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ESTIMATION OF MACHINING ERRORS ON GLEASON BEVEL GEAR CUTTING

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Abstract: *In order to calculate the machining errors it is necessary to develope a mathematical model for the calculation of machining errors. The gears cut by Gleason gear cutting method has measured with CMM. The results of coordinate neasurements must be transformed into deviations of the real surface represented in the direction of the surface normal.*

Key words: *machining error model, bevel gear cutting, measuring on CMM*

1. INTRODUCTION

Gleason gear cutting machine has been very popular in the bevel gear manufacturers. Even the machines settings for obtaining desirable tooth bearing gears are given by Gleason's instruction sheets, a gear pair with a desirable tooth bearing is difficult to obtain at the first cut because of the inherent errors.

The technological aspects of the problems are as follows[1]:

a) The deviations of the real toothe surfaces are inevitable due to the surface distortion by heat treatment, errors of initial machine tool setings, deflection by manufacturing and so on.

b) Aplication of additional finishing operation for elimination of the deviations would be too expensive inn comparision with the approach based on corrections of initially applied machine-tool settings.The advantage of this approach is the possibility of using the same equipment to correct the the deviations . The disadvantage is that the approach will be succesful only if the deviations are repetable.

c) The coordinate maesurements must be performed with high precision, wich currently prohibits them from being performed simultaneously with the manufacturing. Therefore, the coordinate measurements are performed after manufacturing, but only the first gear of the whole gear set to be manufactured is tested.

d) In some cases, master-gears are used and the coordinate measurements provide the informations about deviations from the master-surface for the surface being tested.This approach is considered less efectiveas compared to computerised determination of surface deviations and corrections of machine-tool settings. Minimizing the deviations of real tooth surfaces results in a reduction in the level of transmission errors that cause gear noise and vibration.

2. MATHEMATICAL MODEL OF THE MACHINING ERORS

The pinion and gear tooth surface can be expressed matematically when the generation process is described according to the kinematic theory. In case of the Gleason gear cutting method, the tooth surface of the gear and of the pinion is able to see the transcription of the tool surfaces. That means that the numerical formula of the tooth surface can be expressed by the numerical formula of the tool surfaces.

Generating surfaces and coordinate systems [1,2.3]

Fig.1 shows a head cutter that is used for the generation of Gleason's spiral bevel gears. This tool is provided with blades having straight-lined profiles. These profiles being rotated about C-C form two cones that cut both sides of the tooth. Thus the generating surface is a cone surface.

In the process of generation the followings motions are performed (Fig.2): (1) a rotational motion of the head cutter about axis C-C, that provide the desired velocity of cutting and (2) a rotational motion of the head cutter about axis O-a_g while the gear to br generated rotates about axis O-a_i. This generation of gear-tooth surfaces is based on application of two tool surfaces, Σ_f and Σ_p, which generate gears 1 and 2, respectively. The generating surfaces (generating cones do not coincide; they have different cone angled, $\Psi_c^{(f)}$

and $\Psi_c^{(p)}$ and different mean radii, $r_c^{(f)}$ and $r_c^{(p)}$ (Fig.1)). Special machine-tool settings, ΔE_1 and ΔL_1 must be used for the generation of the pinion.

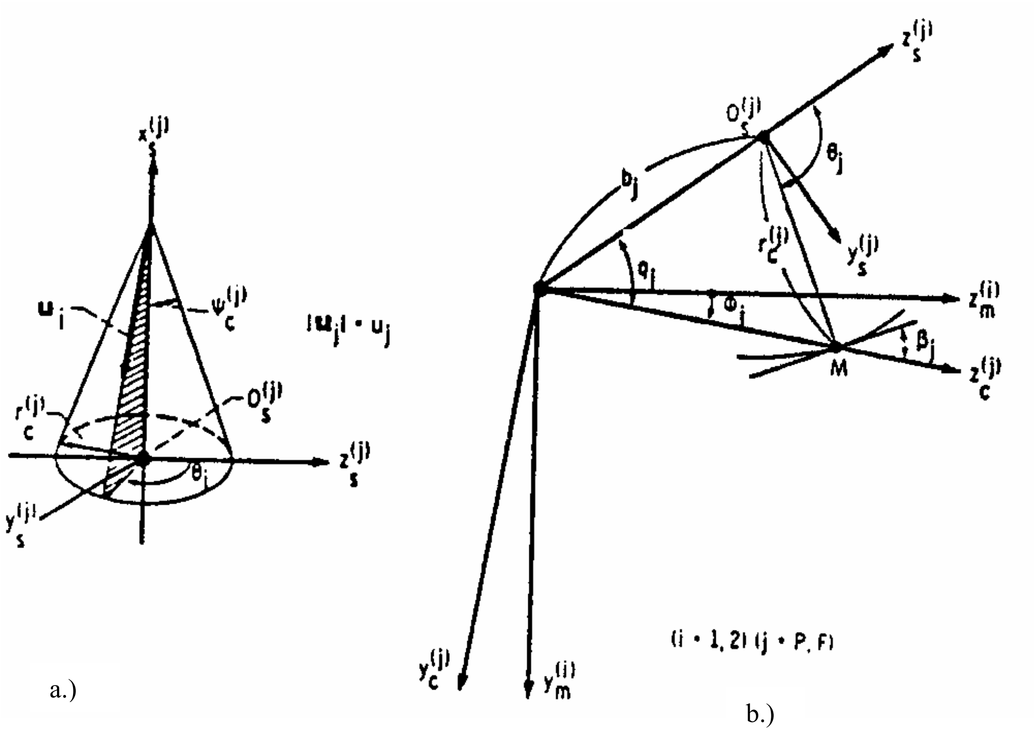

Fig.1. Coordinate system [2]

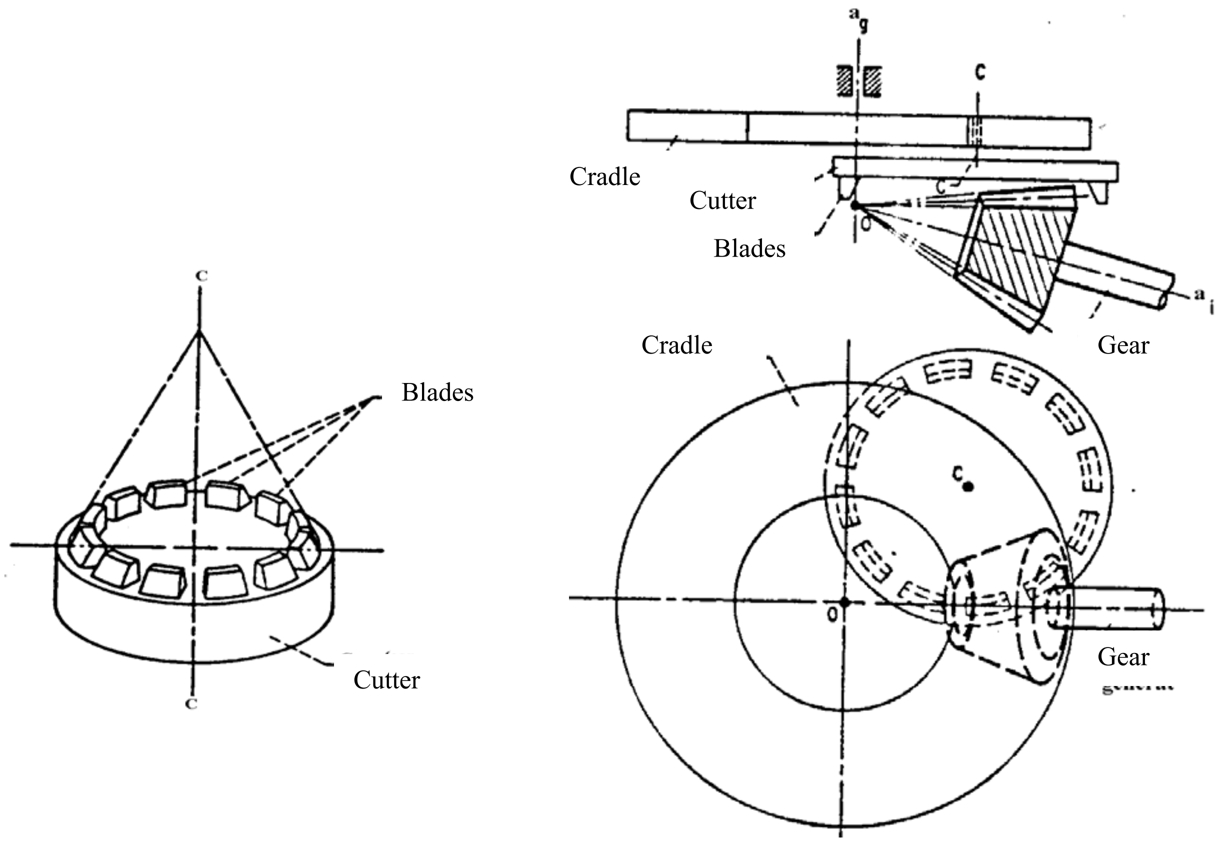

Fig.2 Cutter axis. Generating of the gear [2].

Considering the generation of the gear 2 tooth surface we use the followings coordinate systems:

(1) $S_c^{(p)}$ which is rigidly connected to the generating surface Σ_p (Fig.1.b).

(2) The fixed coordinate system $S_m^{(2)}$, that is rigidly connected to the frame of the cutting machine

(3) The coordinate system S_2, which is rigidly connected to gear 2 (Fig.3.b)

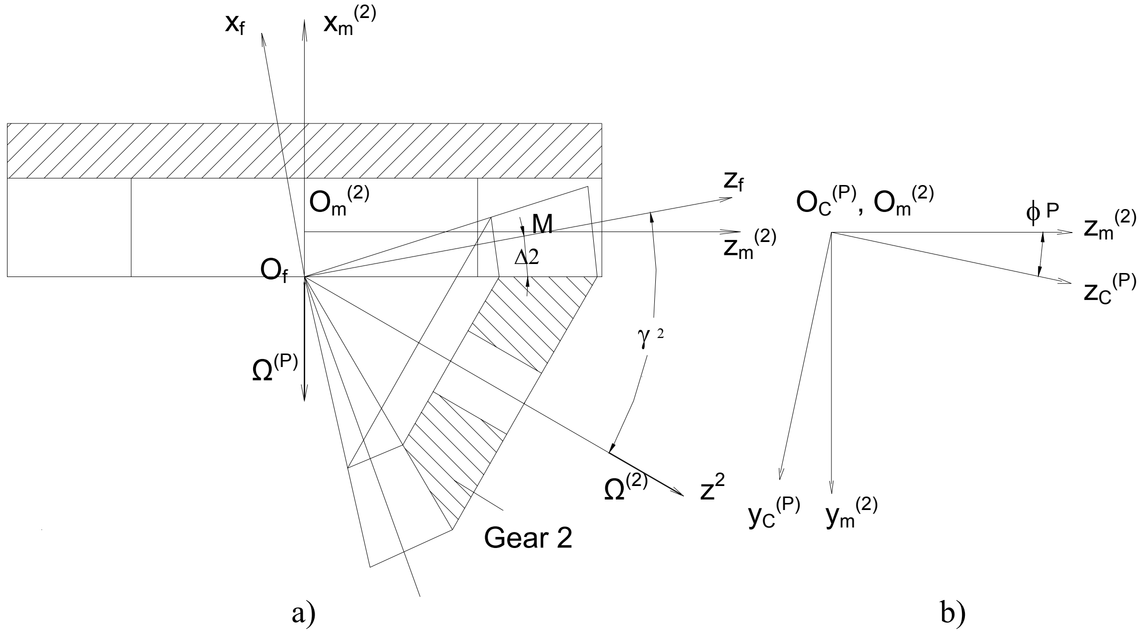

Fig.3. Coordinates of the gear2

In the process of generation, the generating surface rotatea about the $X_m^{(2)}$ axis with angular velocity $\Omega^{(P)}$, while the gear blank rotates about the Z_2 axis with the angular velocity $\Omega^{(2)}$. Axes $X_m^{(2)}$ and Z_2 intersect each other and form the angle $90°+ \Upsilon_2- \Delta_2$, where Δ_2 is the dedendum angle for gear 2. Axis $X_m^{(2)}$ is perpendicular to the generatrix of the root cone of gear 2. The coordinate system S_f shown in Fig.3 is rigidly connected to the housing of the gears and will be used for analysis of conditions of meshing of the gears.

Considering the generation of the pinion, we use the following coordinate systems:

(1) $S_C^{(F)}$, that is rigidly connected to the generating surface Σ_F

(2) $S_m^{(1)}$, that is rigidly connected to the frame of the cutting machine

(3) S_1, that is rigidly connected to the pinion (gear 1) (Fig. 4)

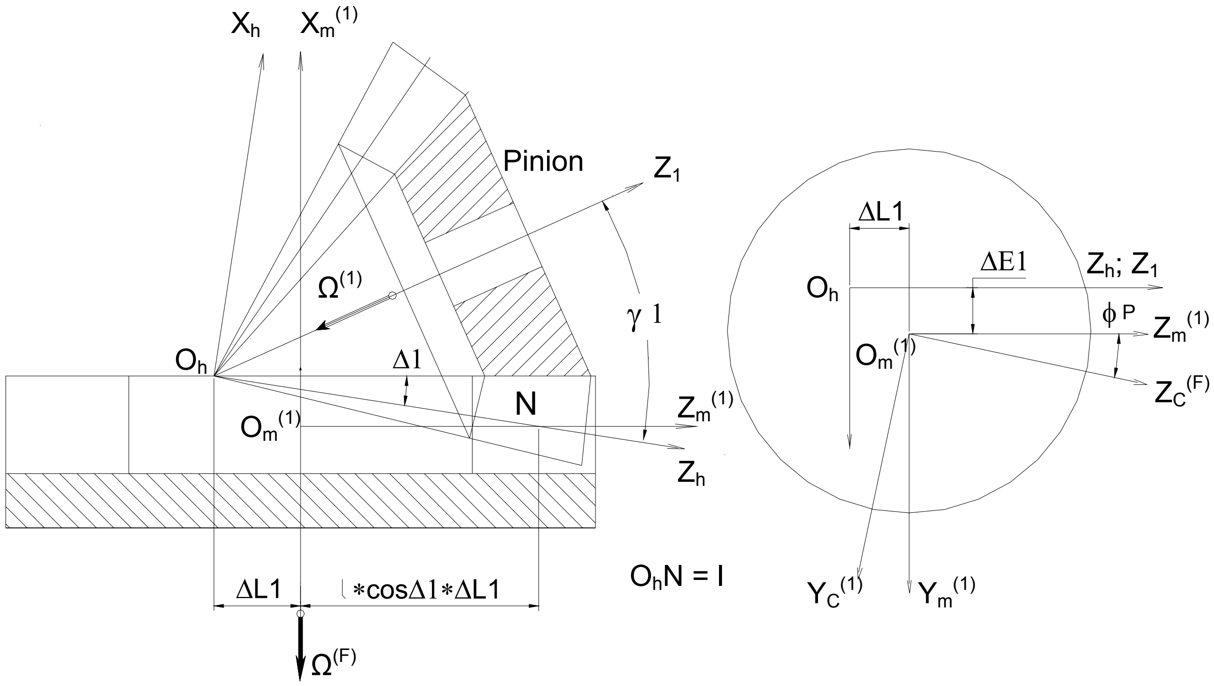

Fig.4 Coordinate system of the pinion

Axes $X_m^{(1)}$ şi Z_1 do not intersect but cross each other; ΔE_1 and ΔL_1 are the corrections of machine-tool settings that are used for the improvement of the meshing of the gears. In the process of generation, the generating surface rotates about the $X_m^{(1)}$ with angular velocity $\Omega^{(F)}$, while the gear 1 blank rotates about the axis Z_1 with angular velocity $\Omega^{(1)}$. Axes $X_m^{(1)}$ and Z_1 form the angle $90°-\Upsilon_1+\Delta_1$, where Δ_1 is the dedendum angle of gear1 and axis $X_m^{(1)}$ is perpendicular to the generatrix of the root cone of gear1.

Generating tool surface

The tool surface is a cone and is represented in the coordinate system $S_S^{(j)}$ as follows (Fig. 1.a):

$$\begin{bmatrix} x_{S(j)} \\ y_{S(j)} \\ z_{S(j)} \\ 1 \end{bmatrix} = \begin{bmatrix} r_{c(j)}\cdot \mathrm{ctg}\,\Psi_{c(j)} - u_j\cdot\cos\Psi_{c(j)} \\ u_j\cdot\sin\Psi_{c(j)}\cdot\sin\theta_j \\ u_j\cdot\sin\Psi_{c(j)}\cdot\cos\theta_j \\ 1 \end{bmatrix} \quad (j = F, P) \qquad (1)$$

where u_j and θ_j are the surface coordinates..

The coordinate system $S_c^{(j)}$ (j= F,P) is an auxiliary coordinate system that is also rigidly connected to the tool (Fig.1.b). To represent the generating surface Σ_F and Σ_P in coordinate system $S_c^{(j)}$ we use the following matrix equation (a left-hand generating gear is considered):

$$\begin{bmatrix} x_{c(j)} \\ y_{c(j)} \\ z_{c(j)} \\ 1 \end{bmatrix} = \left| M_{cS(j)} \right| \cdot \begin{bmatrix} x_{S(j)} \\ y_{S(j)} \\ z_{S(j)} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos q_j & -\sin q_j & -b_j \cdot \sin q_j \\ 0 & \sin q_j & \cos q_j & b_j \cdot \cos q_j \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{S(j)} \\ y_{S(j)} \\ z_{S(j)} \\ 1 \end{bmatrix} \qquad (2)$$

Here b_j and q_j are parameters that determine the location of the tool in coordinate system $S_C^{(j)}$. Ecuations 3.1. and 3.2. yield:

$$\begin{aligned} x_{c(j)} &= r_{c(j)} \cdot \operatorname{ctg} \Psi_{c(j)} - u_j \cdot \cos \Psi_{c(j)} \\ y_{c(j)} &= u_j \cdot \sin \Psi_{c(j)} \cdot \sin\left(\theta_j - q_j\right) - b_j \cdot \sin q_j \\ z_{c(j)} &= u_j \cdot \sin \Psi_{c(j)} \cdot \cos\left(\theta_j - q_j\right) + b_j \cdot \cos q_j \end{aligned} \qquad (3)$$

where j = (F,P).

The unit normal to the generating surface Σ_j (j=F;P) is represented by:

$$\mathbf{n}_{c(j)} = \frac{\mathbf{N}_{c(j)}}{\left|\mathbf{N}_{c(j)}\right|} \quad \text{where} \quad \mathbf{N}_{c(j)} = \frac{\mathbf{d}}{\mathbf{d}\theta_j}\mathbf{r}_{c(j)} \times \frac{\mathbf{d}}{\mathbf{d}u_j}\mathbf{r}_{c(j)} \qquad (4)$$

Using ecuations (3.3) şi (3.4.) (provided $u_j \sin\Psi_C^{(j)} \neq 0$) we obtain:

$$n_{c(j)} = \sin\Psi_{c(j)} \cdot i_{c(j)} + \cos\Psi_{c(j)} \cdot \left| \sin\left(\theta_j - q_j\right) \cdot j_{c(j)} + \cos\left(\theta_j - q_j\right) \cdot k_{c(j)} \right| \qquad (5)$$

Estimation method of the machining errors

The tooth surface can be expressed mathematically as explained in the previous chapter using variable parameters u and θ. This tooth surface equation, machining errors such as cutting errors, wear of cutter head, heat treatment distortions, etc. are included.

Generally the tooth surface is represented as follows, using constant machine settings, in order to eliminate the machining errors, values C_1 to C_n respectively [4]:

$$\mathbf{X}(u,\theta;\ C_1+\Delta C_1,\ C_2+\Delta C_2,\ \ldots,\ C_n+\Delta C_n). \tag{6}$$

where $\Delta C_1, \Delta C_2, \ldots , \Delta C_n$ indicates small different value from the given value by summary of each machine settings respectively:

Next the tooth surface selected arbitrary is measured by CMM. The equation (6) is represented as follows by measured point clouds **M** on the tooth surface:

$$\mathbf{M} = \mathbf{X}(u,\theta;\ C_1, C_2, \ldots, C_n) + \frac{\partial X}{\partial C_1}\Delta C_1 + \frac{\partial X}{\partial C_2}\Delta C_2 + \ldots + \frac{\partial X}{\partial C_n}\Delta C_n \tag{7}$$

By the transposition of matrix **X,** following is represented:

$$\mathbf{M} - \mathbf{X}(u,\theta;\ C_1, C_2, \ldots, C_n) = \frac{\partial X}{\partial C_1}\Delta C_1 + \frac{\partial X}{\partial C_2}\Delta C_2 + \ldots + \frac{\partial X}{\partial C_n}\Delta C_n \tag{8}$$

As the different values from summary, $\Delta C_1, \Delta C_2, \ldots , \Delta C_n$ are very small and the relationship of each is independent and linear each other, the equation (8) can be represented as follows by the method of superposition:

$$
\begin{aligned}
M - X &= \frac{\partial X}{\partial C_1}\Delta C_1 \qquad \Delta C_2 = \Delta C_3 = \ldots = \Delta C_n = 0\\
M - X &= \frac{\partial X}{\partial C_2}\Delta C_2 \qquad \Delta C_1 = \Delta C_3 = \ldots = \Delta C_n = 0\\
&\;\;\vdots\\
M - X &= \frac{\partial X}{\partial C_n}\Delta C_n \qquad \Delta C_1 = \Delta C_2 = \ldots = \Delta C_{n-1} = 0
\end{aligned}
\tag{9}
$$

The different value ΔC_j at j turn can be calculated by each measured point data which construct the clouds **M** in the equation (9). In practice, however, each different value at j turn is not always equal, that is there is dispersion in the different values. Therefore firstly, in the cequation about C_j in the equation (9), calculate the ΔC_j which minimize the sum of the square of each residual and standard deviation at the calculation of j turn. Lastly find out the parameter C_j which shows the smallest standard deviation among each parameter and estimate the real machine setting parameter as $C_k+\Delta C_k$. In this case, the theoretical tooth surface $\mathbf{X}(u,\theta;\ C_1, C_2, \ldots, C_k+\Delta C_k, \ldots, C_n)$ is the best fitted the measured points clouds **M**. And continuing the same way on the remaining n-1 equations, find out the real machine setting parameter which shows the minimum standard deviation by utilizing the founded real machine setting parameters before. When smaller standard deviation is not found, the remaining parameters are estimated as the same value as in the summary.

3.MEASURING POINT DATA BY CMM

The gear is set on the table of the coordinate measuring machine (CMM) arbitrarily. The position of the gear axis and the datum plane must be determined by measurement independent of the tooth surface measurement because the gear is set arbitrarily [5]. The results of coordinate neasurements must be transformed into deviations of the real surface represented in the direction of the surface normal.

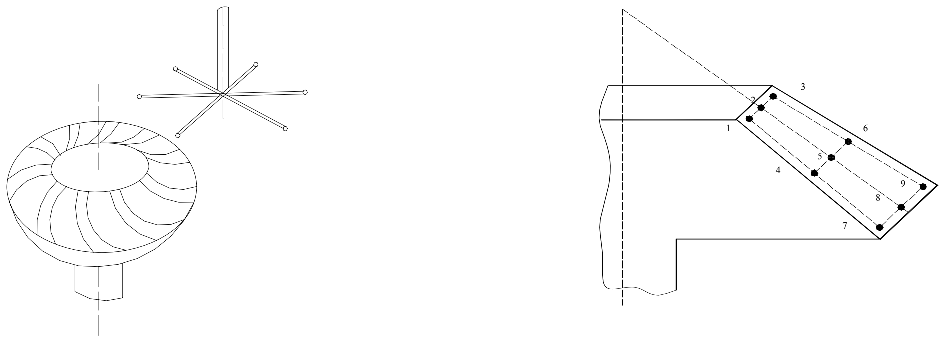

Fig.5. Measuring scheme by CMM

4. CONCLUSIONS

A mathematical model for estimation of machining errors on Gleason gear cutting was proposed. The gears have measured with CMM and the deviations of the real surface can be determined.

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