LUBRICANT FILM THICKNESS VARIATION INSIDE THE MESHING AREA FOR DOUBLE ENVELOPING WORM GEAR

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Abstract: Double enveloping worm gear is well known for its high load carrying capacity that is due mainly to its double instantaneous contact lines, which enclose a "pocket" of lubricant. Because of the globoid shape of the worm, the meshing area is not a planar shape, featuring different contact conditions from engaging to exit. Accordingly, the lubricant film thickness is expected to vary between first and last contact line, which may result in different lubrication regimes for the same gear, with obvious repercussions for carrying capacity and not least wear. The same variation is expected alongside the contact line itself, too.

This paper is aiming to analysing these two kinds of variations by involving the thermo-elasto-hydrodynamic (TEHD) equations in determination of lubricant film thickness. A useful and innovative method for solving these equations, which form a highly non - linear system, is also presented.

Keywords: TEHD, lubrication, double enveloping worm gear, film thickness

1. TEHD EQUATIONS

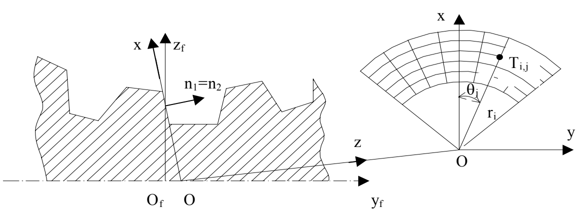

Fig.1
The cylindrical coordinate system attached to the globoid worm gear

Theoretically, in case of globoid worm gear the contact between mating surfaces should be linear, but in reality it is punctual, or partly linear, because of the numerous errors that occur, like: manufacturing errors, assembly errors, local thermal deformations, elastic deformations of worm thread, as well as of wheel tooth.

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Accordingly, the punctual TEHD equations are employed here. The coordinate system attached to the globoid worm gear is local cylindrical one (it is different from one contact point to another) (fig. 1).Oz axe coincides with the normal direction of the considered contact point, whilst xOy plane is tangent to mating surfaces in that specific point.

Therefore, the TEHD equations in cylindrical coordinates become:

1. Reynolds equation [1]:

$$\frac{\partial}{\partial \theta}\left(F_2 \frac{\partial p}{\partial r}\right)+\frac{F_2}{r}\frac{\partial p}{\partial r}+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(F_2 \frac{\partial p}{\partial \theta}\right)=V_r \frac{\partial p}{\partial r}\frac{F_3}{F_0}+\frac{V_\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{F_3}{F_0}\right)-\rho V_z , \tag{1}$$

where F_0, F_1, F_2, F_2 are Dowson functions [2].

2. Energy equation:

$$\begin{aligned}&\rho c_p\left(v_r \frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}+v_z \frac{\partial T}{\partial z}\right)-k_0\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right)=\\&=\alpha_T T\left(v_r \frac{\partial p}{\partial r}+\frac{v_\theta}{r}\frac{\partial p}{\partial \theta}\right)+\eta\left[\left(\frac{\partial v_r}{\partial z}\right)^2+\left(\frac{\partial v_\theta}{\partial z}\right)^2\right]\end{aligned} . \tag{2}$$

3. Elasticity equations. The elastic deformation of the mating surfaces in contact point, in cylindrical coordinates, due to pressure p(r, θ), is (after Timoshenko şi Godier [3]):

$$d(r,\theta)=k_d \int_{R_1}^{R_n}\int_{\zeta_1}^{\zeta_n}\frac{p(r,\zeta)\; dr\; d\theta}{\sqrt{(r\cos\zeta-R\cos\zeta)^2+(r\sin\zeta-R\sin\zeta)^2}} . \tag{3}$$

4. Lubricant viscosity variation with pressure and temperature, as proposed by Roelands (1963) is [4]:

$$\eta=\eta_0 \exp\left\{[\ln(\eta_0)+9.67]\cdot\left[-1+\left(1+5.1\cdot 10^{-9}\cdot p\right)\right]^{\zeta}+\left(1-\frac{T_{\max}}{T_0}\right)\right\} . \tag{4}$$

5. Lubricant density variation with pressure and temperature, as proposed Dowson şi Higginson (1966) is [5]:

$$\rho=\rho_0\left(1+\frac{0.6\cdot 10^{-9}\cdot p}{1+1.7\cdot 10^{-9} p}\right)\cdot\left(1+\beta\cdot T_0\left(1-\frac{T_{\max}}{T_0}\right)\right). \tag{5}$$

2. LUBRICANT FILM THICKNESS

The minimum film thickness is given by the following relation:

$$h(r,\theta)=h_0+d(r,\theta)+s(r,\theta) , \tag{6}$$

where $s$is the geometrical separation of the mating surfaces, and h_0 is the film thickness inside the centre of contact area due to HD effect. Minimum film thickness is given by the minimum value of the $h(r, \theta)$ matrix.

3. NUMERICAL SOLVING OF THE TEHD EQUATIONS

The TEHD equations form a highly non - linear system of equations, difficult to be solved using the conventional methods. Therefore, an innovative method (AMGLUB) [1] was developed based on the following two software packages: MathCAD 2000 and MathConnex2000 Professional. Basically, each of the Reynolds and energy equations are solved in MathCAD files for any of thermal/isothermal cases in part. Levenberg-Marquardt iterative method was involved for solving of Reynolds and equations energy every time.

4. NUMERICAL APPLICATION

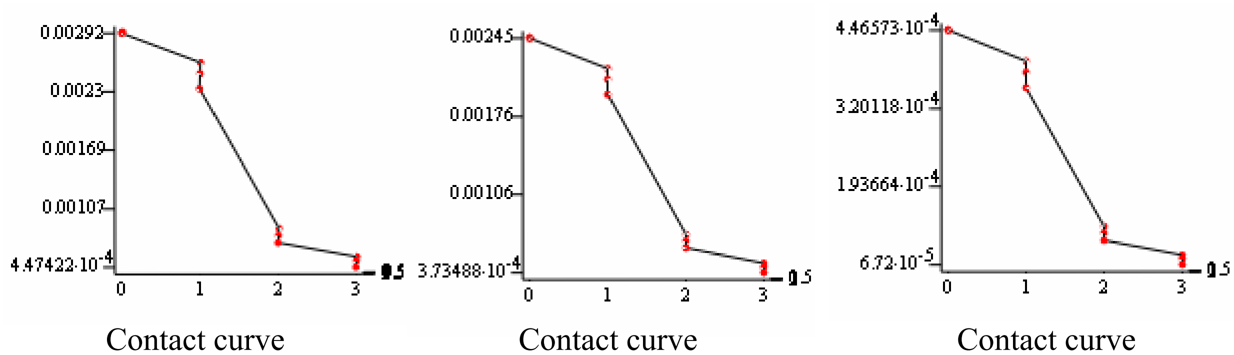
It is considered a Cone double enveloping worm gear having the centre distance A=125 mm, ratio i=9,75, z_1=4, n_1=1500 rot/min, M_{t2}=100 daN*m. Three types of lubricant are used successive:

a) Shell Tivella WB, ν_{40^0C} = 234 cSt,
b) Shell Morlina 10, ν_{40^0C} = 10 cSt,
c) Petrom T90 EP2, ν_{40^0C} = 195 cSt.

After running –in roughness is R_{a1}=0,63 μm for the globoid worm and R_{a1}=0,32μm for the wheel. The minimum film thickness alongside the current four instantaneous contact curves belonging to the meshing area differ from one to another due to the different local contact conditions. The minimum film thickness decreases alongside the meshing area with the decreasing of the worm diameter, as can be seen from the fig.2.

Because the film thickness was calculated taking into account the thermal effect, there is no need to be corrected with the thermal factor ϕ_{Th} [6].

The minimum film thickness decreases as the diameter of the globoid worm gear decreases. The inlet meshing area is characterised by the harshest contact conditions in terms of the thinnest film thickness (here, the relative curvature has the lowest value, too).



Contact curve
a) Shell Tivella WB

Contact curve
b) Petrom T90 EP2

Contact curve
c) Shell Morlina 10

Fig. 2
Minimum film thickness h_{min},[mm]

Along the contact curve, film thickness decreases from the bottom towards the top in a parabolic manner. The effect of lubricant viscosity upon the film thickness is shown in fig.3 and 4.

It can be noted that h_{min} decreases as viscosity increases, with a higher rate for higher viscosity values, for both directions considered: along the contact curve, as well as alongside the meshing area.

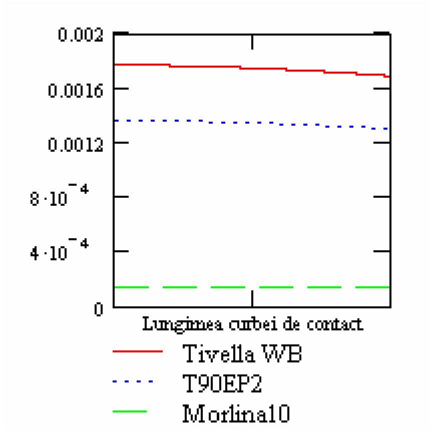


Fig.3
The variation of the minimum film thickness against viscosity along the contact

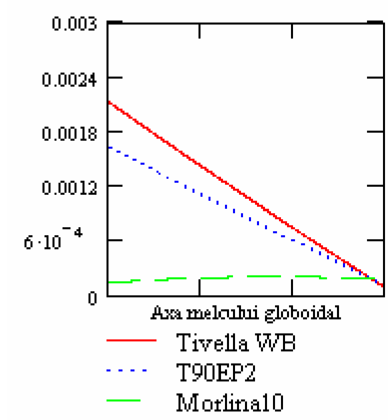


Fig.4
The variation of the minimum film thickness against viscosity along the meshing area

If it were to analyse the influence of viscosity upon the film thickness in contact points for which $u=ct.$ (such points: C11, C21, C31 and C41, where first figure -1,2,3 and 4 - denote the contact curve number and the second one –1-, the first contact point from that contact curve), it can say that there is a certain increase of h_{min}, due to combined effect of separation

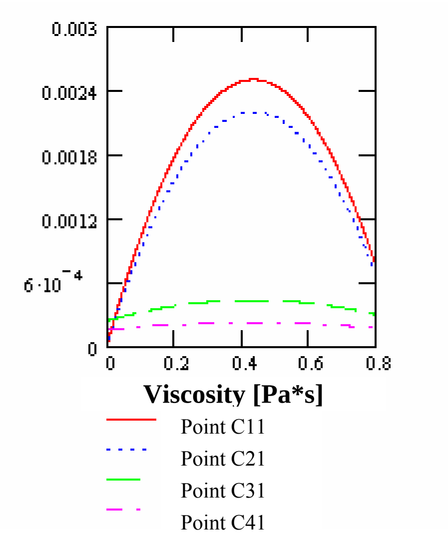


Fig. 5
The variation of the minimum film thickness against viscosity in instantaneous contact points having *u=ct.*

of mating surfaces and Hertz pressure, which lead to increase of elastic deformations. Afterwards, h_{min} decreases due to increases of thermal effect (fig.5).

There is possible to obtain an increase of film thickness by using a higher viscosity lubricant, but this is effective only in the first part of meshing, where rolling velocity is greater and contact force is low.

5. CONCLUSIONS

Determination of film thickness in case of double enveloping worm gear, likewise for any other machine element, carries a significant importance in terms of lubrication regime, having a direct impact on efficiency, durability and energy dissipation due to friction.

From the point of view of durability, it must say that the weakest part of the double enveloping worm is that with the minimum diameter, where film thickness has the lowest value.

Whilst the film thickness can be increase at the beginning of meshing by using a higher viscosity lubricant (fig. 4), this effect is drastically diminished towards the minimum diameter zone of the worm, because of the combined negative effect of the contact geometry and thinning of the lubricant film due to instantaneous increase of temperature. Developing new cinematic alternatives and decreasing of surfaces roughness are deemed to be some of the potential solutions.

6. REFERENCES

1. Fedorciuc-Onişa, C., *The Influence of Some Tribological Factors on The Globoid Worm Gears*, PhD thesis, Technical University of Cluj - Napoca, 2001
2. Dowson, D., *A Generalized Reynolds Equation for Fluid Film Lubrication*, International Journal of Mechanical Science, Pergamon Press Ltd., vol. 4, 1962

3. Timoshenko, S. and Godier, J. N., *Theory of Elasticity*, McGrow –Hill, New York, 1951
4. Kim., K., Sadeghi, F., *Three–Dimensional Temperature Distribution in EHD Lubrication: Part II – Point Contact and Numerical Formulation, Transactions of the ASME, Journal of Tribology, January, 1993, vol.115, pp.36-45*
5. Hsu, C. H., Lee, R. T., *An Efficient Algorithm for Thermal Elastohydrodynamic Lubrication Under Rolling / Sliding Line Contacts*, Transactions of the ASME, Journal of Tribology, October, 1994, vol.116, pp.762-769
6. Olaru, N. D., Tribologie. *Elemente de bază asupra frecării, uzării şi ungerii,* Universitatea Tehnică "Gh,. Asachi", Iaşi, 1995
7. Tallian, T. E., *The Theory of Partial EHD Contacts*, Wear, 21, 1972
8. Lubrecht, A.A., ş.a., *Multigrid, An Alternative Method for Calculating Film Thickness and Pressure Profiles in Elastohydrodynamically Lubricated Line Contacts*, Transactions of the ASME, Journal of Tribology, vol. 108, 1986

Notations:

α = pressure exponent from viscosity relation, *Pa*$^{-1}$,
α_1 = pressure coefficient from density relation, *Pa*$^{-1}$,
α_T = lubricant thermal coefficient, *K*$^{-1}$,
β_1 = pressure coefficient for density relation, *Pa*$^{-1}$,
β = temperature exponent from viscosity relation, *K*$^{-1}$,
$C_{m,n}$ = *m* contact point belonging to *n* contact curve,
c_p = lubricant specific heat, $J \cdot kg^{-1} \cdot K^{-1}$,
d = elastic displacement of mating contact surfaces, *mm*,
E =Young modulus, *Pa.*
η_0 = lubricant dynamic viscosity at ambient temperature, $Pa \cdot s$,
h_0 = film thickness due to HD effect,
h_{min} = minimum film thickness due to EHD effect,
k_0 =lubricant thermal conductivity, $W \cdot m^{-1} \cdot K^{-1}$,
k_{OL} =steel thermal conductivity, $W \cdot m^{-1} \cdot K^{-1}$
k_{Bz} = thermal conductivity of brass, $W \cdot m^{-1} \cdot K^{-1}$,
ν_1, ν_2 = Poisson coefficients,
p = pressure , *MPa,*
p_H = maximum Hertz pressure, *Mpa,*
r, θ, z = cylindrical coordinates, *mm,*
ρ = lubricant density, kg/m^3,
s = geometrical separation of the mating surfaces, *mm*,
T = instantaneous temperature inside the lubricant film, ^0K,
T_0 = lubricant temperature at inlet contact zone, ^0K,
u = linear parameter of the generated surface *[mm],*
v_r, v_θ, v_z = velocity components of elementary lubricant particle, *mm/s, rad/s, mm/s,*
V_r, V_θ, V_z = relative velocity components of the mating surfaces, *mm/s, rad/s, mm/s,*
ξ = elasticity parameter
ζ = Roelands exponent from viscosity relation