# ON THE GEARING SIMULATION USING FINITE ELEMENTS METHODS


***Prof. Dr. Eng. George DOBRE, Dr. Eng. Radu Florin MIRICĂ,***
***Dr. Eng. Ştefan SOROHAN,***

University POLITEHNICA of Bucharest, 313 Splaiul Independentei, 77206 Bucharest,
Phone and fax +01/411 46 93, mobile 095/490 664, geo@meca.omtr.pub.ro



**Abstract.** The paper contains an analysis of the using finite element methods of the spur gears with the consideration of most profile forms (tip profile correction) and profile deviations. The analysis is developed for spur gears because of available resources. The deviation considered is the one of profile form, with a multiple variation along the tooth profile. The paper offers and discusses results of tooth and contact stress that are carried out by finite elements analysis.


## 1. Introduction

The strain and stress analysis of the gear tooth using finite elements methods (FEM) has been the subject of various researches by well-established groups from around the world. It is a difficult undertaking on account of two main aspects: a) the geometrical definition of the gearing; and b) the required support for finite elements analysis (FEA).

The geometrical definition of gearing is generally considered in the classical way. This classical geometrical definition of gear toothing means: a) the use of the reference rack-cutter and the relative rolling motions between this rack (tool) and the half-finished wheel (described analytically in great detail by Litvin - see for example [2]); b) generally not including gearing deviations. The main result of this modelling is the existence of the straight line of contact between conjugated flanks at involute spur and helical gears. The idealised contact line is considered in the first papers using FEA for gears (for example Bong [1]). The distribution of the load was established on this idealised contact line. But this must be regarded as an approximation, because the line of contact turns into a path (contact surface). Thus the correct analysis of the contact surface between the conjugated flanks needs to look at the contact modelling with a good degree of precision in the FEM analysis, using corresponding hardware and software support.

It is clear therefore that the consideration of the gearing deviations and the precise contact FEM analysis complicate the gear modelling for FEM analysis and results in a need for greater resources. Indeed the three-dimensional modelling of the gear toothing leads generally to a very great volume of calculations and needs considerable hardware resources. For example the contact modelling has to be carried out on elements of small dimensions, so resulting models with about $10^5 \ldots 10^6$ degrees of freedom which will be analysed with non-linear calculus procedures.

Elaborating on these aspects, the main objectives of the present paper are the stress and strain state analysis of the gear toothing by FEM, by the consideration:

a) of the most general tooth profile form (tip profile correction, tooth root with undercut because of the protuberance of the tool);
b) of the profile form deviation.

The resources available permitted the analysis of spur gears in 2D, in the conditions the model dimension is about $10^3$ - $10^4$ degrees of freedom at the o judicious 2D modelling. Also, it is mentioned that the analysis is carried out for two gear models: with and without profile form deviations.

## 2. Simulation of the gearing without deviations

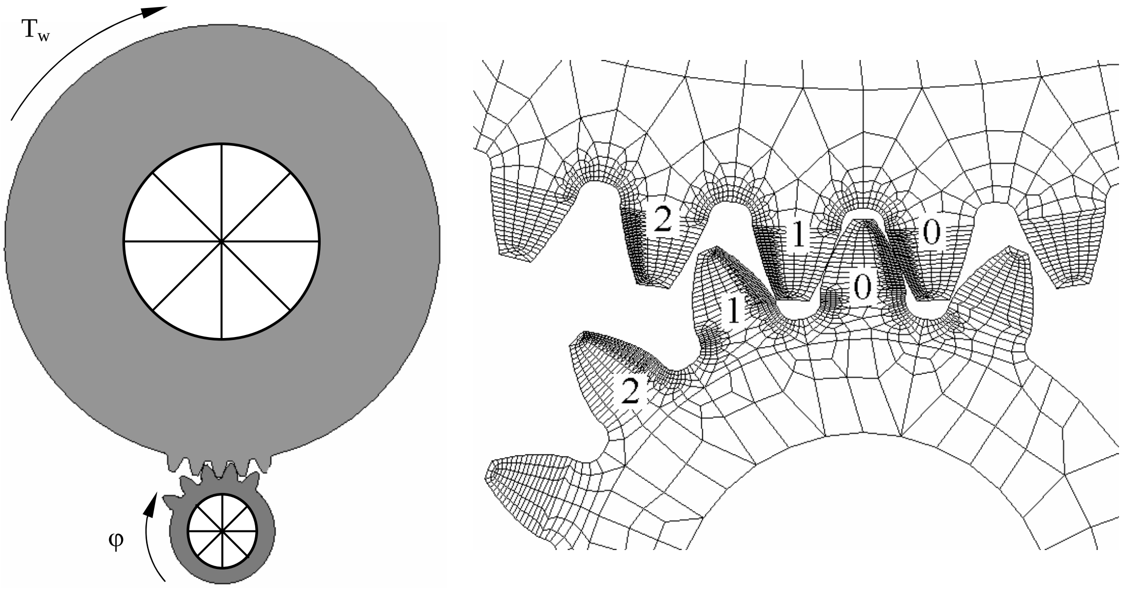
### 2.1. Description of the FEA model and method

**The 2D-gear model** of the simplified macro-geometry is suggested. It reduces the complexity of calculation without affecting its precision (fig. 1, a):

a) the wheels contain 5 teeth (both pinion and wheel);
b) the rest of the wheels is without toothing.

**The FEA method** is developed looking at the gearing simulation from the point of view of gear stress distribution. The following criteria were considered:

a) the mesh is very refined in the contact zones of the flanks entering in contact and less in the rest of the wheel body (fig. 1, b);
b) finite elements are of quadrilateral type, with 4 or 8 nodes;
c) the contact elements of line type are introduced between the conjugated operating flanks;
d) the boundary conditions are given by the task of FEM analysis. For this the centers of the two wheels are fixed with permeating rotation of the wheels around their axis;
e) depending of these boundary conditions, the FEA model is completed with very rigid plane beams elements fasting the wheel center (fig. 1, a). In this way the rotations of the wheel centers are monitored by the rotation freedom degree of the nodes from these centers.




*a)* *b)*

***Fig. 1.*** *The macro-geometrical gear model (a)*
*and a detail of the FEA mesh gear model without deviations in the case of starting position (b)*

This **FEA model is used** thus:

a) the pinion is turned with the angle $\varphi$ increasing progressively;

b) the wheel is loaded with a constant resistant torque $T_w$;

c) the starting analysis position corresponds to the vertical axis of the pinion tooth which is given the number 0.

### 2.2. Application

The gear data are listed in the table 1. Geometrical data of the counterpart rack are represented on the fig. 2. For this gear data, the FEA calculus model contains 3815 nodes and 3999 finite elements. The finite elements dimension in the probable contact flank zones is about 0.15 mm. For stress state simulation, the angle $\varphi$ of rotation has an increment of 2 degrees. The total iterations number was of 93 for 21 angular steps of calculation for $\varphi$. The obtained penetration was under 7 $\mu m$, which is acceptable. Also, an increase of the result precision imposes a very fine gear mesh, which was not possible with the available hardware resources used to carry out this research.

**Table 1.** Geometrical data of the gear analysed by FEM

| Symbol | Description | Value, unit |
|---|---|---|
| $a_w$ | Working center distance | 400 mm |
| $d_1$ | Reference diameter of pinion | 160.880 mm |
| $d_2$ | Reference diameter of wheel | 623.415 mm |
| $h^*_{aP0}$ | Relative addendum of tooth of the generating rack tool | 1.4 |
| $h^*_{fP0}$ | Relative dedendum of tooth of the generating rack tool | 0.945 |
| $h^*_{KfP0}$ | Relative high of root profile correction of the generating rack tool | 0.3 |
| $m$ | Module | 10 mm |
| $p$ | Pitch | 3.141 mm |
| $X_1$ | Profile shift coefficient of pinion | 0.50 |
| $X_2$ | Profile shift coefficient of wheel | 0.34 |
| $Z_1$ | Number of teeth of pinion | 16 |
| $Z_2$ | Number of teeth of wheel | 62 |
| $pr^*_{P0}$ | Relative protuberance of the generating rack tool | 0.026 |
| $\alpha_{prP0}$ | Protuberance pressure angle of the generating rack tool | 8 degree |
| $\alpha_{KP0}$ | Root profile correction pressure angle of the generating rack tool | 21.5 degree |
| $\alpha_{P0}$ | Pressure angle of the generating rack tool | 20 degree |
| $\rho_{aP0}$ | Tip radius of the generating rack tool | 0.4 |

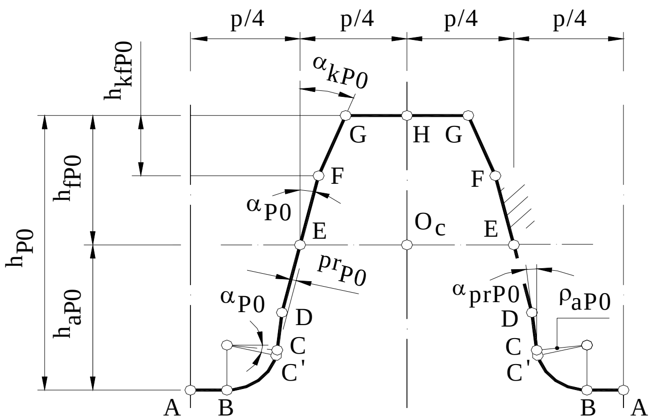



***Fig. 2.*** *Geometrical elements of the counterpart rack profiles*

The variations of maximum tensile and compressive stress $\sigma_{F1}$ and, respectively, $\sigma_{F3}$ on the root of the tooth are shown in fig. 3. The stress state is valid for the tooth pair 1 marked on fig. 1, b. The representation of stress state from fig. 2 may be interpreted thus:

a) the compressive stress $\sigma_{F3}$ as absolute value is greater than the tensile stress $\sigma_{F1}$;
b) there are clearly zones of single and double gearing seen by the size of the stress values over the zone of action (path of contact);
c) the tip profile corrections at each wheel determine a quasi-linear variation of the stress along the double gearing zones with the reducing of the gearing shocks;

d) the diagrams of each type of stress ($\sigma_{F1}$ or $\sigma_{F3}$) for both the wheels are approximately symmetrical in comparison with the middle of the zone of action; so the variation mode is similar for each of the conjugated wheels of the gear.

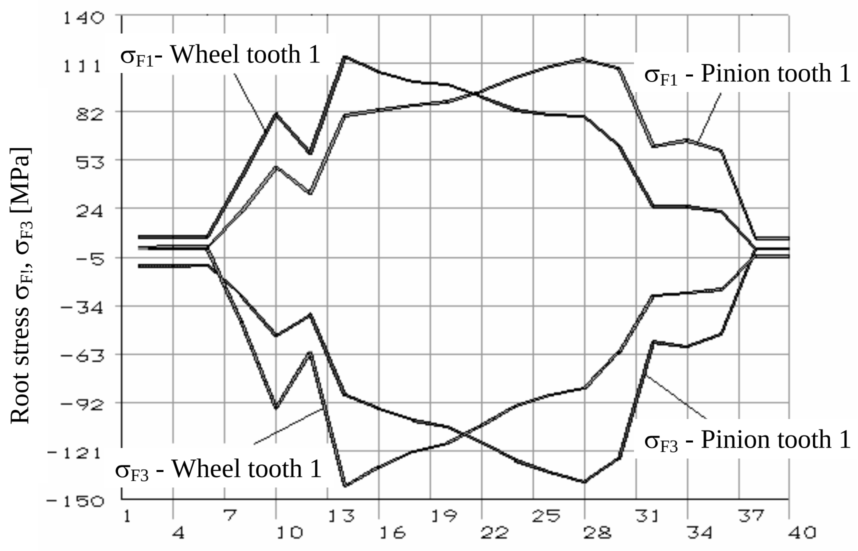



***Fig. 3.*** *The stress state (S1, S3) along the zone of contact for the tooth pair 1 (marked on the fig. 1, b)*

## 3. Simulation of the gearing with deviations

### 3.1. Description of the FEA model and method

The same **2D-gear model** containing 5 teeth on each wheel was used for the FEA in the case of existing tooth profile form deviations. The consideration of this deviation is justified by the intention to see its effect on the strain and stress state. The profile deviation was described along the profile by a sinusoidal law (an image of the geometry form of the teeth with profile deviation is made in the fig. 4). The deviation amplitude is chosen taking into account the profile deviation tolerance values for the ISO precision class 6, which are specific to a fine final finishing of the toothing; it is 13 $\mu$m for the pinion and 16 $\mu$m for the wheel.

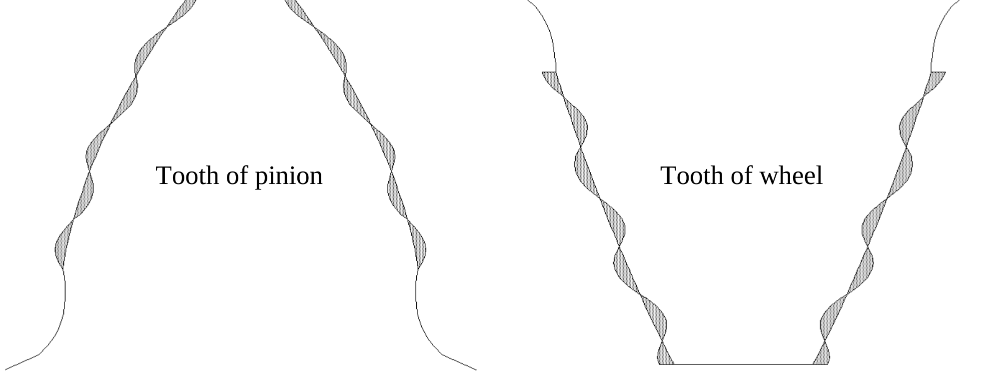



***Fig. 4.*** *The model of the profile deviation of pinion (a) and wheel tooth (b) – the representation is magnified 50 times*

The **FEA method** is realised taking into account the main task of the FEM analysis: the influence of the gear deviations over the stress contact stress state and the relative rotation of the pinion. The following observations can be made:

a) the mesh in the contact zones is much finer than in the case of the previous root stress analysis. The reason for this is the larger number and smaller dimensions of contact elements;
b) the boundary conditions include the fixation of the wheel on the boring and the loading of the pinion with a given torque (fig. 5);
c) the penetration has to be controlled in a major mode by using of a solving algorithm based on the penalty functions augmented with Lagrange multipliers.

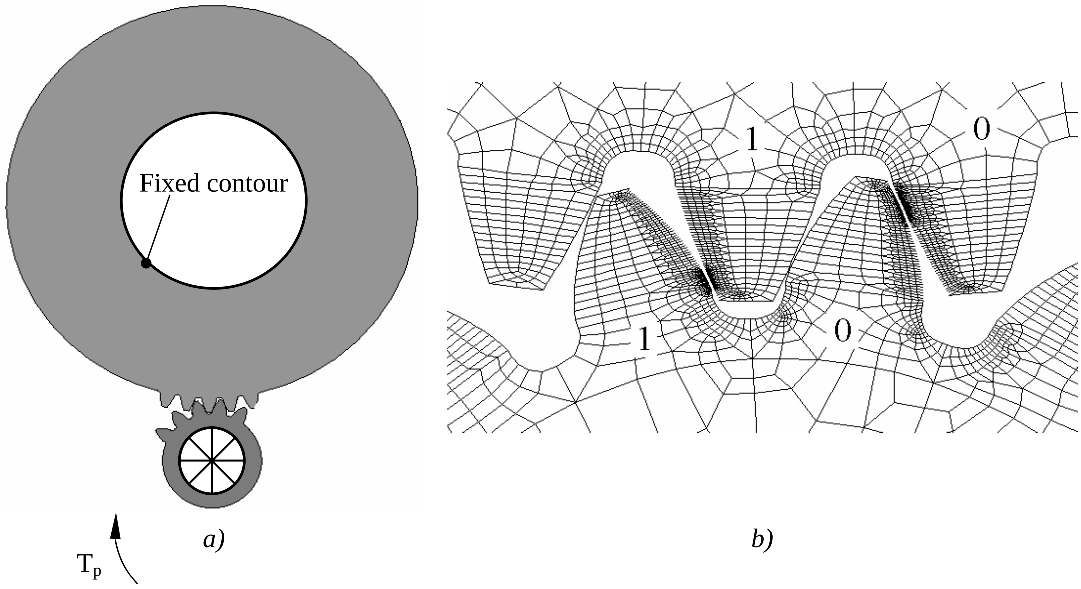



***Fig. 5.*** *The macro-geometrical gear model (a)*
*and a detail of the FEA mesh gear model with profile deviations in a current position (b)*

**The FEA model** is used in the way:

a) the loading is carried out in two steps: firstly the conjugated teeth were put in contact in a desired gearing position by a turning of the pinion and after then the pinion torque was applied;
b) the analysis was carried out for two main gearing positions of the tooth pair 1: close the particular point A (theoretical entering in double gearing, defined by the pinion rotation angle $\varphi$ =8 degrees on the zone of action) and close the particular point C (pitch point, at $\varphi$ =16 degrees on the zone of action);
c) the stress analysis was developed for both types of gearing – with and without profile deviations.

### 3.2. Application

The gear data are the same used for the first application (table 1). The pinion torque is $T_p$ =4011 Nm.

For this gear data, the FEA calculus model is characterized by the following values:

a) the dimension of the elements was under 0.1 mm in the contact zones, in order to have a very fine mesh;

b) the penetration value obtained in this way was under 0.5 $\mu$m ;

c) the total number of iterations for convergence was 14 for the loading (in the case of this better penetration).

A part of results obtained by FEA are given in table 2 (root and contact stress, contact path diameter) and fig. 6.

The interpretation of the results given in table 2 is as follows:

*a) regarding tooth stress state:*

- the effect of multiple gearing is seen either for the case of the wheels without or with profile deviations: the tension values are smaller in the case of the double gearing;
- accidentally the profile deviations produced greater values of the tooth stress than in the case of the absence of these deviations;

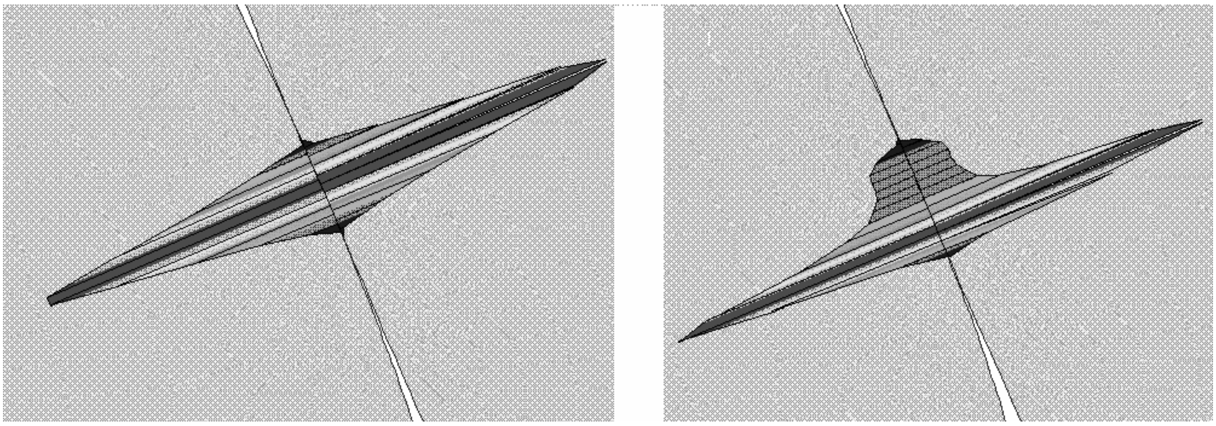
*b) regarding contact stress state:*

- the double gearing leads to the essential modification (decreasing) of the contact stress $\sigma_H$ in comparison with the single gearing;
- the profile deviations lead to greater values of the contact stress in comparison with the gearing without profile deviations.

**Table 2.** Values of the main sizes determined by FEA

<table>
<tr><th rowspan="3">Size, observations</th><th rowspan="3">Symbol, unit</th><th colspan="4">Tooth pair 1 close to point:</th></tr>
<tr><th colspan="2">A (at $\varphi = 8$ degrees as position), double gearing</th><th colspan="2">C (at $\varphi = 16$ degrees as position), single gearing</th></tr>
<tr><th>Without profile deviations</th><th>With profile deviations</th><th>Without profile deviations</th><th>With profile deviations</th></tr>
<tr><td rowspan="2">Root stress, pinion tooth 1</td><td>$\sigma_{F1}$ [MPa]</td><td>29.4</td><td>37.6</td><td>87.2</td><td>90.2</td></tr>
<tr><td>$\sigma_{F3}$ [MPa]</td><td>-35</td><td>-44.8</td><td>-92.7</td><td>-98.4</td></tr>
<tr><td rowspan="2">Root stress, wheel tooth 1</td><td>$\sigma_{F1}$ [MPa]</td><td>54.8</td><td>65.5</td><td>105.8</td><td>113.5</td></tr>
<tr><td>$\sigma_{F3}$ [MPa]</td><td>-58</td><td>-73.6</td><td>-132</td><td>-139</td></tr>
<tr><td>Contact stress, tooth pair 1</td><td>$\sigma_H$ [MPa]</td><td>406</td><td>550</td><td>1001</td><td>1069</td></tr>
</table>

The distribution of the contact stress represented in the fig. 6 (for the single gearing) shows that profile deviations modify considerably the classical elliptical form of the Hertz pressure.



a) b)

***Fig.6.*** *Contact stress distribution in the single gearing, without (a) an with profile deviations (b)*

## 4. Conclusions

- The FEA of spur gears is possible in 2D in the case of limited resources (PC of moderate specification widely available on the market nowadays). The model dimension is about $10^3$ - $10^4$ degrees of freedom for the 2D modelling.
- The tip profile corrections lead to a quasi-linear variation of the tooth stress on the zones of double gearing, that is the gearing shocks are reduced.
- FEA for two gear models (without and with profile form deviations) shows that the results depend on the penetration (the penetration value obtained was very small - under 0.5 $\mu$m - for one of the gear models analysed).
- An important modification rule (increasing) appears for the root and contact stress in the presence of the profile deviations in the double gearing.

**References**

[1] BONG , H. B. Erwiterte Verfahren zur Berechnung von Stirnradgetrieben auf der Basis numerischer Simulationen und der Methode Finiter Elemente. Diss. TH Aachen, 1990.
[2] LITVIN, F. Gear Geometry and Applied Theory, Prentice-Hall, Inc., ISBN 0-13-211095-4, Englewood Cliffs 1994.