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UTILIZATION OF SOME COMPUTER ASSISTED TECHNIQUES IN GENERATING AND STUDY OF THE HYPOCYCLOIDAL FLANKS OF THE SPUR GEAR TEETH STRESS

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Abstract: The curved flanks for the involute tooth gear are generated with hypocycloidal line. The obtained tooth is crowning that means a controlled pitch spot. The paper deals with the spur gear design on the basis of parametric equations of the tooth profile and flank lines established using the kinematics generation method. The profile and flank lines are represented in a CAD environment, the profile is extruded along the flanks lines and the tooth generated as 3D object. Some elements of stress analysis on the tooth root are presented in condition in which some elements of the generating rack profile are modified (pressure angle, fillet on the tooth root, flank line, gear material and heat treatment) for curved teeth with hypocycloidal flanks. Spur gear tooth modelling enables the analysis of the teeth with symmetrical and asymmetrical involute teeth, and also with rectilinear or curved flanks in order to establish the tooth bending stiffness.

Keywords: *involute profile, flank line, generation by rolling, pressure angle, geometric parameters, spur gear, 2D and 3D modelling, 3D FEM analysis, load distribution.*

1. INTRODUCTION

There are important differences between the geometric representation of the involute curve and kinematic generation by rolling on certain machine tools. These are presented in detail in scientific works and known by specialists.

The kinematic generation of the flanks with involute profile in machine tools is achieved applying the rolling method with fixed line and mobile line [1].

The kinematic generation by rolling has on the basis the theory of conjugated curves [7, 9, 10, 11].

The gear flanks, as involute surfaces are extensively used due to some qualities induced in gearing namely kinematic, technological and functional [1, 10, 13, 15].

Regarding the flank line, form this is a part of a cylindrical helix or a cycloidal curve (circle arc, cycloid arc, and hypocycloid arc) or plane spiral curve. The paper deals with the generation of curved flanks with hypocycloidal lines, that, as the involute, are kinematically generated by rolling.

The utilizations of the parametric equations of the involute [7] and hypocycloid curves in advanced CAD environments [20] (CATIA, ProEngineer) drives to accurately 2D and 3D representation of the toothing and gears. This kind of CAD representation are used for FEM studies in order to simulate and evaluate the teeth comportment in composed loads.

On the basis of the 3D model of the tooth the comparative analysis of the teeth in bending is achieved [2, 13] in the following cases: tooth with symmetrical flanks, tooth with asymmetrical flanks, tooth with straight flank line, tooth with hypocycloid flank line.

It is known the fact that the improvement of the tooth behaviour in bending could be achieved by heat treatment or improving the fillet surface quality on tooth root (carburization, shot peening and grinding only for large fillet).

Other researches propose the modification of the generating profile of the cutting tool by combining the involute curve with fillet curves other than arc of circle. The paper proposes the study of the influence of tooth section increase upon the bending stiffness by achieving asymmetrical profile and curved flanks having hypocycloidal lines.

2. THE PARAMETRIC EQUATIONS OF THE INVOLUTES PROFILE

It is considered that the line segments T_s and T_d (Fig. 1) as generating curves attached to the segment *N-N* and also the method of rolling with fixed line. Thus, the segment *N-N* (fixed) rolls with the circle C_r having the radius R_r (rolling radius of the workpiece gear).

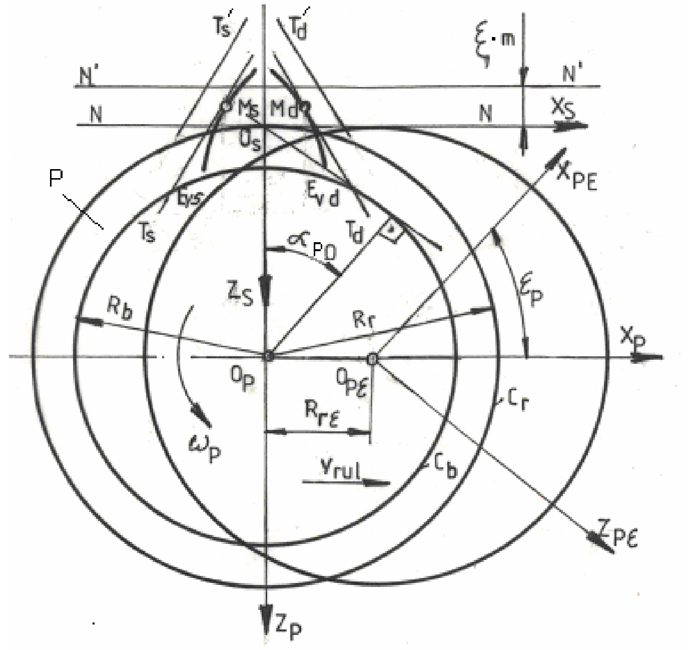


Fig. 1. Involute generation through generating motion with fixed line

The kinematics condition is expressed by relation:

$$v_{rul} = R_r \cdot \omega_p, \tag{1}$$

where ω_p is the angular velocity of the circle C_r. If the line *N-N* is located in other position *N-N'* displaced with ξm above or beneath position named *N-N* the displaced tooth is generated with the ξ correction ratio.

The two segments T_s and T_d are inclined in opposite directions with the pressure angle α_{P0}, the distance between points M_s and M_d being $M_s M_d = m\pi / 2$.

The length of segments T_s and T_d is established so that these to generate the involute profiles E_{vs} and E_{vd} respectively.

In the reference system S_S ($O_S X_S Y_S Z_S$) the cutting edges T_s and T_d are defined by the parametric equations:

$$\begin{cases} X_S = \pm u \cdot \sin\alpha_{P0} \mp \left(\dfrac{m \cdot \pi}{4} + \dfrac{\xi \cdot m}{2} \sin 2\alpha_{P0} \right) \\ Y_S = 0 \\ Z_S = -u \cdot \cos\alpha_{P0} - \xi \cdot m \cdot \sin^2 \alpha_{P0} \end{cases} \tag{2}$$

The two generating segments generate only the involute part of the profile on each flank. The part of the profile at the tooth root was graphically generated by filleting the involute profile and the tooth root circle considering a simplification of application (radius close that imposed by standards from the geometric point of view). The parametric equations of the generating profile in the most general are described in [4] .

By passing the previous equations (2) from the reference system S_S, in the reference system S_P, results the parametric equations of the generated involute profiles by the two segments. This passing supposes to take in consideration the motion parameter (rolling) expressed by the angle ε_p.

The following relation between the parameter u and ε_p was established:

$$u = \left(\frac{m \cdot \pi}{4} \mp \frac{\xi \cdot m}{2} \cdot \sin 2\alpha_{P0} \right) \cdot \sin \alpha_{P0} \pm R_r \cdot \varepsilon_p \sin \alpha_{P0} - \xi \cdot m \cdot \sin^2 \alpha_{P0} \cdot \cos \alpha_{P0} \qquad (3)$$

Thus, the parametric equations of the involute profiles generated on the workpiece gear have the form:

$$\begin{cases} X_{P\varepsilon} = \left[\left(k\frac{m\pi}{4} - R_r \varepsilon_P \right) \cos\alpha_{P0} + k\xi m \sin\alpha_{P0} \right] \cos\alpha_{P0} \cos\varepsilon_P + \left[\left(\frac{m\pi}{4} - kR_r \varepsilon_P \right) \sin\alpha_{P0} \cos\alpha_{P0} + \xi m \sin^2 \alpha_{P0} + R_r \right] \sin\varepsilon_P \\ Y_{P\varepsilon} = 0 \\ Z_{P\varepsilon} = \left[\left(k\frac{m\pi}{4} - R_r \varepsilon_P \right) \cos\alpha_{P0} + k\xi m \sin\alpha_{P0} \right] \cos\alpha_{P0} \sin\varepsilon_P - \left[\left(\frac{m\pi}{4} - kR_r \varepsilon_P \right) \sin\alpha_{P0} \cos\alpha_{P0} + \xi m \sin^2 \alpha_{P0} + R_r \right] \cos\varepsilon_P \end{cases} \qquad (4)$$

The generation of the two involute profiles of the tooth flanks is simultaneously achieved. The contact between the segments T_s and T_d and the generated profile is accomplished from the root by points of the tooth, or in opposite way, depending on the generating segment T_s or T_d and the rolling motion direction. In the actual generation, the cutting edges are displaced with equal distances s_T (feed) along the reference line *N-N*, generating elemental segments greater or smaller. The envelope of these segments is the theoretical involute profile. Thus, the actual generated profile is different from the theoretical one [9].

The size of the profile roughness of the flank is determined by the generating method by rolling, technological process and by the technological parameter s_T.

3. THE PARAMETRIC EQUATIONS OF THE HYPOCYCLOID LINES

This generating process of the hypocycloidal flanks needs the defining of the following basic elements: generating rack, cutting tool, motions and kinematic chains, and also the possibility of adaptation of method and tool tooth processing machine [8].

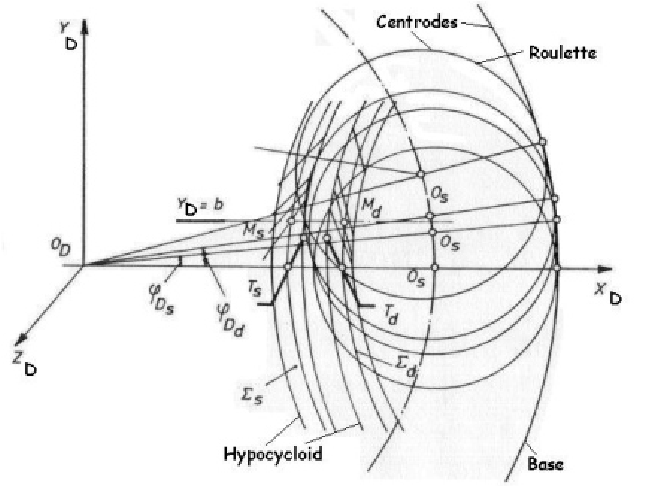


Fig. 2. Generate of the hypocycloid flanks with straight cutting edges.

The cutting edges T_s and T_d are placed in the same plane, perpendiculary on the $X_D O_D Y_D$, which is attached to the roulette. The roulette's center is moving along a circle concentrical with the base. This moving's parameter is the φ_D angle. The roulette has a radius, R_R, and the base, R_B. Their rapport ($R_B/R_R=i_H$) has an integer value. The cutting edges T_s and T_d are those indicated and represented in figure 1.

Their role is to generate the involute profile. In the roulette plane, the distance between the cutting edges T_s and T_d corresponds to a half of the tooth pitch. The two cutting edges are adapted through a support at one of the basic elements of the tool named roulette (*R*), which rolls inside of another fixed element named base (*B)*. A point belonging to the roulette generates a normal hypocycloid considering i_H=3,4,5 or 6. Points belonging to the cutting edge T_s generate shorted hypocycloides, also, for $T_{d,}$ there are oblonged hypocycloides. These hypocycloides are part of the Σ_d, respectively, Σ_s surfaces.

The assembly formed by *B* and *R* constitutes a simple planetary mechanism. The assembly constituted by *B*, the i_H groups of two cutting edges and the supports port tools (not represented) is named milling head [8, 16]. It is adapted in tooth processing machines with hobbing cutter on the tangential saddle (port tool). Therefore, the tool is a milling head multicutters having i_H groups of two cutters.

The parametric equations of the hypocycloid in the reference system S_D $(O_DX_DY_DZ_D)$ have the following form:

$$\begin{cases} X_D = (R_B - R_R)\cdot\cos\varphi_D + r\cdot\cos\left(\dfrac{(R_B - R_R)}{R_R}\varphi_D\right) \\ Y_D = (R_B - R_R)\cdot\sin\varphi_D - r\cdot\sin\left(\dfrac{(R_B - R_R)}{R_R}\varphi_D\right) \\ Z_D = -u\cdot\cos\alpha_{P0} \end{cases} \tag{5}$$

where $r = -k\cdot u\cdot\sin\alpha_{P0} - R_r + k\cdot m\cdot\dfrac{\pi}{4}$, u = (-1.25…1.25)m, φ_D is the motion parameter of *R* in *B*.

The workpiece gear *P* defined through the tooth number z_p and module *m* is positioned with its axis vertical, its mean plane corresponding to the plane $X_DO_DZ_D$ when $\varphi_D = 0$.

The workpiece gear *P* achieves the required rotation motion of continuous dividing and for supplying the rolling with the generating rack flanks. The workpiece rotation angle ε_p (see fig. 1) corresponding to the milling head rotation with the angle φ_D in radians is determined with relation:

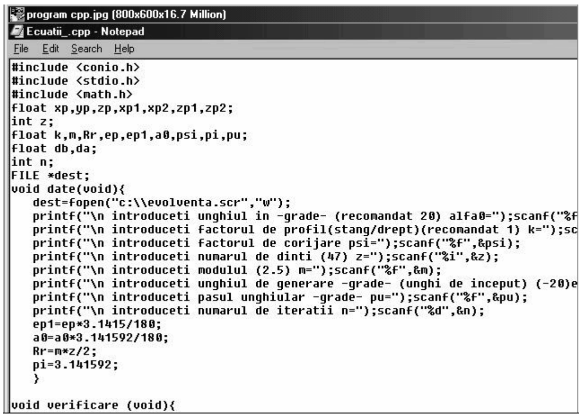
$$\varepsilon_p = \varphi_D \frac{i_H}{z_p}\left(1 + \frac{s_T}{2\cdot\pi\cdot R_r}\right), \text{ grade} \tag{6}$$

from which the required kinematic connections between workpiece (ε_p), saddle (s_T), and milling head (φ_D) derive.

4. INVOLUTE PROFILE AND FLANK WITH HYPOCYCLOID LINE MODELING

For the representation and study of involutes profiles and hypocycloid flanks lines, a calculation programmes in C++ (fig. 3) were developed. It achieves the calculation of the profile and flank lines point coordinates based on the parametric equations of the both curves (fig. 4).

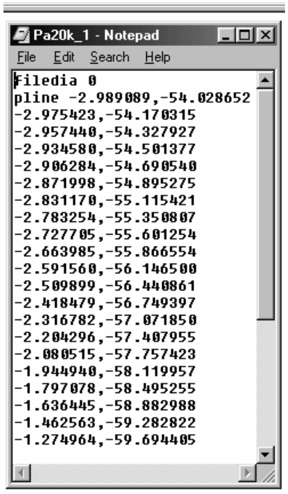
The coordinates are used in CATIA environment for graphic representation of the two flanks lines and profile of the tooth. In addition, one can achieve calculation regarding the measurable and nonmeasurable elements of the teeth in different frontal planes and cylindrical sections.



```
#include <conio.h>
#include <stdio.h>
#include <math.h>
float xp,yp,zp,xp1,xp2,zp1,zp2;
int z;
float k,m,Rr,ep,ep1,a0,psi,pi,pu;
float db,da;
int n;
FILE *dest;
void date(void){
   dest=fopen("c:\\evolventa.scr","w");
   printf("\n introduceti unghiul in -grade- (recomandat 20) alfa0=");scanf("%f
   printf("\n introduceti factorul de profil(stang/drept)(recomandat 1) k=");sc
   printf("\n introduceti factorul de corijare psi=");scanf("%f",&psi);
   printf("\n introduceti numarul de dinti (47) z=");scanf("%i",&z);
   printf("\n introduceti modulul (2.5) m=");scanf("%f",&m);
   printf("\n introduceti unghiul de generare -grade- (unghi de inceput) (-20)e
   printf("\n introduceti pasul unghiular -grade- pu=");scanf("%f",&pu);
   printf("\n introduceti numarul de iteratii n=");scanf("%d",&n);
   ep1=ep*3.1415/180;
   a0=a0*3.141592/180;
   Rr=m*z/2;
   pi=3.141592;
   }

void verificare (void){
```

Fig. 3. A part of written programme(C++).

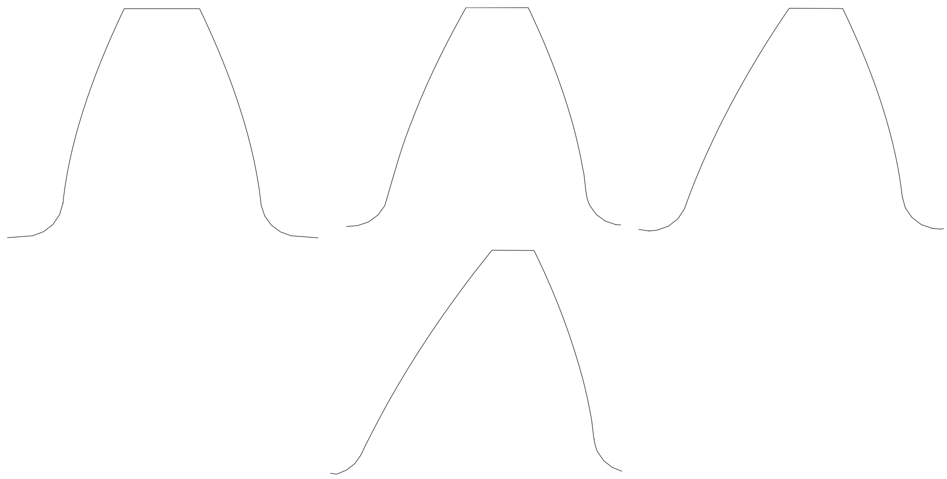


```
Filedia 0
pline -2.989089,-54.028652
-2.975423,-54.170315
-2.957440,-54.327927
-2.934580,-54.501377
-2.906284,-54.690540
-2.871998,-54.895275
-2.831170,-55.115421
-2.783254,-55.350807
-2.727705,-55.601254
-2.663985,-55.866554
-2.591560,-56.146500
-2.509899,-56.440861
-2.418479,-56.749397
-2.316782,-57.071850
-2.204296,-57.407955
-2.080515,-57.757423
-1.944940,-58.119957
-1.797078,-58.495255
-1.636445,-58.882988
-1.462563,-59.282822
-1.274964,-59.694405
```

Fig. 4. Numerical values of profile coordinates.

According to relations (4), the geometric parameters that influence the form and accuracy of the generated involute profile are: the pressure angle α_{P0} or $\alpha_{P0,m}$, pitch ($p_d = m\,\pi$), correction ratio (ξ) and rolling radius of the workpiece gear ($R_r = z_p \cdot m/2$).

The rolling angle ε_p is a kinematic parameter and it is related to the technological parameter s_T (see 6).



a. $\alpha_{P0}/\alpha_{P0} = 20°/20°$ **b.** $\alpha_{P0}/\alpha_{P0,m} = 20°/25°$ **c.** $\alpha_{P0}/\alpha_{P0,m} = 20°/30°$ **d.** $\alpha_{P0}/\alpha_{P0,m} = 20°/35°$

Fig. 5. Symmetrical (*a*) and asymmetrical (*b*, *c*, *d*) tooth profiles (m = 3 mm).

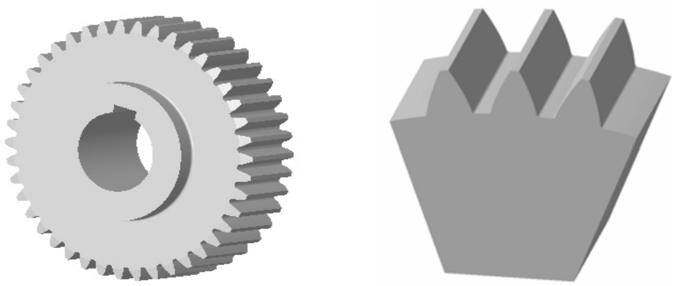
From all these parameters, the radius R_r and rotation angle of the workpiece ε_p can induce errors due to some adjusting imprecision.

In figure 5, the normal profile and profiles with the modified pressure angle are represented in the reference system of the workpiece gear. Also, the following geometric elements can be represented: basis circle, dividing circle, point circle, and root circle, corresponding to a spur gear having z_p= 38 teeth and m = 3 mm.

On the basis of point coordinates belonging to the flank profiles, one can determine, using known calculation relations [6, 9, 12, 14] values for the following geometric elements of the tooth: constant chord, height at constant chord, deviation of thickness and height on constant chord.

For 3D representation, the determined profile was used in AutoCAD environment and extruded along a line (straight tooth - fig. 7 a, b) and also along an arc of hypocycloid (see fig. 8) for a tooth width of 30 mm. For the curved tooth the following elements of the milling head were considered: R_B = 240 mm, R_R = 45 mm. By multiplying the tooth profile on the dividing circle and extruding, the 3D representation of a spur gear can be achieved.

The 2D and 3D models of the profiles and teeth of the gears represented in this way are the geometric support for the FEM analysis for determining the stress and deformation in tooth bending. This method was used also in other researches [2, 12, 14].



a b

Fig. 7. Tooth 3D representation.

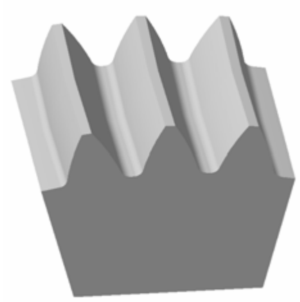
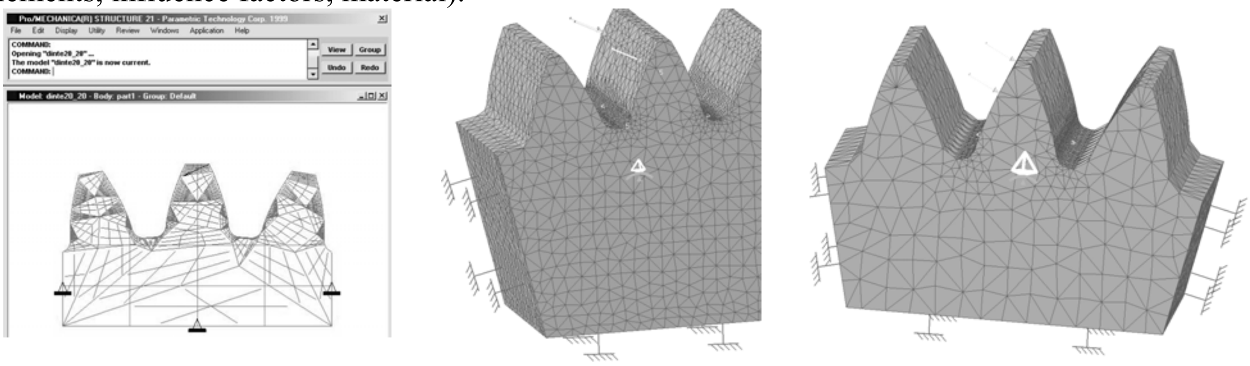


Fig. 8. Gear 3D representation.

In figure 9 is shown the tooth mesh of a tooth with symmetrical flanks for FEM application. That tooth belongs to a spur gear loaded in specific conditions (forces, geometric elements, influence factors, material).



a. b. c.

Fig.9. Boundary of the 3D tooth model.

a.ProEngineer model; **b**.Hypocycloid flanks line tooth 3D CATIA model; **c**.Straight line tooth 3D CATIA model

The constraint section of the tooth in the wheel body is considered as in speciality literature [12, 14]. For the curved line flanks the constraints sections are more favorable in the bending stress.

In figures 10, 11 and 12 the results of the numerical analysis in ProEngineer for three cases which differ through the tooth profile; one symmetrical and two asymmetrical are presented.

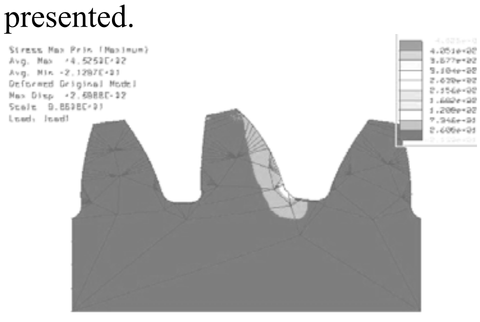


Fig. 10. Root stress for symmetrical profile (α_{P0}/α_{P0} = 20°/20°).

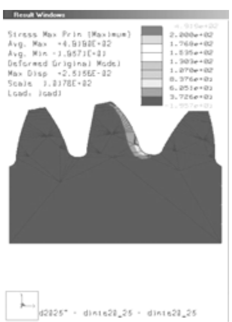


Fig. 11. Root stress for asymmetrical profile ($\alpha_{P0}/\alpha_{P0,m}$= 20°/25°).

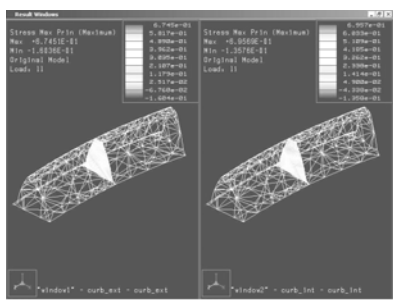


Fig. 12. Root stress for hypocycloidal flank line

In figures 13, and 14 are presented the results of the numerical analysis in CATIA for the two case. It is an analyses of a involute tooth with straight line flanks, respectivelly for the involute tooth with hypocycloid line flanks with load on convex and concav flank.

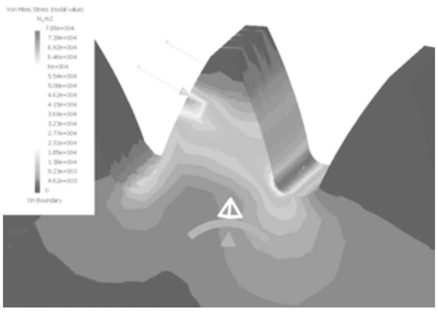


Fig. 13. Analysis of the root stress for a involute tooth with straight line flanks

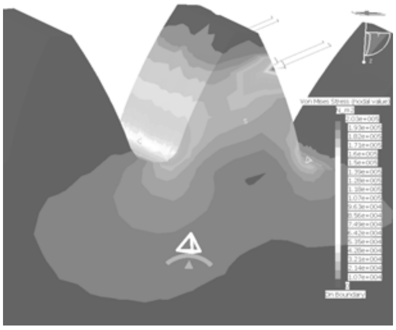


Fig. 14 a. Analysis of the root stress for a convex tooth with hypocycloid line flanks

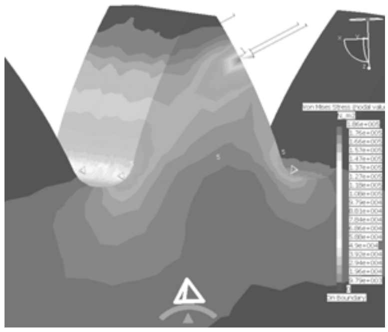


Fig. 14 b. Analysis of the root stress for a concav tooth with hypocycloid line flanks

It is considered a decrease of the stress at the tooth root as the pressure angle increases on one flank. Thus, for an increase of the pressure angle from 20° to 35° can be observed that the stress at tooth root is reduced by about 20%. The analysis is extended also for the comparison between the straight and curved teeth (fig. 14 a, b). For the curved tooth the evaluation of the stress was considered in the medial plane due to the position of the contact path in that area.

5. CONCLUSIONS

The parametric equations of the involute profile are established on the basis of the kinematics generation method by rolling. For the generation of hypocycloidal curve flanks, the flank line is also generated by kinematics and construction of a milling head with multicutters adapted on a tooth-processing machine. The machine tool assures a cinematic generation of the hypocycloidal flanks.

The numerical and computer aided design studies enables the profile and flank line point coordinates and their geometric representation. The equation parameters allow modifying the tooth profile (asymmetry), the curve radius of the flanks, the profile correction, in order to model the tooth used in FEM analysis.

On the basis of the method presented, the influence of the flank asymmetry and curved flank line on the stress in bending at the tooth root were established. The increase of the critic section of the tooth root by asymmetric profiles and curved flank lines leads to the primary evaluations such as a decrease up to with 20% of the stress.

The study results are only to observe the important areas of the stress. The main conclusion is that the tooth with hypocycloid flanks has a better stress behaviour than the straight flank tooth with approximately 25-30%, according with the curve radius of the hypocycloid.

REFERENCES

[1] Botez, E., - (1969), *Maşini-unelte. Vol. 1. Cinematica*. Editura Tehnică, Bucuresti.

[2] Brinck, P., - (1989), *Zahnfusstragfähigkeit Oberflächenehärteter Stirnräder bei Lastrichtungsumkehr,* Dissertation, Technische Universität München.

[3] Deng, G., Nakanishi, T., - (2001), *Enhancement of Bending Load Carrying Capacity of Gears Using an Asymetric Involute Tooth*, JSME International

Conference on Motion and Power Transmissions, Volume II. Fukuoka, Japan, p. 513-517.
[4] Dobre, G., Ghionea, A., Mirică, R., - (2002), *On Defining the Counterpart Rack Flanks for Modeling the Hypocycloidal Curve Cylindrical Toothing*, Romanian Journal of Technical Sciences. Applied Mechanics, Tome 47, special number, Editura Academiei Romane, Bucuresti, p. 113-116, ISBN 973-27-0932-4.
[5] Dobre, V., Mirica, Fl., R., Sorohan, St., - (2003), *On the Gearing Simulation using finite elements methods*. The International Meeting of the Field of Gears, 4th Edition, Stientific Bulletin, Serie C, vol XVI, North Univetsity of Baia Mare, Romania, ISSN 1224-3264. p 65-72.
[6] Dobre, V., Barbu, V., - (1998), *Indrumator pentru proiectarea angrenajelor şi bazele teoretice de calcul*. OID-ICM, Bucharest. ISBN 973-9187-63-3.
[7] Ghionea, A., Maşala, I., - (1998), *Contributions to the Kinematic Generation Study on the Involute Profile*, TCMM nr. 34, Editura Tehnică, Bucuresti, p. 245-250, ISBN 973-31-1238-0.
[8] Ghionea, A., - (1996), *Cap de frezat adaptabil la maşinile de danturat FD – Cugir*. Tehnologii, Calitate, Maşini, Materiale. Editura Tehnică, Bucureşti, p. 192-197.
[9] Ghionea, A., Constantin, G., Ghionea, I., - (2003), *Analytical Study of the Profile Roughness of the Flanks of the Spur Gear Curved Teeth*, International Conference "Power Transmission", Varna, Bulgary, ISBN 954-90272-9-5.
[10] Henriot, G., - (1999), *Engrenajes. Conception. Fabrication*. Mise en oeuvre. 7th édition, DUNOD, Paris, ISBN 210-003903-2.
[11] Litvin, F. L., - (1994), *Gear Geometry and Applied Theory, Prentice*-Hall Inc., Englewood Cliffs, USA, ISBN 0-13-211095-4
[12] Marunic, G., - (2000), *Tooth fillet stress area in case of non-uniform load distribution*. Proceedings of the 11th International DAAAM Symposium, Vienna, Austria, ISBN 3-901509-13-5, pp.501-502.
[13] Rădulescu, Gh., Miloiu, Gh., Gheorghiu, N., Muntean, C., Vişa, F., Ionescu, N., Popovici, V., Dobre, G., Raşeev, M., - (1986), *Îndrumar de proiectare în construcția de maşini.* Vol. III, Editura Tehnică, Bucuresti.
[14] Tanaka, M. (2001), *Analytical solution for the three dimensional contact problem of mating gears*. The JSME International Conference on Motion and Power Transmissions, MPT 2001, Fukouko, Japan.
[15] Wächter, K., (1987), *Konstructionslehre fur Maschineningeneure. Grundlagen, Konstruktions und Antriebselemente.* VEB. Verlag Technik, Berlin, ISBN 3-341-00045-3.
[16] *** *Cap de frezat.* Brevet de invenție. Int. Cl. B 23 F 21/12, 28.10. 1995. OSIM, Romania, autori: Ghionea, A., Oprean A.
[18] *** STAS 12222-84, STAS 12268-84, DIN 3990/2,3, MAAG Taschenbuch
[19] *** ISO 6336, - (1996), Calculation of Load Capacity of Spur and Helical Gears. Part 1: basic principles, introduction and general influence factors.
[20] *** CATIA V5R8, ProEngineer, AutoCAD 2000 documentation.