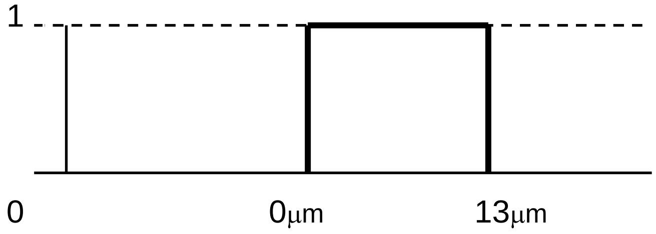
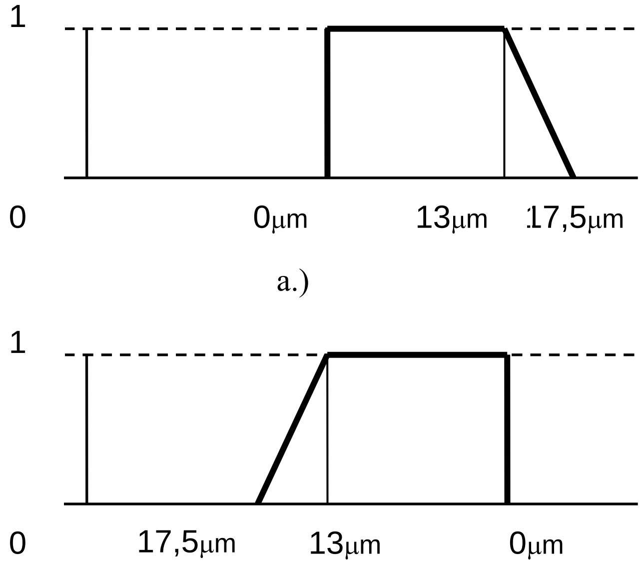


Fig. 2. Characteristic function

The deviation at the flank form within the tolerance limits of f_f=13 μm is exact, accurate, according to a characteristic function presented in fig. 2. This is the context in which the quality evolution system operates today.

In paper [2], we experimentally demonstrated that an improvement of tooth profile deviation after the first hours of operation occurs. We also admit into the assembling flow gears with a profile form deviation higher than 35-45% recommended according to tolerance standards, for the smooth operation condition. It would mean that we can consider all the gears with a tooth profile form deviation up to 13x1,35=17,5 µm.



a.)

b.)

Fig. 3 Membership function

From a technical point of view, there is an imprecision area, admissible by arbitration, based on the kinematic reconfiguring of the flank form after running in. The form of the membership function can also depend on the aspects related to the deviation sense of the flank form (thinning at the tooth edge with thickening at the basis, or thickening at the tooth edge and thinning at the basis, and the appartenence function will also be symmetrical (fig. 3 a and b). In this context, if ten evaluators evaluate the parts in a batch, supposing that a number of 8 evaluators are aware of the problem of flank form improvement, and only 8 state that the part meets the gearing function specific to stage/degree 7 for the criterion of smooth operation, we can state that the appartenence function takes into account the feeling (here, rationing based on knowledge) apparently subjective (apart from the prescriptions of quality evaluation that confirm conformity with documentation) of this group of evaluators. Generally, it is not easy to accept defining a concept on the basis of subjective considerations, or apparently subjective ones, but on the basis of knowledge. There are proposals to introduce more objective and frequent definitions of the truth value. Thus, the truth value for

$\mu_A(x)$ is sometimes understood as the ratio of a sufficient number of evaluators that agree with the statement of $x \in A$). For the case above:

$$\mu_A(x) = 0{,}8 : \tag{14}$$

In the example provided, the truth value of the statement: „The gear flank form deviation ensures the operation of the gearing in the precision stage 7” is presumed to be a viable value of the corresponding fraction of population. Within the number of evaluators, each evaluator has only two possible answers: „The deviation of the gear flank form ensures the operation of the gearing in stage 7 of precision”, and” The deviation of the gear flank form does not ensure operation of the gearing in stage 7 of precision” basing on the subjective admitting of the gear’s improving the flank form within a few hours of operation. The fuzzy logic users have tried to build up the appartenence functions on the basis of their own feelings and rationing. The subjective ascribing of the real truth values is not an easy task , but it seems sufficiently accurate and exact for interhuman communication and decision taking, techological this time, that means implications including manufacturing costs. We also have to take into account the knowledge degree of evaluators, as rationing takes knowledge into account.

4. Conclusions:

1. A field of approach for the problem of tooth profile precision issue opens up, field that can be founded on mathematical statistical criteria, in mass and series production, on grinding machines with abrasive worm tool, that can be considered a technological system with normal, stable operation, with Gauss Laplace distribution of deviations of tooth flank profile.

2. Analysis premises are founded on the statistical bases of values of tooth profile deviation, that can be increased in order to reduce costs, without influencing the operational precision of the gearing from the viewpoint of tooth profile deviation, in the case of gears ground by an abrasive worm tool.

3.The sets and the fuzzy logic find their application in the technological decision regarding the evaluation of tooth profile deviation, having as basis the evaluation based upon the complex knowledge of the phenomenon.

5. References

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