

## SOME THEORETICAL ASPECTS APPLICABLE TO THE RADIAL HYDRODYNAMIC WORKING BEARINGS UNDER HARD SHOCKS

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**Abstract:** *The paper presents some theoretical considerations and hypotheses in relation to the elastohydrodynamic lubrication in the case of bearings functioning under hard shocks, that have hydrodynamic lubrication under normal conditions. We state and provide determination relations of the pressure and thickness of the lubricating layer, taking into account the elastic deforming of the pin and bush.*

*Key words:* bearing, elastohydrodynamic lubrication, elastic deforming, pin and bush.

### INTRODUCTION

In the case of the great Hertz pressure forces, in friction coupling heavily stressed, the elastohydrodynamic lubricating working condition develops. This is frequently met with rolling bearings, sliding bearings with plastic bushings etc.[5],[6].

The way to solve the lubrication issue for a radial bearing subject to heavy loads comprises the solving of the Reynolds equation, of the energy equation, of the equation the elastic deformation of the surfaces, and the equation of variation of viscosity and density of the lubricant with the pressure and the temperature, which together form a non-linear integro-differential system [1],[3].

We consider a system of coordinates in rotation, with the centre of the bushing, the centre of the journal performing a certain movement limited only by the constant stroke between the journal and the bushing (Fig. 1). The journal rotates with the angle speed  $\Omega_1$ , the angle speed of the bushing  $\Omega_2 = 0$ .

The equations that describe the pressure, the temperature and the form of the lubricating film are:

**A. Reynolds equation**, in cylindrical coordinates is [2]

$$\frac{\partial}{\partial \theta} \left( \frac{h^3}{\eta} \rho \frac{\partial P}{\partial \theta} \right) + R_1^2 \frac{\partial}{\partial z} \left( \frac{h^3}{\eta} \rho \frac{\partial P}{\partial z} \right) = 12R_1^2 \left[ \frac{\partial \rho h}{\partial t} - \frac{1}{2} (\dot{\phi} - \Omega_1) \frac{\partial \rho h}{\partial \theta} \right]. \quad (1)$$

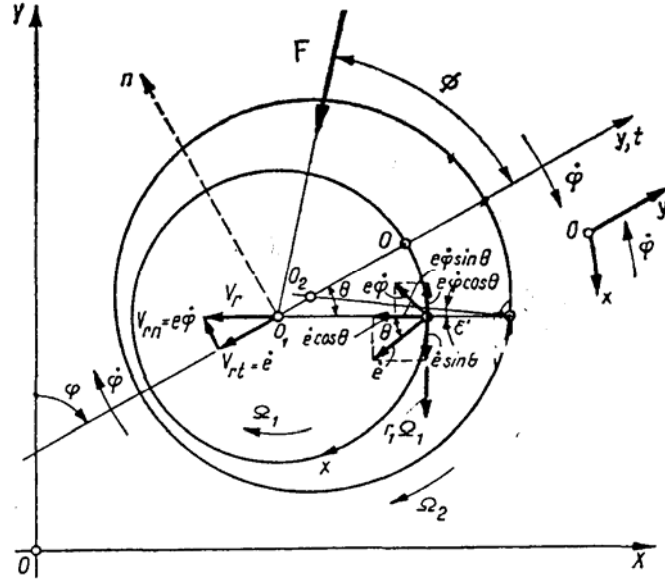


Fig. 1. The system of coordinates for the journal bearing

The limit conditions of the pressure are:

$$P(R_{\min}, \theta) = P(R_{\max}, \theta) = P(R, \theta_{\min}) = P(R, \theta_{\max}), \quad (2)$$

and

$$P = 0 \text{ for } \frac{\partial P}{\partial \theta} = 0. \quad (3)$$

The expression for the rotating speed of the line of the centre is [2],[9]

$$\dot{\varphi}(t) = -\dot{\phi}(t), \quad (4)$$

where

$$\dot{\phi} = \Omega_1 - \frac{\psi^2(1+\varepsilon)(1-\varepsilon^2)(2+\varepsilon^2)F(t)\cos\phi}{72\eta\lambda R_1\varepsilon^2}. \quad (5)$$

**B. The energy equation**, in cylindrical coordinates is [4]

$$\rho c_F \left( \frac{\partial T}{\partial t} + \frac{u}{R_1} \frac{\partial T}{\partial \theta} + v \frac{\partial T}{\partial y} \right) = k_F \frac{\partial^2 T}{\partial y^2} + \eta \left( \frac{\partial u}{\partial y} \right)^2. \quad (6)$$

The components of the speed within the film are given by the expressions

$$u = \frac{1}{R_1} \frac{\partial P}{\partial \theta} \left( I - \frac{I_2}{J_2} J \right) + \Omega_1 R_1 \left( I - \frac{I_2}{J_2} J \right), \quad v = -\int_0^h \left( \frac{1}{R_1} \frac{\partial u}{\partial \theta} + \frac{\partial w}{\partial z} \right) dy, \quad (7)$$

where

$$I = \int_0^y \frac{\sigma}{\eta} d\sigma, \quad J = \int_0^y \frac{1}{\eta} d\sigma, \quad I_2 = \int_0^h \frac{y}{\eta} dy, \quad J_2 = \int_0^h \frac{1}{\eta} dy. \quad (8)$$

The limit conditions for the speeds in a stationary guide mark connected to the bushing are:

- at the surface of the journal:  $u_1 = \Omega_1 R_1$ ,  $v_1 = \frac{\partial h}{\partial t}$ ,  $w_1 = 0$  ; (9)

- at the surface of the bushing:  $u_2 = v_2 = w_2 = 0$ . (10)

### C. The equation of elasticity.

The elastic deformation of the contact surfaces, due to the pressure  $P(R, \theta)$ , in cylindrical coordinates, is [3]

$$v(R, \theta) = \frac{2}{\pi E'} \int_{R_1}^{R_2} \int_{\xi_1}^{\xi_2} \frac{P(R, \xi) dR d\theta}{\sqrt{(R \cos \xi - r \cos \xi)^2 + (R \sin \xi - r \sin \xi)^2}}. \quad (11)$$

The minimum thickness of the lubricating film is defined by the relation [7]

$$h(R, \theta) = h_0 + v(R, \theta) + \delta(\theta). \quad (12)$$

The thickness of the lubricating film also comprises a thermoelastic component,  $\delta(\theta)$  [4]

$$\delta(\theta) = U_C(\theta) - U_A, \quad (13)$$

where the dilatation of the journal  $U_A$ , is calculated by using the Timoshenko relation, for a long cylinder without exterior forces [4]

$$U_A(t) = \frac{2(1 + \nu_A) \alpha_A}{R_1} \int_0^{R_1} [T(r, t) - T_0] r dr. \quad (14)$$

The termic deformations of the bushing,  $U_C(\theta)$  is calculated numerically, with matrix notations, the relations are [4]

$$\{\sigma\} = [D]\{\varepsilon\} - [D']\{\varepsilon_0\}, \quad (15)$$

where  $\{\sigma\} = [\sigma_x \sigma_y \tau_{xy}]^T$ ,  $\{\varepsilon\} = [\varepsilon_x \varepsilon_y \gamma_{xy}]^T$ ,  $\{\varepsilon_0\} = \alpha_C T [1 \ 1 \ 0]^T$ ,

$$[D] = \frac{E_C}{(1 + \nu_C)(1 - 2\nu_C)} \begin{bmatrix} 1 - \nu_C & 0 & 0 \\ 0 & 1 - \nu_C & 0 \\ 0 & 0 & \frac{1 - 2\nu_C}{2} \end{bmatrix}, \quad [D'] = (1 + \nu_C)[D]$$

### D. The equation of the heat within the radial bearing.

The bi-dimensional equation of the heat in polar coordinates for the bushing is [4]

$$\rho_C c_C \frac{\partial T}{\partial t} = k_C \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} \right], \quad (16)$$

and for the journal, considering the uniform temperature on a circumferencial direction, results the unidimensional equation

$$\rho_A c_C \frac{\partial T}{\partial t} = k_A \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right). \quad (17)$$

On the exterior surface of the bushing the heat eliminated through convection is considered,

$$k_C \frac{\partial T}{\partial r} \Big|_{r=R_2'} = h(T - T_0) \quad (18)$$

### E. The equations of the density and viscosity of the lubricant with the pressure and the temperature

The equation of variation of the density of the lubricant with the pressure and temperature, proposed by Dowson and Higginson (1966), is [8]

$$\rho = \rho_0 \cdot \left( 1 + \frac{0,6 \cdot P \cdot 10^{-9}}{1 + 1,7 \cdot P \cdot 10^{-9}} \right) \left( 1 + \beta \cdot T_0 \left( 1 - \frac{T_{\max}}{T_0} \right) \right), \quad (19)$$

where  $\rho_0$  represent the density at the reference temperature  $T_0$ .

The variation of the lubricant viscosity with the pressure and the temperature is given by the exponential relation[3]

$$\eta = \eta_0 \exp[\alpha P - \beta(T - T_0)], \quad (20)$$

where  $\eta_0$  represent the viscosity at the reference temperature  $T_0$ .

### The general organization graphic for the numerical solving of the lubrication of the radial bearing

We consider that the bearing works in a hydrodynamic working condition with static charge G and the number of rotation n. We consider as known the values for  $R_1$  the journal radius,  $D_1$  the journal diameter, L the bushing width, C the radial stroke in functioning, and  $\rho_0, \eta_0$  the density respectively the viscosity of the lubricant at the reference temperature  $T_0$ . We determine the angle speed of the journal  $\Omega_1$ , the attitude angle  $\varphi$  and the excentricity  $\varepsilon$ , obtaining the thickness of the lubricating film  $h_{\min}$ , in hydrodynamic working conditions.

In these conditions, we consider the elastic deformation  $v_{1,2} = 0$ ,  $P=0$ ,  $T = T_0$ ,  $T_1 = T_{1,initial}$ ,  $T_2 = T_{2,initial}$ ,  $T = T_1 = T_2$ , and we establish the convergency criteria

$$\frac{\sum_i \sum_j |P_{i,j_n} - P_{i,j_{n-1}}|}{\sum_i \sum_j P_{i,j_n}} \leq 0,005 \quad (21)$$

$$\frac{\sum_i \sum_j \sum_k |T_{i,j,k_n} - T_{i,j,k_{n-1}}|}{\sum_i \sum_j \sum_k T_{i,j,k_n}} \leq 0,005 \quad (22)$$

The simplified scheme of the algorithm of calculation is presented in Fig. 2 [1].

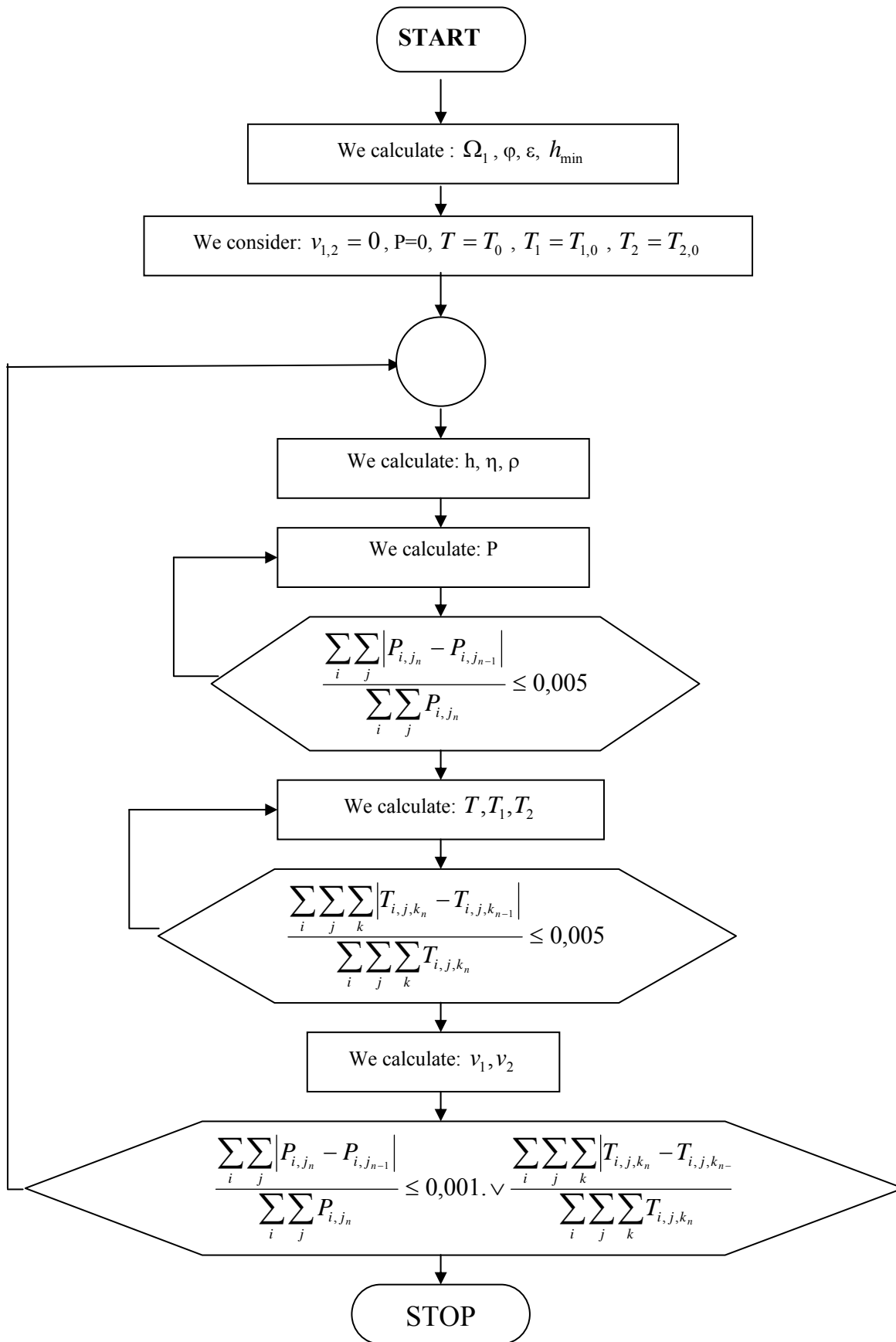


Fig. 2 The general organisation graphic for the numerical solving of the lubrication of the radial bearing [1]

## CONCLUSIONS

The method of experimenting the radial bearings subject to heavy load, presented in this paper, comprises some theoretical aspects that can be found in the functioning of these bearings.

Theoretical research dwell on the elaboration of non-linear integro-differential system formed of the Reynolds equation, the equation of energy, the equation of the elastic deformation of the pin and of the bushing, the equations of the variation of the density and viscosity of the lubricant film with the pressure and the temperature.

It is required to verify in real experimental conditions the behaviour of the bearings with hydrodynamic lubrication subject to hard shocks, in order to determine the form of the lubricating film and its thickness, aswell as the value of the pressures within the lubricating film.

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