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THE INFLUENCE OF THE ADDENDUM MODIFICATION ABOUT DYNAMIC FACTOR OF THE SPUR GEARING WITH INVOLUTE PROFILE HAVING THE CENTRE DISTANCE IMPOSED

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Abstract *The prediction of the spur gear's dynamic behavior and gear noise has always been a major concern in gear design. An interesting solution is the computer simulation of the mesh process using mathematical models to approximate the real process. The work proposes to analyze some aspects of the theoretical dynamic behavior of the spur gearing with involute profile using a program made-up in Matlab6 that allows the theoretical calculation of the dynamic factor.*

1. Introduction

The addendum modifications have a decisive influence about the dynamic behavior of the spur gear. The correctly choice of the addendum modifications is very important, because these decide in a big way the geometrical and resistance characteristics of the spur gearing.

As for the spur gearing with involute profile having the centre distance free, existed prescriptions as to the choice of the addendum modification coefficients in term of functional requirements, in the case of the spur gearing with involute profile having the centre distance imposed is much harder because the sum of the addendum modification coefficients are given. For this choice we must know the gear's dynamic behavior, a useful method is the dynamic simulation using a mathematical model.

In the specialty literature, the dynamic factor is defined in many ways. So, in [3], the mathematical definition is:

$$K_v = 1 + \frac{M_{t\_din}}{M_{tn}} = 1 + \frac{F_{t\_din}}{F_{tn}} \tag{1}$$

Another definition for the dynamic factor is given based on the static and dynamic stress on the teeth base [5]:

$$K_v = \frac{\sigma_{din}}{\sigma_{sta}} \tag{2}$$

Because the exact calculus (theoretical) of the dynamic factor is laborious, on the grounds of simplicity was adopted empirical formula.

In 1927, A. A. Ross introduced the following empirical formula for the dynamic factor K [4]

$$K = \frac{78}{78 + \sqrt{\upsilon}} \tag{3}$$

where υ is the pitch line speed measured in [ft/min]. This expression received acceptance as a standard factor used by the American Gear Manufacturer's Association (AGMA).

In 1959, a similar factor for use with higher precision gears was introduced by Wellauer [4]:

$$K = \sqrt{\frac{78}{78 + \sqrt{\upsilon}}} \tag{4}$$

Buckingham has developed an expression for the dynamic load in term of the pich line speed and the applied load. His formula is [4]:

$$F_{din} = F + \sqrt{f_a\left(2f_b - f\right)} \tag{5}$$

where: F_{din} is the dynamic load,

F is the applied load,

$f_a = \frac{f_b \cdot f_c}{f_b + f_c}$,

$f_b = 0{,}0555 \cdot E \cdot b + F$,

$f_c = 0{,}00025 \frac{R_1 + R_2}{R_1 \cdot R_2} M\upsilon$,

E is the elastic constant,

b is the face width,

R_i are the pitch radii of the gears,

M is the effective mass of the gears

In this expression the units are [pounds] and [inches], except for the pitch line speed which is measured in [ft/min].

Another developed method for the calculus is the usage of diagrams where are three calculus domains: sub critical, resonance and supra critical [3].

2. The mathematical simulation

In the literature of specialty existed several mathematical models which approximated the gear meshing.

The mathematical model used in the dynamic analysis of the gear meshing is a single degree of freedom model. It can yield accurate results provided that important dynamic effects are considered. The most important effects that should be considered in the model are non-linearity of mesh stiffness, mesh damping and the excitation due to gear errors [5].

The differential equation must to solve is:

$$m_e\ddot{x} + c_m\dot{x} - c_1\dot{e}_1 - c_2\dot{e}_2 + k_m x - k_1 e_1 - k_2 e_2 = F_n, \qquad (6)$$

where: $m_e = \dfrac{I_1 I_2}{I_1 R_{b2}^2 + I_2 R_{b1}^2}$ is equivalent mass representing the total inertia of a gear pair, $F_n = \dfrac{M_{n1}}{R_{b1}} = \dfrac{M_{n2}}{R_{b2}}$ is the static mesh load, c_m is the viscous damping of the gear mesh (total), c_i is the viscous damping coefficient on the *i*th tooth pair in mesh, k_m is the stiffness of the gear mesh (total), k_i is the stiffness of the *i*th tooth pair in mesh, e_i is the displacement excitation representing the relative gear errors of the *i*th tooth pair meshing teeth pair.

The *loaded static transmission error* can be obtained from equation (6) by neglecting the dynamic terms:

$$x_s = F_n / k_m + (k_1 e_1 + k_2 e_2)/k_m \qquad (7)$$

The *loaded dynamic transmission error* can be obtained from equation (6) by considering the dynamic terms.

Because both errors can be obtained from the same equation, result that is possible to determined the *loaded dynamic transmission error* in terms of the *loaded static transmission error*.

The dynamic mesh load is given by:

$$F_d = c_m\dot{x} - c_1\dot{e}_1 - c_2\dot{e}_2 + k_m x - k_1 e_1 - k_2 e_2 \qquad (8)$$

the *loaded dynamic transmission error* can be written:

$$x = F_d / k_m + (k_1 e_1 + k_2 e_2)/k_m - c_m\dot{x}/k_m + (c_1\dot{e}_1 + c_2\dot{e}_2)/k_m \qquad (9)$$

The simulation program made-up in Matlab6 take determine the variations of the maximum relative slip between flanks, of the static and dynamic stress at the teeth base and the variations of the load and stress dynamic factor in terms of the repartition of the addendum modifications coefficients.

3. Results of the simulation

For study, I considered a spur gearing with involute profile, characterized by the initial data presented in Table1:

Table 1

Parameter	Pinion	Wheel
Number of teeth	21	41
Module	4	
Pressure angle	20^o	
Face width [mm]	28	28
Backlash [mm]	0,1	
Center distance [mm]	127	
Modulus of elasticity [N/mm^2]	$2,1\cdot10^5$	$2,1\cdot10^5$
Poisson's coefficient	0,29	0,29
Density [kg/mm^3]	$7,87\cdot10^{-6}$	$7,87\cdot10^{-6}$
Input torque [N·m]	800	
Input speed [rot/min]	1000	

In figure 1 and 2 is presented the variations of the maximum relative slip between flanks in terms of the pinion's addendum modifications coefficients.

The value of the pinion's addendum modification coefficient for an equal maximum relative slip between flanks for both gears is $x_1 = 0.43778534591944$

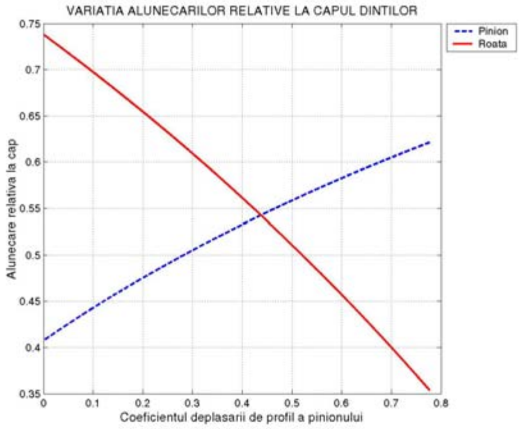

Fig.1

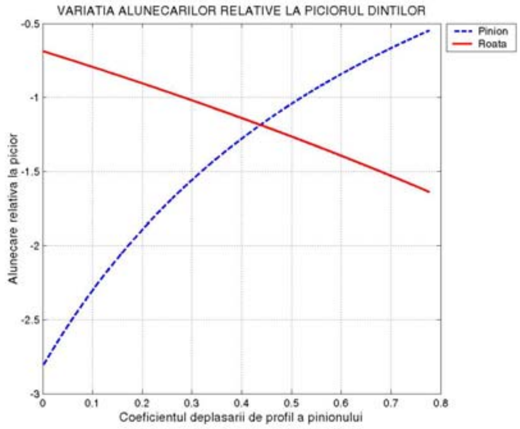

Fig.2

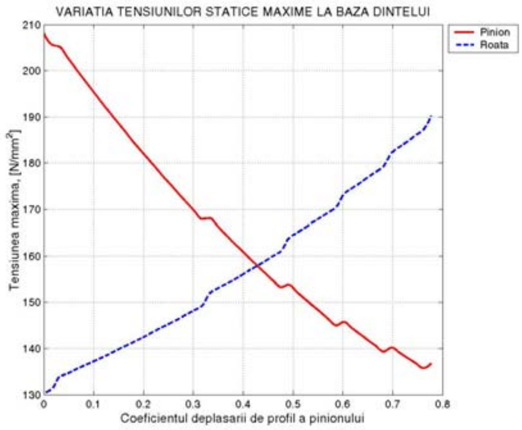

Fig.3

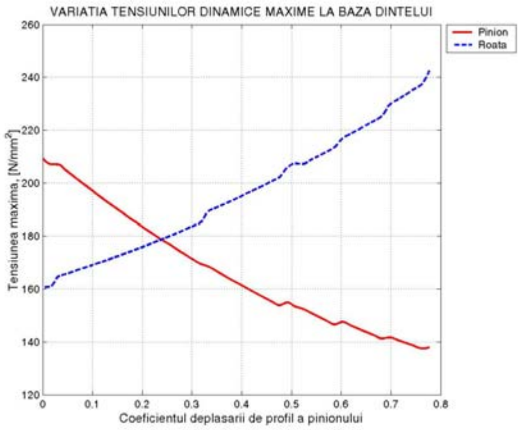

Fig.4

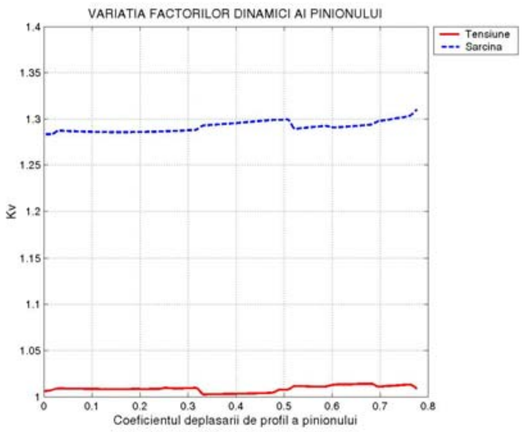

Fig.5

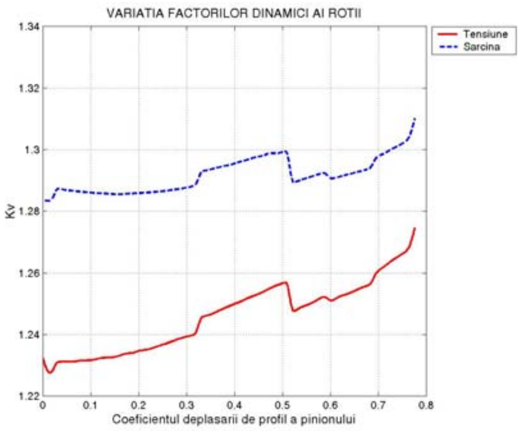

Fig.6

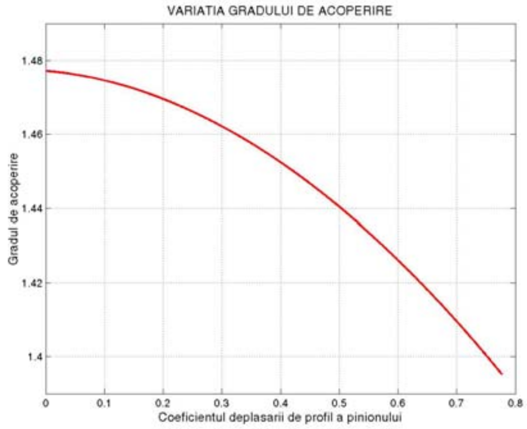

Fig.7

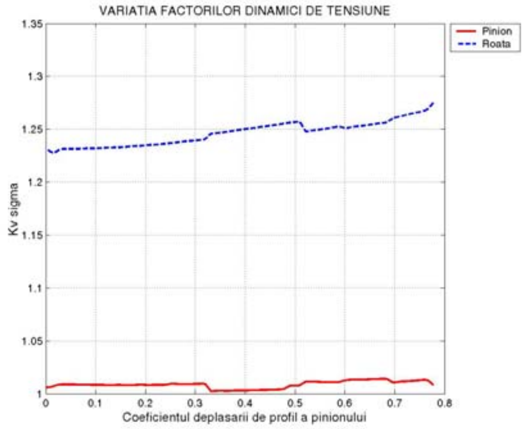

Fig.8

In figures 3 and 4 is presented the variations of the static and dynamic stress at the teeth base in terms of the pinion's addendum modifications coefficients.

The value of the pinion's addendum modification coefficient for an equal dynamic stress at the teeth base for both gears is $x_1 = 0.23794372264893$

In figures 5 and 6 is presented the variations of the static and dynamic stress at the teeth base in terms of the pinion's addendum modifications coefficients for the pinion, respectively the wheel.

In figures 5 and 6 is presented the variations of the dynamic stress at the teeth base in terms of the pinion's addendum modifications coefficients for the pinion, respectively the wheel.

In figure 7 is presented the contact ratio in terms of the pinion's addendum modifications coefficients

In figures 8 is presented the variations of the dynamic stress at the teeth base in terms of the pinion's addendum modifications coefficients for the pinion and the wheel.

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