DETERMINATION OF PRINCIPAL ROUGHNESS PARAMETERS BASED ON FRACTAL – STATISTIC DUAL THEORY

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Abstract: In case of limit or dry lubrication of a tribo-couple it is necessary to consider the real surface of tribo - couples for correctly estimation of the friction losses. In order to this, the principals parameters of roughness have to be known.

This paper deals with the establishing of the principals parameters of roughness by aid of a relatively new mathematical method, the fractals one, as well as of the statistically one. It has the advantage of requiring practical accessible roughness parameters for calculations, as R_a or R_y.

Keywords: roughness, friction, tribology, contact area

1. INTRODUCTION

One of the requirements involved when it comes to evaluate the energy dissipations through friction in case of a tribo – couple working in dry/limit lubrication conditions is to consider the surfaces' real topography. Very often this is not easy to achieve due to numerous parameters and/or laborious measurements methods. Therefore, a new approach was developed, with the main advantage of needing only one roughness parameter (either arithmetic mean roughness or CLA, R_a, or max. peak-valley height, R_y). The other ones are inferred from this, based on the dual fractal – statistics theory.

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2. CONTENT

According to [1], the current usual machining processes lead to obtaining a final surface characterised by a Gaussian distribution of asperities. The semi - spherical model of peaks is considered here, having the same curvature radius, but different heights [1].

The statistical principal roughness parameters are: curvature radius of asperity, slope RMS slope of the profile, average wavelength and maximum height of the profile. Starting from the arithmetic mean roughness values for these two tribo – couple surfaces, R_{a1}, R_{a2}, the other parameters are calculated in accordance with the following algorithm, based on the dual fractal - statistic theory:

1. *Average frequency of asperities ω_{ap} [μm^{-1}]* :

$$\omega_{ap} = \frac{\dot{y}_{ap}}{R_{ap}} \, , \tag{1}$$

where $\dot{y}_{ap}$ represents mean slope asperity, which can be calculated from the Abbot – Firestone bearing curve or extracted from tables, function of machining process [1] and R_{ap} represents RMS mean roughness ($R_{ap} \approx 1,22 R_a$).

2. *Wavelength of profile, λ_{ap}*:

$$\lambda_{ap} = \frac{2\pi}{\omega_{ap}}. \tag{2}$$

3. *Average density of profile, D_0*:

$$D_0 = \frac{2}{\lambda_{ap}}. \tag{3}$$

4. *Density of extreme points (max and min)*(after Eksler, quoted by A. Tudor in [1]): depending on the roughness group of the tribo – couple surface. (*$D_{e1}=0.9$ and $D_{e2}=1.1$* for N6, respective N5 roughness precision classes after STAS 5730/2-85, roughness precision class after STAS 5730/2-85),

5. *Curvature radiuses of spherical modelled asperity are given by* :

$$\rho_1 = \frac{1 + (\pi D_{01} R_{ap1})^2}{2\pi D_{01} D_{e1} R_{ap1}}, \quad \rho_2 = \frac{1 + (\pi D_{02} R_{ap2})^2}{2\pi D_{02} D_{e2} R_{ap2}}, \tag{4}$$

6. *The equivalent curvature is, therefore:*

$$\nu = \frac{\nu_1 \cdot \nu_2}{\nu_1 + \nu_2}; \quad \omega_{ap} = \frac{2\omega_{ap1} \cdot \omega_{ap2}}{\omega_{ap1} + \omega_{ap2}}; \quad b = \frac{b_1 \cdot b_2}{b_1 + b_2}; \quad R_{ap} = \frac{R_{ap1} \cdot R_{ap2}}{R_{ap1} + R_{ap2}}; \tag{5}$$

$$R_{\max} = \frac{R_{\max 1} \cdot R_{\max 2}}{R_{\max 1} + R_{\max 2}}; \quad \Delta = \frac{\Delta_1 \cdot \Delta_2}{\Delta_1 + \Delta_2}; \quad \sigma = \sqrt{\sigma_1^2 + \sigma_2^2}; \quad \rho = \frac{\rho_1 \cdot \rho_2}{\rho_1 + \rho_2}$$

where ν, b şi Δ are Abbot – Firestone bearing curve's parameters, which can be extracted from the table [3], depending on the machining process and material, R_{max} is maximum height of profile ($R_{max} \approx 0.4 R_a$), and σ - standard deviation of these two roughness ($\sigma = (0.7...0.8)\exists R_{ap}$).

7. *Spectral density function, $S(\omega)$* [1]:

$$S(\omega) = \frac{1}{2\pi} \frac{\pi D_0}{(\pi D_0)^2 + \omega^2}. \tag{6}$$

8. *Fractal parameter, D* [5]:

$$D = \frac{\ln(\sigma \cdot \omega_{ap}^2)}{\ln(\omega_{ap})}. \tag{7}$$

9. *Fractal parameter $G, \mu m$ (it deduces from the spectral function, according to spectral theory* [5]):

$$G = \exp\left[-0.5 \frac{-3 \cdot \ln(\omega_{ap}) + 2 \cdot D \cdot \ln(\omega_{ap}) - \ln(2 \cdot S(\omega_{ap}) \cdot \ln(\gamma))}{D - 1}\right], \tag{8}$$

where γ is the scale factor.

10. *Deformation area of one spherical asperity (according to fractal theory of roughness) $\delta_r, [\mu m^2]$* [5]:

$$a = \exp\left[2 \frac{D \cdot \ln(G) - \ln(G) + \ln\left(\dfrac{\rho}{\pi^2}\right)}{D} \right]$$

(9)

11. *Deformation of one spherical asperity (according to fractal theory of roughness δ_r, [μm^2]):*

$$\delta_r = G^{D-1} a^{\frac{2-D}{2}} .$$

(10)

12. *Reference length of equivalent profile, l_s [μm],*

$$l_s = \frac{1}{\omega_{ap}} .$$

(11)

13. *Real contact area of spherical asperity, A_r [μm^2], obeying a Gauss type distribution* [1]:

$$A_r = \int_0^{l_s} \pi \cdot 1 \cdot x \cdot \left[\frac{1}{\sigma_r^2 \cdot \sqrt{2 \cdot \pi}} \exp\left[-0.5 \cdot \frac{\left(\dfrac{\delta_r}{R_{mx}}\right)^2}{\sigma_r^2} \right] \right] \cdot dx$$

(12)

Alongside the reference length, calculated with relation (12), there is one asperity.

14. *Greenwood – Williamson plasticity index* [3] (for assessing the prevalent deformation type):

$$\psi_G = 0.7 \frac{E^*}{HB_{min}} \sqrt{\frac{R_{ap}}{\rho}} ,$$

(13)

E^* being the Young equivalent modulus for the tribo –couple.

15. *Real contact pressure, p_r [$N/\mu m^2$]:*

$$p_r = \frac{N}{A_r} .$$

(14)

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3. NUMERICAL APPLICATION

Let's consider the case of a tribo - couple having $R_{a1}=0,63\mu m$ (N6 roughness precision class after STAS 5730/2-85), respective $R_{a2}=0,32\mu m$ (N5 roughness precision class after STAS 5730/2-85), measured by a Taylor – Hobson roughness- meter (range: $R_a=0...30\mu m$, accuracy $\pm0,002\mu m$. According to the above-described algorithm, the principal roughness parameters are presented in the table 1.

Table 1. **The principal roughness parameters**

No.	Parameter	units	Rel.	Numeric value	
				Surf. 1	Surf. 2
1	Mean slope asperity, $\dot{y}_{ap}$	deg	Table1 [1]	2	1
2	Wavelength of profile, λ_{ap}	[μm]	(2)	2,375	2,413
3	Average density of profile, D_0	[μm]	(3)	0,842	0,829
4	Density of extreme points, D_e	[μm]	Table1 [1]	0,9	1,1
5	Curvature radiuses of spherical modelled asperities, ρ	μm	[4]	1,389	0,909
6	Fractal parameter, D	-	[7]	1,829	
7	Fractal parameter, G	μm	[8]	0,07296	
8	Deformation area of one spherical asperity, a	μm²	[9]	0,003961	
9	Deformation of one spherical asperity, δ_r	μm²	[10]	0,071	
10	Reference length of equivalent profile, l_s	μm	[11]	0,381	
11	Real contact area of spherical asperity, A	μm²	[12]	0,204	
12	Greenwood – Williamson plasticity index, ψ_G	-	[13]	78,869	

Because of the value of plasticity index is higher than one, it results that the plastic deformations prevails during real time contact of these two tribo - elements.

4. CONCLUSIONS

The fractal - statistic dual theory proposed above proves efficiently and handy when one needs to establish the principal roughness parameters of a tribo – couple. Moreover, it allows determining the coefficient of friction for limit/dry lubrication, alongside assessing the real surfaces geometry. Nevertheless, a wider range of applications is required in order to improve the reliability of this new method.

5. REFERENCES

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