THE RELATIONS OF ACCURACY PARAMETERS OF CYLINDRICAL SURFACES

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This paper is aimed at obtaining the relations of accuracy parameters of cylindrical surfaces of machined parts. The relations are based on the fact that manufacturing factors responsible for errors of the surface form and position are chiefly common for different kinds of errors. These relations will be useful at least for:

production engineers to recognize the fact (often ignored) that manufacturing errors of the surface form and position of machined parts are essential (they make up generally 30…50 per cent of the total manufacturing errors) and don't depend on the actual surface size within the limits of tolerance. Thus it will assist to concentrate their efforts on improving production processes through reducing surface form and position errors;

production engineers and tooling designers in estimating accuracy and reliability of machining operations and tooling to be planned. By applying the relations, any accuracy parameter could be calculated if one of them is known.

In our consideration, the following assumptions (commonly accepted) will be used:

1. Manufacturing errors of surface form and position are random. That is they are caused by a considerable number of independent manufacturing factors, none of them is not prevailed. There is no need to make any additional assumptions about the manufacturing factors;
2. Form deviation of the actual surface in cross section of a cylinder, namely circularity deviation, in respect to the radius (i. e. expressed in terms of radius) is normally distributed. Probability of occurrence, or density, y(r) of form deviation is described by the equation

$$y(r) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{r^2}{2\sigma^2}}, \quad (1)$$

where r is a random value of the form deviation,

σ is a standard deviation,

the practical range of a deviation scatter that includes 99.73 per cent of all possible deviations, or an error of the surface form ε_{fR} expressed in terms of radius, is

$$\varepsilon_{fR} = 6\sigma ; \quad (2)$$

3. Form deviation of a cylinder in terms of radius may be presented by the equation (1)

$$\varepsilon_{fR} = k_R \cdot \varepsilon , \quad (3)$$

where ε is the total manufacturing error of the cylinder diameter,

k_R could be called a coefficient of form accuracy in terms of radius. National standard GOST 24643-81 assumes three levels of relative geometrical accuracy: A, B, and C. For these levels k_R equals 0.30, 0.20 and 0.12 respectively.

Form accuracy in terms of radius and diameter. There are practical advantages of presenting form deviation in terms of diameter (ε_f), not radius (ε_{fR}). Thus let us consider a random cross section of a cylinder and find out a relation between ε_f and ε_{fR}.

It is clear that the circularity deviation in respect to the diameter includes two random and independent form deviations in terms of radius. Since distribution of a sum of two magnitudes with a Gaussian distribution is a Gaussian one then the practical range of scatter for the deviation in terms of diameter is

$$\varepsilon_f = K\sqrt{\varepsilon_{fR}^2 + \varepsilon_{fR}^2} = \sqrt{2}K \cdot \varepsilon_{fR} , \quad (4)$$

where K is a coefficient of a relative distribution. Using K enables to increase the reliability of calculations when actual distributions differ from the Gaussian ones. K does not exceed 1.2 for distributions close to normal [2].

If we substitute (3) and accepted value of K in (4) we obtain

$$\varepsilon_f = 1.7k_R \cdot \varepsilon , \quad (5)$$

where $1.7k_R$ may be denoted as k_1 and called a coefficient of relative form accuracy. For the levels of relative geometric accuracy A, B, and C, k_1 equals 0.51, 0.34, and 0.20

respectively. After rounding these values of k_1 to the nearest numbers of the row of prevalent numbers R 10/2, we obtain a row for k_1: 0.50, 0.30, and 0.20 (see table 1).

The relations of form accuracy in terms of radius and diameter have been received in respect to a cross section of a cylinder and its form error – a circularity deviation. A profile deviation of a longitudinal section of a cylinder can be considered as the result of a random occurrence of a circularity deviation in its random position along the cylinder axis. Consequently, the relation (5) can be foundly extended to longitudinal profile deviation of a cylinder and its cylindricity deviation as a whole. Thus, relative accuracy level of cylindricity as well as circularity and longitudinal profile of a cylinder can be characterized by k_1, but straightness – by k_R.

Note that the presented approach is sufficiently true for relatively short cylinders with L/D < 2 when systematically acting manufacturing factors (e. g. machine tool accuracy, cutting tool wear and thermal deformation, etc) are not essential. For long cylinders (L/D = 2…10), systematically acting manufacturing factors are prevailing and cause decreasing form accuracy by 1…2 degrees [2].

Table 1. The levels and coefficients of relative geometrical accuracy of cylindrical surfaces

Accuracy parameters and coefficients	Coefficients (k) for the levels of relative accuracy:		
	A	B	C
Form accuracy in terms of radius, $k_R = \frac{\varepsilon_{fR}}{\varepsilon}$	0.30	0.20	0.12
Form accuracy in terms of diameter and radial run-out, $k_1 = \frac{\varepsilon_f}{\varepsilon} = \frac{\varepsilon_\uparrow}{\varepsilon}$, $\varepsilon_f = \varepsilon_\uparrow$	0.50	0.30	0.20
Cylinder axis deviation, $k_2 = \frac{\varepsilon_\Delta}{\varepsilon}$, $k_2 = 0.5k$,	0.25	0.16	0.10
Coaxiality deviation for cylinders machined at a single location, $k_3 = \frac{\varepsilon_\Delta}{\varepsilon}$, $k_3 = k_R$	0.30	0.20	0.12

Axis deviation. Let us examine fig.1. It shows that random and independent form deviations on the opposite ends of the diameter cause the axis deviation of a cylinder with respect to its nominal position. A random axis deviation Δ in terms of diameter equals double deviation Δ_R in terms of radius and depends on form deviations r_1 and r_2

$$\Delta = 2\Delta_R = |r_1 - r_2|$$

or

$$\Delta = \begin{cases} r_1 - r_2 & if \quad r_1 > r_2 \\ r_2 - r_1 & if \quad r_2 > r_1 \\ 0 & if \quad r_1 = r_2 \end{cases} \tag{6}$$

For such a model, the probability density y(Δ) of the axis deviation Δ is equal to the product of the probability of occurrence r and (r±Δ) along the whole range of r scatter. Thus,

$$y(\Delta) = \int_{-\infty}^{+\infty} y(r)(y(r+\Delta) + y(r-\Delta))dr\,. \tag{7}$$

Using (1) we receive

$$y(\Delta) = \frac{1}{2\pi\sigma^2}\int_{-\infty}^{+\infty} e^{-\frac{r^2}{2\sigma^2}}(e^{-\frac{(r+\Delta)^2}{2\sigma^2}} + e^{-\frac{(r-\Delta)^2}{2\sigma^2}})dr\,. \tag{8}$$

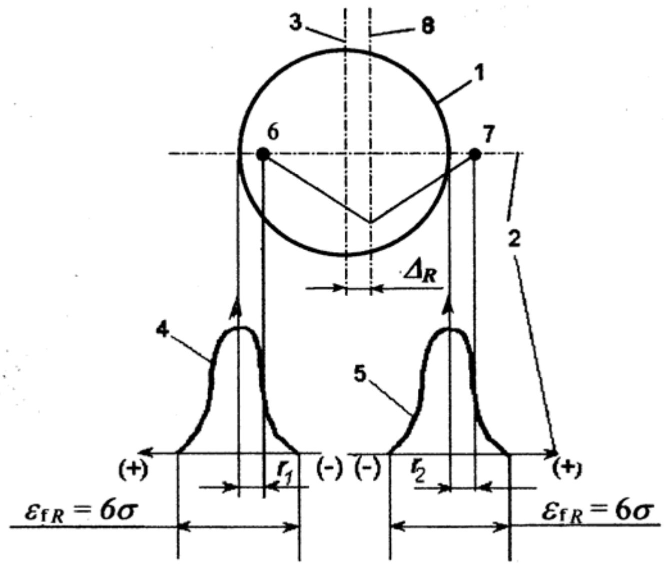


Figure 1. Illustrating the cylinder axis deviation:

After integrating (8) over r, we obtain the distribution function (or density) of Δ

$$y(\Delta) = \frac{1}{\sigma\sqrt{\pi}} e^{-\frac{\Delta^2}{4\sigma^2}}, \tag{9}$$

where axis deviation Δ≥0.

1 – a cross section of a cylinder with true form and nominal position; 2 - a random diameter direction in cross section 1; 3 - a nominal axis position of the cylinder; 4 and 5 - distribution plots of the form error ε_{fR} in terms of radius; 6 and 7 - random form deviations; 8 - an offset axis of the cylinder (symmetrically positioned to points 6 and 7).

Distribution function (9) has following characteristics:

- the average value M(Δ) of the axis deviation

$$M(\Delta)=\int_{0}^{+\infty}\Delta\cdot y(\Delta)\cdot d\Delta=\frac{1}{\sigma\sqrt{\pi}}\int_{0}^{+\infty}\Delta\cdot e^{-\frac{\Delta^2}{4\sigma^2}}d\Delta=\frac{2\sigma}{\sqrt{\pi}}=1.1284\sigma, \tag{10}$$

- the dispersion D(Δ)

$$D(\Delta)=\int_{0}^{+\infty}(\Delta-M(\Delta))^2\cdot y(\Delta)\cdot d(\Delta)=2\sigma^2\left(1-\frac{2}{\pi}\right)=0.7268\sigma^2, \tag{11}$$

- the standard deviation σ(Δ)

$$\sigma(\Delta)=\sqrt{D(\Delta)}=\sigma\sqrt{2\left(1-\frac{2}{\pi}\right)}=0.8525\sigma, \tag{12}$$

- the practical axis deviation scatter ε_Δ including 99.73 per cent of all possible deviations

$$\varepsilon_\Delta=4.256\sigma. \tag{13}$$

The diagram of the cylinder axis deviation (9) in normalized form (with $\delta=\Delta/\sigma$) is shown in fig.2.

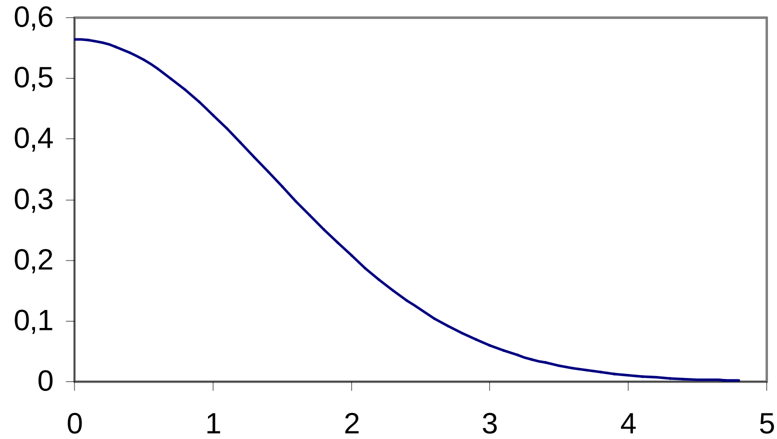


Figure 2. The distribution of the cylinder axis deviation.

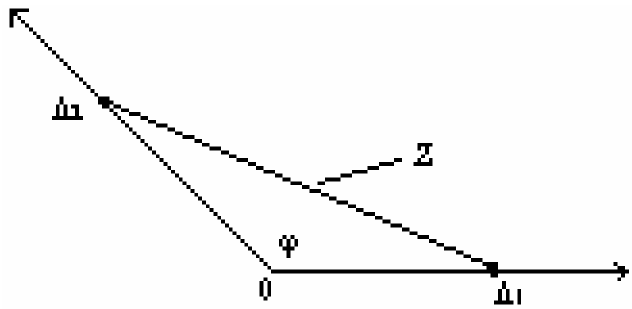
By using the coefficient K (equals 1.2) and equations (2) and (3), we convert (13) into the following form

$$\varepsilon_\Delta=1.2\frac{4.25}{6}k_R\cdot\varepsilon=0.85k_R\cdot\varepsilon, \tag{14}$$

where $0.85k_R$ may be denoted by k_2 and called the coefficient of relative cylinder axis deviation. For the levels A, B, and C of relative geometric accuracy, k_2 is equal to 0.25, 0.17, and 0.10 respectively. After rounding these values to the nearest numbers of the row of prevalent numbers R5, we get a number row for k_1: 0.25, 0.16, and 0.10.

It is necessary to note that a cylinder axis deviation can appear after infeed grinding or turning of a workpiece. After traverse cutting, a cylinder axis deviation takes place in any cross section along the cylinder axis. But these deviations are random in their angular position. Thus, the resulting deviation should be equal to zero. Nevertheless in this case a straightness deviation of the cylinder axis appears. It is easy to show that its value can be described by k_2 as well.

Coaxiality. A coaxiality deviation Z of two cylindrical surfaces machined at a single location (without a location error) can be considered as the result of two random cylinder axis deviations Δ_1 and Δ_2 with random angle φ between them shown in fig.3.



$$Z = \sqrt{\Delta_1^2 + \Delta_2^2 - 2\Delta_1\Delta_2 \cos\varphi}$$

$$y(Z) = \int\limits_{S(\Delta_1,\Delta_2,\varphi)} y(\Delta_1,\Delta_2,\varphi,Z)\,dS$$

Figure 3. Illustrating the coaxiality deviation Z:
0 – true position of two cylinder axes.

This coaxiality deviation changes from zero (when the two cylinder axis deviations are equal ($\Delta_1 = \Delta_2$) and coincide in direction ($\varphi = 0$)) up to the sum of the two deviations (when their directions are opposite - $\varphi = \pi$). Using numerical methods, we found out that the distribution function of the coaxiality deviation can be described by the equation of the following kind

$$y(Z) = a \cdot Z^b \cdot e^{-cZ^d}\,. \tag{15}$$

The plot of the distribution function y(Z) in the normalized form (when $\sigma = 1$) is displayed in fig.4. In this case equation (15) gets the following form

$$y(Z) = 0.592 \cdot Z^{0.7085} \cdot e^{-0.374Z^{1.623}}\,. \tag{16}$$

The main features of the distribution are:

- the average value of the coaxiality deviation M(Z) = 1.80;
- the standard deviation $\sigma(Z) = 2.18$;
- the practical coaxiality deviation scatter $\varepsilon_\Theta = 5.65$.

Taking into account the coefficient K (K = 1.2) and the equations (2) and (3), we conclude that the practical coaxiality deviation scatter can be presented in the form

$$\varepsilon_{\Theta} = 1.2\frac{5.65}{6}k_R \cdot \varepsilon = 1.13k_R \cdot \varepsilon, \tag{17}$$

where $1.13k_R$ may be denoted by k_3 and called the coefficient of relative coaxiality deviation. For the levels A, B, and C of relative geometric accuracy, k_3 is equal to 0.34, 0.22, and 0.14 which are close to the numbers of the row of prevalent numbers R10/2(…0.20…), namely, 0.30, 0.20, and 0.12.

It is worth to remind of the restriction previously considered. For the same reason relation (17) can be applied only for the surfaces obtained by using infeed cutting.

When coaxial surfaces are machined at two locations then the additional error (the location error) should be taken into account.

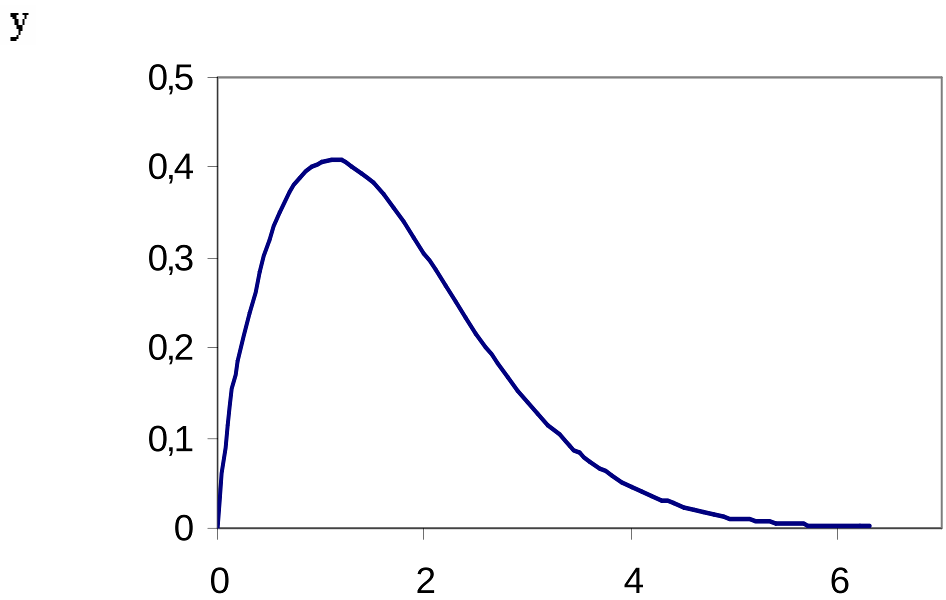


Figure 4. The distribution of the coaxiality deviation.

Radial run-out. It is quite clear that radial run-out $\varepsilon_{\uparrow}$ of a cylindrical surface can be considered as the sum of two errors: its form deviation ε_{fR} in terms of radius and its coaxiality deviation ε_{Θ} with respect to the datum

$$\varepsilon_{\uparrow} = \sqrt{(K \cdot \varepsilon_{fR})^2 + \varepsilon_{\Theta}^2} . \tag{18}$$

The coefficient K of relative distribution is used only in the first expression of (18) because the second one (ε_{Θ}) has already been received by applying K.

Taking into account the equations (2), (3), (17), and (18) and that K = 1.2, we find out that

$$\varepsilon_{\uparrow} = \sqrt{(1.2k_R \cdot \varepsilon)^2 + (1.13k_R \cdot \varepsilon)^2} = 1.65k_R \cdot \varepsilon\,, \tag{19}$$

where $1.65k_R$ may be called the coefficient of the relative radial run-out accuracy. For the levels A, B, and C of relative geometric accuracy, it makes up 0.50, 0.33, and 0.20 respectively and coincide closely with the values of the coefficient k_1.

In conclusion it is worth to note that the relations have been presented in this paper are in a good agreement with a great deal of experimental data [1]. Besides, they are very simple. Therefore these relations are suitable for practical application.

References

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Abstract

The paper presents the relations of accuracy parameters for cylindrical surface of machined parts, namely the deviations of circularity, axis, and coaxiality and also radial run-out. The relations are based on a statistical approach and unity of main manufacturing factors causing the form and position errors of cylindrical surfaces.