GENERATING KINEMATICS AND THE NUMERICAL STUDY OF THE TOOTHING WITH FLANK HYPOCYCLOID LINE AT THE RACK BAR

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Abstract This paper presents some fundamental aspects concerning the generating kinematics and the elements of the milling head used withine hypocycloid toothing process at the rack bar. There are also defined analytically and numerically examined profile, line and the crowning of the flanks generated. Diferent values of the constructive parameters defining the tool used and the generating toothing are taken into account. Some programs of calculation and of representation are used for the characteristic lines of the generated flanks.

Keywords: curved toothing, base, rolling, generating kinematics, tool, cutting edges, hypocycloid line flank, radius of curved shape, crowning.

1. INTRODUCTION

The rack-and pinion gear or cylindrical ones with curved and crowned flanks have appeared and have been extended from the neccessity to increase the fatigue strength at the teeth bending aspects proved by the practice too.

The researches concerning hypocycloid curved toothing gear complied with this requirement. The analysis has comprised different aspects, from the theoretical bases of the generating process [2], up to defining the parameters which determine the shape of the generated flanks; it is adapted the milling process or tppthing machines from FD-Cugir class [3], to some experimental researches regarding the industrial aplication of the tooth construction and of the hypocycloid toothing gear. The base characteristic defining the hypocycloid toothing consists of its curved convex shape, respectively a concave shape of the tooth flanks Σ_1 and Σ_2 (fig.1).

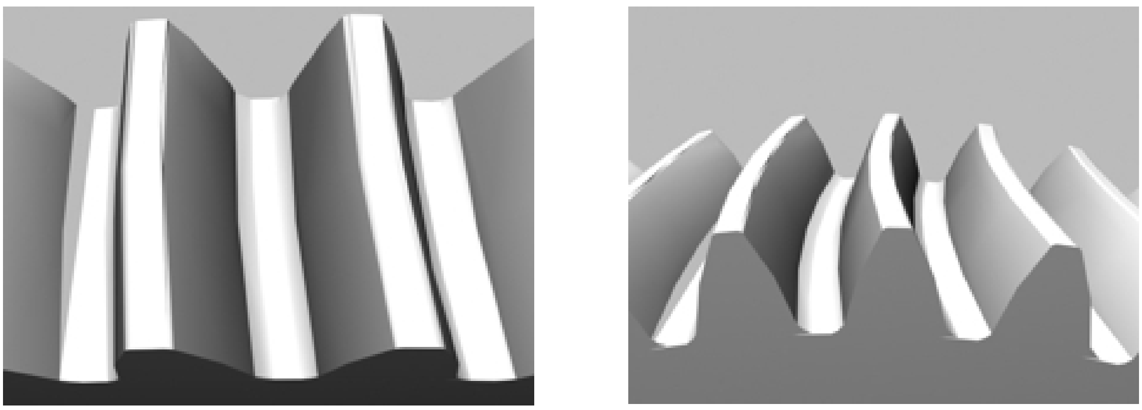

Fig.1, a *Rack bar teeth shape* **b** *toothed wheel*

The flanks with D_1 and D_2 lines are arcs of hypocycloid lengthened and shortened, and the profile G is a segment of straight, namely involute. Two toothed wheels with hypocyloid flanks make up an outside parallel cylindrical involute gearing.

The wheels have rack bar of commun reference with surfaces and profile according to STAS 821. The flanks are defined by curved hypocycloid lines with different radiuses curved shape in the plane of reference and in the plane parallel to them.

2. DEFINITION AND KINEMATIC GENERATING OF THE HYPOCYCLOID

One considers a curent point M which belongs to the loop h_1h_2 (fig.2) of a normal hypocycloid having a number 3,4,5 or 6 loops. Within the base B rolls a number i_H of rollings R disposed equidistanly to the angular spacing y_R=360/i_H. a point M is attached, in a definite position, to each rolling. The notations R_B and R_R represent the radiuses of the base rolling circles, respectively of rollings.

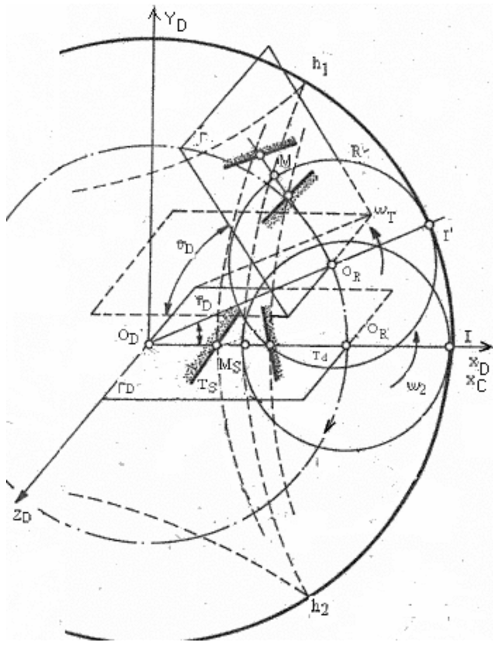

Fig.2 *Base elements and kinematics of the milling head*

In the system of reference S_D, noted ($X_DO_DY_DZ_D$), the parametric equations of the hypocycloid are defined on the basis of some simple calculations:

$$\begin{cases} x_D = (R_B - R_R)\cdot\cos\varphi_D - R_R\cdot\cos\dfrac{R_B - R_R}{R_R}\varphi_D \\ y_D = (R_B - R_R)\cdot\sin\varphi_D + R_R\cdot\sin\dfrac{R_B - R_R}{R_R}\varphi_D \ , \\ z_D = 0 \end{cases} \qquad (1)$$

in which φ_D represents circular motion parameter of the rolling axis within the base.

If the angle $\varphi_D = (\pi / i_H)\ ...\ (-\pi / i_H)$ is generated kinematically, as a path of M point, the loop h_1h_2 is disposed simmetrically to the axis O_DX_D.

During a comlete rotation (φ_D=360°) of O_R centre in relation with O_B, are generates a whole number $i_H=R_B/R_R$ of identical loops, which belongs to a closed hypocycloid.

The length of a loop is determined by the relation $l_{h_1h_2} = 8(R_B\text{-}R_R)/i_H$.

The following values of the constructive parameters are considered as: R_B=240 mm şi R_R=40, 48, 60, 80 mm.

The radius of the curved shape of the normal hypocycloid h_1h_2 is determined by the relation:

$$\rho_H = 2^{\frac{3}{2}}\frac{R_R(R_B - R_R)}{R_B - 2R_R}\left(1+\cos\frac{R_B}{R_R}\varphi_D\right)^{\frac{1}{2}}, \text{ mm} \qquad (2)$$

For the actual generating of the hypocycloid, each circle B and R is associated with a toothed wheel having z_B and z_R teeth, making up are inner gearing. A simple planetary gear come be associated with the base B and with each rolling R.

The rotation axes of the rollings (O_R) are supported by driving disc (D_a) which axis of rotation coincides with the base center (O_D).

In order to generate the hypocycloid loops i_H, the following continuous motions of rotation are made simultaneously: centers rotation O_R in relation with O_D centre, with Angular speed ω_D and rollings rotation around their own axes with a speed ω_R, between the two speeds being settled the condition: $\omega_R = \omega_D i_H$.

A tool holder support is attached to each rolling where a set of two tools is mounted. The rectiliniar cutting edges T_s and T_d of the tool edges are contained in the plane Γ_M positioned perpendiculary on the plane $X_D O_D Y_D$ where the hypocycloid is generated.

During the hypocycloid loop generating $h_1 h_2$, the plane Γ_M changes continously its position. The angle between this and the plane $X_D O_D Z_D$ is dependent on the angular parameter φ_D, in accordance with the relation $\theta_D = (i_H - 1)\varphi_D$ (3). The plane position defining Γ_M is important for the study of the surfaces generated by the two maine cutting edges and for the active and constructive geometry estabished for each tool.

Fig.3 *Multi-tool milling head*

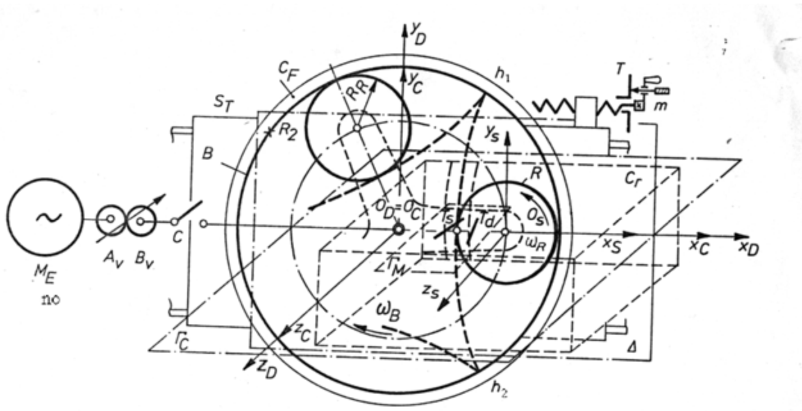

Fig.4 *Structure of the kinematic chains*

The ensemble made up of base, drive disc, rollings, tool-holder and set of tools represents multi-tool milling head [3]. This ensemble (fig.3) is mounted by means of some connecting pieces at a slide of the toothing machine making a motion of translation, of positioning or a continous one.

3. THE STUDY OF THE HYPOCYCLOID FLANKS ON THE RACK BAR

The cutting edges T_s and T_d make a main cutting movement (fig.4). The rollings R drive being a part of the milling head C_F in the planetary motion and, therefore, that motion if the tools on the hypocycloid $h_1 h_2$ paths is carried out by the maine kinematic chain adjusted with wheels of exchange A_V and B_V. A kinematic feed chain ensures the piece movement C_r in the direction of the tooth addendum generated (axis $O_C Z_C$) but a kinematic chain through a discontinous dividing ensures the tool holder slide S_T motion in the axis direction $O_C X_C$ with the distance $p=m\pi$, or that of the piece holder slide (non-represented). To generate a symmetrical toothing on the worckpiece width, it is necessary that the mean plane Γ_C of the workpiece to be positioned so that it coincides with the plane Γ_M, positive in wich it reches, only if the parameter φ_D=0.

The parametric equations of the generated flanks are determined by the transfer of the cutting edges T_S and T_D equations from the coordinate system S_S in the system S_D [4], on which the system S_C is considered overlapped.

Thus, it is written:

$$[\mathbf{X}_C]=[\mathbf{X}_S][\mathbf{M}_{CS}] \tag{4}$$

where the matrix [$\mathbf{M}_{CS}$] carries out the transformation between S_S and S_C systems and has the form:

$$[M_{CS}] = \begin{bmatrix} \cos\frac{R_B - R_R}{R_R}\varphi_D & \sin\frac{R_B - R_R}{R_R}\varphi_D & 0 & (R_B - R_R)\cos\varphi_D \\ -\sin\frac{R_B - R_R}{R_R}\varphi_D & \cos\frac{R_B - R_R}{R_R}\varphi_D & 0 & (R_B - R_R)\sin\varphi_D \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \qquad (6)$$

The cutting edges in the S_C system are defined by the equations:

$$\mathbf{X}_S: \begin{cases} x_S = -\left(R_R + k\frac{m\pi}{4}\right) + ku\sin\alpha_0 \\ y_S = 0 \\ z_S = -u\cos\alpha_0 \end{cases}, \qquad (5)$$

the calculation are made in the relation (4) and it rezults:

$$\mathbf{X}_C: \begin{cases} x_C = (R_B - R_R)\cos\varphi_D + \left[k\cdot u\cdot\sin\alpha_0 - \left(R_R + k\frac{m\pi}{4}\right)\right]\cos\frac{R_B - R_R}{R_R}\varphi_D \\ y_C = (R_B - R_R)\sin\varphi_D - \left[k\cdot u\cdot\sin\alpha_0 - \left(R_R + k\frac{m\pi}{4}\right)\right]\sin\frac{R_B - R_R}{R_R}\varphi_D \\ z_C = -u\cos\alpha_0 \end{cases} \qquad (7)$$

where the parameter $\varphi_D \in(-15°\ldots15°)$ and $u \in(-1{,}25\ m/\cos\alpha_0 \ldots 1{,}25\ m/\cos\alpha_0)$, m- the toothing modulus in mm, α_0- the pressure angle of reference in degrees.

The parameters u and φ_D, vary independently each other and have values determined by the toothing modululs, respectively by the rack bar width b and by the constructive characteristics R_B and R_R of the milling head.

The line and the profile of the generated flanks are determined by the intersection of the defined surfaces with the relations (7), with profile front planes $X_CO_CY_C$ ($z_C=h$), respectively with front planes $X_CO_CZ_C$ ($y_C=-b/2 \ldots b/2$) on the work piece width.

Thus, the *flanks line* is defined by the following parametric equations:

$$\begin{cases} x_C = (R_B - R_R)\cos\varphi_D + r\cos\frac{R_B - R_R}{R_R}\varphi_D \\ y_C = (R_B - R_R)\sin\varphi_D - r\sin\frac{R_B - R_R}{R_R}\varphi_D \\ z_C = h \end{cases}, \qquad (8)$$

which are obtained by the removal of the parameter u between the equations (7). In the relations (8) the factor r is determined by the relation:

$$r = \left[-kz_C\,\mathrm{tg}\alpha_0 - \left(R_R + k\frac{m\pi}{4}\right)\right]. \qquad (9)$$

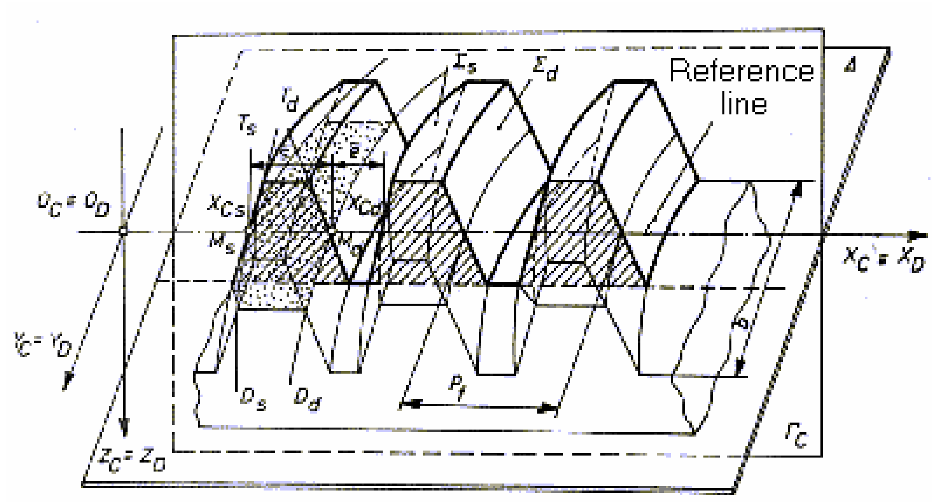

Fig.5 *Flanks with hypocycloid line of the rack*

The dimension z_C, coresponding to the teeth addendune (fig.5), it is established the following range of variation (-1,25…1,25)*m*. For *k*=1, points of the hypocycloid linegthened D_s are established on the coordinate system S_C, and *k*=-1 for the shortened hypocycloid D_d.

There it is written the second relation of (7), under the form:

$$\left(R_B - R_R\right)\sin\varphi_D - \left[ku\sin\alpha_0 - \left(R_R + k\frac{m\pi}{4}\right)\right]\sin\frac{R_B - R_R}{R_R}\varphi_D - y_C = 0\,, \qquad (10)$$

which for the parameter *u* different values are considered in the mentioned range.

The relation (10) is used for the values calculation corresponding to the parameter φ_D. Pairs of values *u* and φ_D established in this way, are introduced in the relatins (7) and the coordinates x_C and z_C of the profile in the plane $X_CO_CZ_C$ (y_C=const.) are determined correspondingly to the rack width.

The solution of the trigonometrical equation (10) presents some difficulties, a reasone for which the following method of calculation has been established:

- One considers this equation under the general form:

$$C_1\sin^n\varphi_D + C_2\sin^{n-1}\varphi_D + ... + C_n\sin\varphi_D + C_{n+1} = 0\,; \qquad (11)$$

- The coefficients $C_1,\ldots,C_{n+1}$, have different values determined by the ratio$(R_B-R_R)/R_R$ notated RAP, and the degree of the equation. The study is made for four values of the ratio RAP=2;3;4 and 5, considering R_B and R_R as the values above mentioned. For each value of the RAP ratio, the equation has different forms, its degree (NR) resulting 4,3,8 respectively 5; If one considers the pair of values RAP=4 and NR=8, on the basis of some simple trigonometrical transformations the following are established: C_1=1, C_2=0, C_3=-2, C_4=0, C_5=5/4, C_6=0, $C_7=(C^2-R_R^2)/4C^2$,$C_8=R_Rb^*/8C^2$ and $C_9=b^{*2}/64C^2$. The factor b^* represents the value considere for $y_C\in[-b/2...b/2]$ and the factor $C=ku\sin\alpha_0-(R_R+km\pi/4)$.
- The ecuation (11) is solved easily on computer, using Math CAD program.

The *profile of the flanks* are obtained by intersection the flanks generated Σ_s and Σ_d (fig.5) defined by the parametric equation (7) with the plane $X_CO_CZ_C$ ($y_C=b^*$), considering for b^* a value coresponding to the rack width *b*.

Thus, from the second equation, a relation of connecting is established between the parameters *u* and φ_D, under the form :

$$u = -\frac{1}{k\sin\alpha_0}\left[\frac{b-(R_B-R_R)\sin\varphi_{Ds,d}}{\sin\frac{R_B-R_R}{R_R}\varphi_{Ds,d}} - \left(R_R + k\frac{m\pi}{4}\right)\right]. \qquad (12)$$

Replacing the relation (12) in the other two equations of (7), it results:

$$\begin{cases} x_C = (R_B-R_R)\cos\varphi_{Ds,d} - [b-(R_B-R_R)\sin\varphi_{Ds,d}]\mathrm{ctg}\frac{R_B-R_R}{R_R}\varphi_{Ds,d} \\ y_C = \pm b/2 \\ z_C = \frac{1}{k}\mathrm{ctg}\alpha_0[\frac{b-(R_B-R_R)\sin\varphi_{Ds,d}}{\sin\frac{R_B-R_R}{R_R}\varphi_{Ds,d}} - \left(R_R + k\frac{m\pi}{4}\right)] \end{cases}, \qquad (13)$$

which represent the parametric equations of the flanks profile. The parameter $\varphi_{Ds,d}$ is determined depending on the rack width b and are the coefficient k.

4. TOOTHED RACK THIKNESS CALCULATION

Points of the flanks profile generated in a some plane $X_CO_CZ_C$ ($y_C=b^*$) are obtained intersecting the respective profile with the plane $X_CO_CY_C$ ($z_C=h$).

The analytical study of the tooth thickness and of the toothing gap is difficult, because the plane Γ_M (fig.4) containing the tool cutting edges is inclined in relation with the plane $X_CO_CZ_C$ ($y_C=b^*$), with the angle θ_D (relation 3). A numerical method of calculation has been elaborated for the points coordinates M_d and M_s (fig.5), disposed on the flanks (Σ_d) on the right and (Σ_s) on the left, in the same plane $X_CO_CZ_C$ ($y_C=b^*$) .

One consideres that the point M_d belongs to the edge T_D, for the position corseponding to a value of the angle φ_{Dd}, but $M_S\in T_S$, for an angle φ_{Ds}, which is different of φ_{Dd}.

For a certain value of the dimension z_C, on the tooth high, in the limits established by the equation of this form the relations (7) written under the form (10), one detemines a pair of actual values for the parameter φ_D, either this φ_{Dd} or φ_{Ds}.

The form and the degree of the equation (10) are established depending on the values of the factor (R_B-R_R) and the ratio (R_B-R_R)/R_R. Thus, after some transformations, the respective equation becomes under the form (11) , presented in chapter 3.

The program of calculation elaborated determines the pair of values φ_{Dd} and φ_D and the coordinates x_{Cd} and x_{Cs} (fig.5) wich belong to the cutting edges T_d respectively T_s in the points of intersection of the two planes $X_CO_CZ_C$ ($y_C=b^*$) and $X_CO_CY_C$ ($z_C=h$). The points M_d and M_s placed on the flanks on the right and on the left have the coordinates (x_{Cd}, b^*), respectively (x_{CD}, b^*) in a certain plane $X_CO_CY_C$ ($z_C=h$). The tooth thickness variation is established by calculating lengths of the segment M_dM_s.

The size $\overline{e}$ of the free space between the flanks on the tooth thickness $\overline{s}$ in the plane $X_CO_CZ_C$ and in some plane $X_CO_CZ_C$ ($y_C=\pm$ b*), they are determined by the relations $\overline{e} = x_{Cs} + p_f - x_{Cd}$, respectively $\overline{s} = x_{Cd} - x_{Cs}$.

The tooth crowning results in its thickness modification in the longitudinal direction (fig.6) beginning from the mean part (plane $X_DO_DZ_D$) .

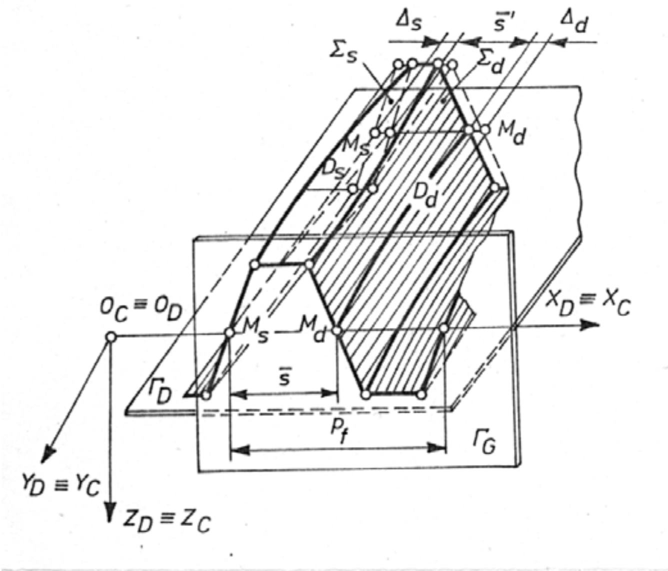

Fig.6 *The tooth crowning*

Thus, with the first relation (13) are calculated:

$$\bar{s}-\bar{s}'=\left(R_B-R_R\right)\left(\cos\varphi_{Dd}-\cos\varphi_{Ds}\right)-\frac{b}{2}\left(\operatorname{ctg}\frac{R_B-R_R}{R_R}\varphi_{Dd}-\operatorname{ctg}\frac{R_B-R_R}{R_R}\varphi_{Ds}\right)+$$
$$+\left(R_B-R_R\right)\left(\sin\varphi_{Dd}\operatorname{ctg}\frac{R_B-R_R}{R_R}\varphi_{Ds}-\sin\varphi_{Ds}\operatorname{ctg}\frac{R_B-R_R}{R_R}\varphi_{Ds}\right), \quad (14)$$

which represents the tooth thickness difference in the mean plane $\Gamma_G \equiv X_D O_D Z_D$ and in the plane corespoding to $y_C=b/2$, in the same plane $\Gamma_D \equiv (X_D O_D Y_D)$.

In the relation (14), the parameters φ_{Dd} and φ_{Ds} have different values depending on the plane Γ_D ($z_C=h$), respectively on the plane Γ_G ($y_C=b/2$), which one determines points of the flanks lines D_d, respectively D_s.

There is noted with Δ_d and Δ_s the arrows of the flanks lines in the plane $X_C O_C Y_C$ in the fig.6 (see table 1)

In the table 1 there are presented numerical values determined by the relation (14)

Table 1. Values of the arrow and of the rack tooth crowning

Milling head		**Tooth characteristics**				**Dimension**
		m	**Arrow**		**Crowning**	z_C, mm
R_B, mm	R_R, mm	mm	Δ_d, mm	Δ_s, mm	mm	
240	80	2.5	0.1776	0.1740	0.0036	-3.13
			0.1785	0.1730	0.0056	-1.88
			0.1796	0.1719	0.0076	-0.62
			0.1801	0.1714	0.0086	0
			0.1805	0.1709	0.0097	0.62
			0.1815	0.1698	0.0116	1.88
			0.1824	0.1687	0.0137	3.13

5. RADIUSES OF CURVED SHAPE OF THE RACK BAR FLANKS LINES

By intersecting the flanks generated, defined by the relation (7), with planes $X_C O_C YC$ $z_C=h$, pairs of curved lines result: hypocycloids shortened (D_{Hd}), or lengthened ones (D_{Hs}) (fig.7).Are considered the parametric equations (8) which define the flanks lines, where x_D and y_D represent the coordinates of a current point M to be belonged tro these.

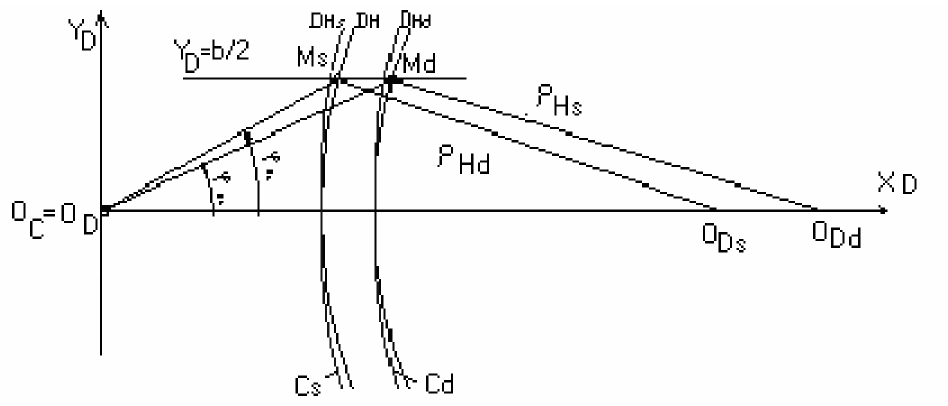

Fig.7 *Hypocycloidal lines approximation with arcs of circle*

The derivates of the respective coordinates are established in the relation with the parameter φ_D. After having made the calculations imposed by the relation (2), results the calculation formula for radiuses of curved shape of the lengthened and shortened hypocycloids as flanks lines. The points O_{Ds} and O_{Dd} are the center of the radius curves. The hypocycloid lines can be aproximate with two circle arcs C_s and C_d.

6. CONCLUSIONS

The change of the tooth flanks for the gear wheels is made in order to increase the gear performations. The involute teeth with hypocycloid flanks, made with rack bar, are crowning. The numerical study, based on parametric equations of the flanks, defines the shape, the profile lines, the crowning and the thickness of the teeth. To define the process which generate the flanks of the rack bar, was established the tool cutting edges of the milling head and the kinematic. The resutls obtained are useful in the prduction and utilization of the hypocycloid teeth wheels and gears.

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