DETERMINATION OF HORIZONTAL AXIS WIND TURBINE PERFORMANCE IN YAW BY USE OF SIMPLIFIED VORTEX THEORY

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Abstract

The paper presents an application of non-iterative lifting line theory of Horizontal Axis Wind Turbine(HAWT) to determination of HAWT's performance. The presented model is developed on the basis of modified Witoszyński's propeller theory presented in [4] . In this model an iterative determination of induced velocity field is avoided thanks to solution method similar to as in Witoszyński's propeller theory. The Goldstein kappa factor or Prandtl tip loss factor are introduced to the equations of momentum and angular momentum for ideal HAWT. Ideal axial force (thrust) and ideal torque are determined using blade element method (BEM) (profile losses are neglected). This values of thrust and torque are compared to those obtained from momentum and torque equation. Thanks to approximation of lift force coefficient vs. angle of attack by sine curve one may obtain a quadratic equation for axial velocity component An extended version of the model including nonuniform inflow is presented. In the theory a quasi-steady approach to blade element characteristics was applied. The calculations are compared with experimental data obtained at Risϕ 100 kW test turbine. Presented results show that, the method described in the paper underestimates performance for low speed winds, whereas for strong winds the power output is slightly overestimated.

Key words: *Wind turbine, Vortex theory, HAWT, Nonuniform inflow, aerodynamics*

1. SIMPLIFIED VORTEX THEORY OF HAWT ROTOR UNDER NONUNIFORM INFLOW CONDITIONS

It is well known that in *real conditions* rotary wings (airplane and marine propellers, helicopter rotors and wind turbines) work in nonuniform inflow. In case of HAWT this nonuniformity is caused by many different conditions .In this, study the following effects were considered:

1. Yaw angle or tilt angle;
2. Wind velocity profile;

It was also assumed that blades are infinitely rigid, so flapping motions, and blade torsion could be neglected.

Momentum and angular momentum for circumferentialy varying velocity field can be expressed in following form:

$$\frac{1}{4\pi}\rho W^2 BcC_L \sin\phi r dr d\psi = \rho V_\vartheta V_X r^2 F dr d\psi \tag{1.1.}$$

$$\frac{1}{4\pi}\rho W^2 BcC_L \cos\phi dr d\psi = 2V_X\left(V_W \cos\phi_y - V_X\right) rF dr d\psi \tag{1.2.}$$

where: $\xi = r/R$, r-current radius, c- blade chord, F-Prandtl tip-loss factor, R-propeller tip radius. The sectional lift coefficient is given by the formula:

$$C_L = a_0 \chi_p \sin\alpha^{(0)} \tag{1.3.}$$

$a_0 = \left(dC_L / d\alpha\right)$ -lift curve slope in its linear portion for airfoil section.

From eqns. (2.1.) and (2.2.) we obtain:

$$\tan\phi = \frac{V_\vartheta}{2\left(V_w \cos\phi_y - V_x\right)} \tag{1.4.}$$

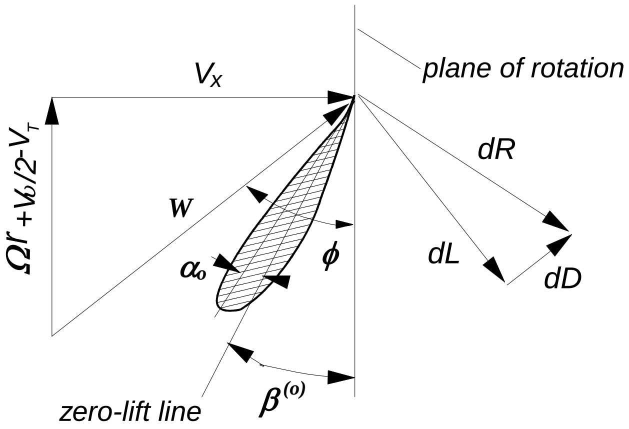

Fig 1.1. Velocity triangles and forces at the blade element at yawed conditions.

Velocity triangle gives us second (see: Fig 1.1.) eqn. for $\tan\phi$:

$$\tan\phi = \frac{V_x}{\Omega r - V_w \sin\phi_y \cos\psi + V_\vartheta/2} \tag{1.5.}$$

By comparison of (1.4.) and (1.5.) one may establish a formula expressing dependence between axial velocity at actuator disc and tangential induced velocity:

$$\overline{V}_\vartheta = \frac{2\overline{V}_x\left(\lambda \cos\phi_y - \overline{V}_x\right)}{\xi - \lambda \sin\phi_y \cos\psi} \tag{1.6.}$$

Angular momentum equation (1.1.) yields:

$$W\sigma C_L = 2V_\vartheta F \tag{1.7.}$$

Wkere: $\sigma = B\bar{c}/(2\pi\xi)$ -local disc solidity

Inserting (1.3.) and (1.6.) to (1.7.) and neglecting higher order term, proportional to square of swirl velocity one obtains quadratic equation for V_x. The solution of this eqn. is:

$$\overline{V}_x = \frac{A-\lambda\cos\phi_y}{2} + \sqrt{\left(\frac{A-\lambda\cos\phi_y}{2}\right)^2 + A\left(\xi-\lambda\sin\phi_y\cos\psi\right)\tan\beta^{(0)}} \tag{1.8.}$$

Where blade element shape coefficient A takes form:

$$A = \frac{\sigma a_0\chi_p}{4}\,\frac{\left(\xi-\lambda\sin\phi_y\cos\psi\right)\cos\beta^{(0)}}{F+\dfrac{\sigma a_0\chi_p}{4}\sin\beta^{(0)}} \tag{1.9.}$$

When V_x is calculated one may easy determine inflow angle:

$$\phi = arc\,tan\frac{\overline{V}_\vartheta}{2\left(\lambda\cos\phi_y - \overline{V}_x\right)} \tag{1.10.}$$

Shaft power coefficient can be calculated as:

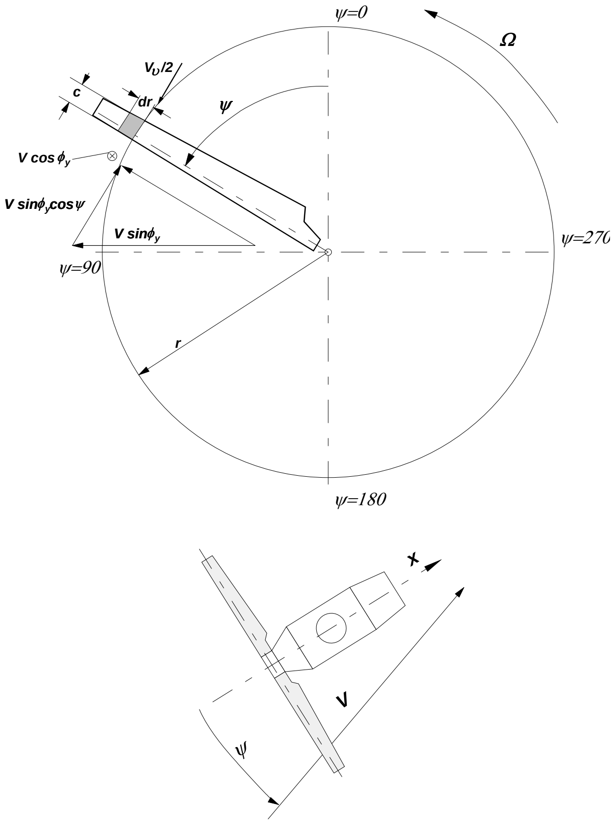

Fig 1.2. HAWT rotor at yawed inflow conditions. Schematic.

$$P_C = \frac{X^3}{2}\int_0^{2\pi}\int_{\xi_h}^{1}\overline{W}^2\sigma\left(C_L \sin\phi - C_D \cos\phi\right)\xi^2 d\xi d\psi \qquad (1.11.)$$

Rotor drag coefficient is given by formula:

$$D_C = \frac{X^2}{\pi}\int_0^{2\pi}\int_{\xi_h}^{1}\overline{W}^2\sigma\left(C_L \cos\phi + C_D \sin\phi\right)\xi d\xi d\psi \qquad (1.12.)$$

where: ξ_h -non-dimensional radius of hub, C_D -blade section drag coefficient (two dimensional airfoil), $X=\omega R/V_w$ is the tip speed ratio, $X=\lambda^{-1}$

2. COMPARISON OF THE THEORY WITH EXPERIMENTAL DATA

To compare presented theory with experiment, *100 kW* Risϕ test turbine experimental data were chosen. One of the blades of the HAWT was instrumented, to provide measurement of agles of attack, as well forces acting on selected blade segments. There were possibility to obtain long time series(up to *600 sec.*) of the data like: angle of attack at $r=0.71R$, wind speed, rotor yaw angle, power output, rotor speed, and normal as well as tangential forces acting on three selected blade segments. The detailed description of the facility as well as experimental data may be found in [3] This facility was chosen because of airfoil characteristics for wide range of angles of attack are available [3]. It was also important that mechanical and electrical power curves were given. The static airfoil data were corrected for stall delay due to rotation. The correction was made by use of empirical stall delay model proposed by J.L. Tangler and M. S. Selig [5]. Figure 2.1. shows mechanical power generated by „constant-speed" *(n=47.5 r.p.m.)* rotor for various wind velocities. Comparison of experimental curve with results obtained for tilted rotor are in good agreement with experimental data. However, for low wind velocities the presented theory underestimates power, but maximum power value is slightly overestimated.It is worth noting that for low wind velocities there is no significant difference between power calculated for tilted rotor and rotor in axial inflow. The appreciable difference appears only for high wind velocities (over *12 m/s*).

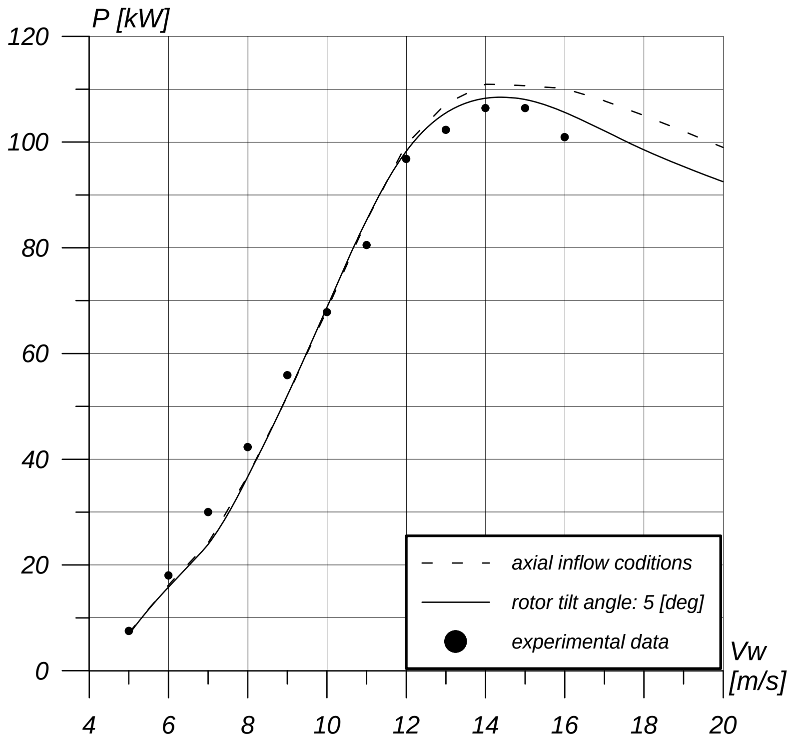

Fig. 2.1. Shaft power curve as a function of wind velocity

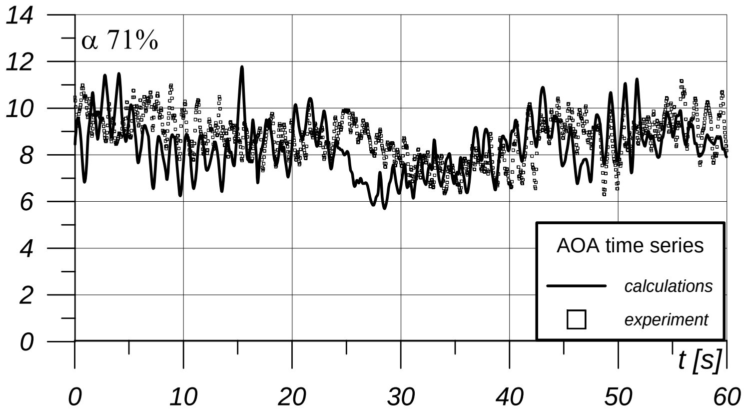

Fig. 2.2. Measured and calculated angle of attack time series $V_w=8.4\ m/s$ (1)

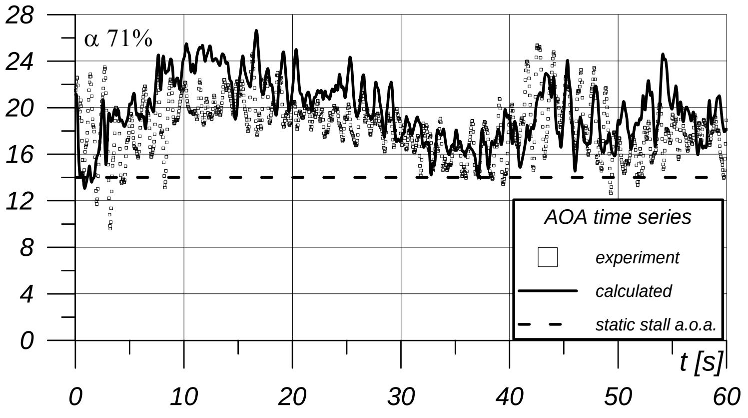

Fig. 2.3**.** Measured and calculated angle of attack time series $V_w=14.1\ m/s$ (2)

Database included to report [3] has provided oportunity to compare calculation results with field rotor measurements. The results of calculations in confrontation with experimental data are depicted in figures 2.2...2.3. As far as wind profile is concerned, it was assumed that power law (2.1.) is fulfilled:

$$V_w(H) = V_w^{(hub)}\left[H/H_{hub}\right]^a \qquad (2.1.)$$

For the presented calculations the value $a=0.26$ has been chosen (terrain covered with numerous small obstacles) .

3. FINAL REMARKS AND CONCLUSIONS

The results presented above show good agreement with experimental data.

However, for angle of attack series one may see that there is a time shift between calculations and experimental data. This may be caused by the fact that five-hole Pitot probe used for measurements was forwarding the blade in azimuth [3]. It was found that the model of HAWT has a limitation ensued from physical conditions: for „constand-speed" Horizontal Axis Wind Turbines the relative induced velocity („induction factor") $a=(V_x - V_w)/V_w$ increases when wind velocity decreases. Calaculations has shown the model fails when for any blade element the induction factor reaches value about $a \approx 0.60$. Of course, the exact limiting value of *a* depends on geometry of blade element. However, for this regime appeares a specific work-state called in helicopter aerodynamics „vortex ring state". Hence, simple theoretical model should not be appllied.

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